Simple method of computing a function in fixed-point arithmetic

S.S. Deo and K.V.S. Hari

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The authors present a simple method, based on the principle of successive approximation, of computing a function using a fixed-point arithmetic processor. As an example, computing the square root function is considered and the performance is compared with Newton's method, in terms of accuracy and the number of instruction cycles required.

Introduction: The method of successive approximation (SA) is most commonly used in analogue to digital converters (ADCs). A simple method, based on this principle, for the computation of functions in fixed-point arithmetic is proposed. The effectiveness of this method in terms of accuracy, number of cycles etc.,

depends on how effectively the *inverse function* is computed. This Letter presents the algorithm and presents the performance of the algorithm for square root computation on a fixed-point DSP processor.

Successive approximation (SA) method for computing a function: Consider a B bit fixed-point arithmetic processor and assume that the values are represented in fractional format as

$$X = b_s.b_0b_1\dots b_{B-2} \quad (1)$$

with b_s denoting the sign bit. Consider the function

$$y = f(x)$$

which needs to be evaluated and let $f^{-1}(\cdot)$ denote the *inverse function*.

Algorithm:

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 $y_0 \leftarrow "b_s = 0, b_0 = 1, b_i = 0, i \neq 0";$  initial guess (usually mid-point)
 $y_1 \leftarrow "b_s = 0, b_1 = 1, b_i = 0, i \neq 1";$  a constant named shift-Val-1
 $y_2 \leftarrow "b_s = 0, b_2 = 1, b_i = 0, i \neq 2";$  a constant named shift-val-2
FOR (LOOP = 1 to  $B-1$ ),
START LOOP
  IF ( $x > f^{-1}(y_0)$ )
     $y_0 \leftarrow y_0$  OR  $y_1$ ;
    shift right  $y_1$  by 1 bit position;
    shift right  $y$ , by 1 bit position;
  END IF
  IF ( $x < f^{-1}(y_0)$ )
     $y_0 \leftarrow y_0$  AND (NOT  $y_2$ ) OR  $y_1$ ;
    shift right  $y_1$  by 1 bit position;
    shift right  $y_2$  by 1 bit position;
  END IF
  IF ( $x = f^{-1}(y_0)$ )
    TERMINATE LOOP;
  END IF
END LOOP
 $f(x) \leftarrow y_0$ ;
STOP

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Remarks:

- (i) The number of iterations required for computation are equal to the number of bits used for representing y .
- (ii) The accuracy and computational complexity of implementing the *inverse function* dictates the accuracy and computational complexity of the **SA** method of function evaluation.

Examples: Square root computation: Usually, the square root of a number is evaluated using the Newton Raphson method given by the iterative formula [1]:

$$y(n) = \frac{1}{2} \left[y(n-1) + \frac{x}{y(n-1)} \right]$$

where n denotes the n th iteration. After some iterations, $y(n)$ approximates to $\sqrt{x}$.

Using the **SA** method, it is easy to see that for square root computation, the inverse function is simply squaring, or multiplication. This can effectively be carried out by a hardware multiplier in one clock cycle, which is part of a digital signal processing (DSP) processor (for e.g. [4]).

Performance comparison: A uniform random number generator was used to generate 100 random numbers between zero and one, and the square root was calculated for each of them using the two methods. The average error was chosen as the performance measure. Table 1 presents the results.

Method	Cycles per iteration	Iterations	Average error
Newton-Raphson	32	40	5×10^{-4}
Successive approx	24	15	2×10^{-5}

Sine inverse computation: The proposed method was also used for computing the arcsin function in [5] and it has been shown that it

performs better than the standard method using a series expansion [2].

Conclusion: A simple method, the **SA** method, is proposed for computing a function in fixed-point arithmetic. The method is general and its power lies in efficient implementation of the inverse function. As an example, it has been shown that using a DSP processor, computing the square root using the proposed method is more accurate and requires less computation compared to the conventional Newton-Raphson method.

References

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