The observer for this system is

$$\dot{z}(t) = (A_0 + \Phi_A(0)KC)z(t) + A_1z(t-\alpha) + A_1 \int_0^\alpha \Phi_A(\xi-\alpha)K \\ \cdot [Cz(t-\xi) - y(t-\xi)]d\xi - \Phi_A(0)Ky(t) + Bu(t).$$

When $u(t)$ is derived from $z(t)$ we have

$$u(t) = P_0z(t) + \int_{-\alpha}^0 P_1(\xi)A_1z(t+\xi)d\xi$$

so that the overall system equations are given by

$$\dot{x}(t) = (A_0 + BP_0)x(t) + A_1x(t-\alpha) + \int_{-\alpha}^0 BP_1(\xi)A_1x(t+\xi)d\xi \\ + BP_0e(t) + B \int_{-\alpha}^0 P_1(\xi)A_1e(t+\xi)d\xi$$

$$\dot{e}(t) = (A_0 + \Phi_A(0)KC)e(t) + A_1e(t-\alpha) + A_1 \int_0^\alpha \Phi_A(\xi-\alpha)Ke(t-\xi)d\xi$$

for which the eigenvalues are the roots of the equation

$$\det \left[\frac{sI - A_0 - BP_0 - A_1e^{-s\alpha} - B \int_{-\alpha}^0 P_1(\theta)A_1e^{+s\theta}d\theta}{0} \quad \begin{matrix} -BP_0 - B \int_{-\alpha}^0 P_1(\theta)A_1e^{+s\theta}d\theta \\ sI - (A_0 + \Phi_A(0)KC + A_1e^{-s\alpha} + A_1 \int_0^\alpha \Phi_A(\theta)Ke^{-s\theta}d\theta) \end{matrix} \right] = 0.$$

It is now clear that the spectral separation property holds also for time-delay systems. Gressang has proved a similar result for the observer equation developed in [3].

CONCLUSIONS

A theory of Luenberger type of observers for time-delay systems has been developed in this short paper. The stability of the observer is established and the observer equations are reduced to the form of integro-differential equations which have not previously appeared in the literature. The structural similarity of the observer as developed here to the Kalman filters for time-delay systems is indicated. The computational problems involved are also discussed.

APPENDIX

Proof of (20): The characteristics for (19) are the family of lines with slope $dt/d\xi = -1$, i.e., the set of lines satisfying $t+\xi=c$, a constant. By integrating the first equation of (19) along these characteristics we find that

$$\frac{d\tilde{z}}{d\xi}(c-\xi, \xi) = -\Phi_A(\xi)KC\tilde{z}(c-\xi, 0) + \Phi_A(\xi)Ky(c-\xi)$$

so that

$$\tilde{z}(c-\theta, \theta) = \tilde{z}(c, 0) - \int_0^\theta \Phi_A(\xi)[KC\tilde{z}(c-\xi, 0) - Ky(c-\xi)]d\xi, \\ \theta \in [-\alpha, 0],$$

i.e.,

$$\tilde{z}(t, \theta) = \tilde{z}(t+\theta, 0) - \int_0^\theta \Phi_A(\xi)[KC\tilde{z}(t+\theta-\xi, 0) - Ky(t+\theta-\xi)]d\xi.$$

Thus,

$$\tilde{z}(t, -\alpha) = \tilde{z}(t-\alpha, 0) - \int_0^{-\alpha} \Phi_A(\xi)[KC\tilde{z}(t-\alpha-\xi, 0) - Ky(t-\alpha-\xi)]d\xi \\ = \tilde{z}(t-\alpha, 0) + \int_0^\alpha \Phi_A(\xi-\alpha)[KC\tilde{z}(t-\xi, 0) - Ky(t-\xi)]d\xi.$$

Now, putting $\tilde{z}(t+\theta, 0) = z(t+\theta)$ in the second equation of (19) yields (20).

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Stochastic Differential Equations for Linear Smoothing Problems

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Abstract—Stochastic differential equations for the linear fixed point, fixed interval, and fixed lag smoothing problems are derived using the martingale representation theory.

I. INTRODUCTION

In this short paper we give simple and rigorous derivations of the stochastic differential equations satisfied by the fixed point, fixed interval, and fixed lag smoothed estimates [5] and the differential equations for the conditional covariances in linear systems using the martingale representation theory as developed for estimation problems by Fujisaki *et al.* [2]. The results are based on a lemma which is proved in Section II. For the sake of completeness the principal results (Theorems 1 and 2) of Fujisaki *et al.* [2] used in the sequel are summarized.

For simplicity only the scalar case is considered. Extensions to finite-dimensional cases are straightforward.

II. STOCHASTIC INTEGRAL REPRESENTATION FOR SMOOTHED ESTIMATES

Let $(\Omega, \mathcal{G}, P)$ be a probability space. The system process $x(t, \omega)$, $t \in [0, T]$ is assumed to take values in a complete separable metric space S . The signal process $h(t, \omega)$ and the observation process $z(t, \omega)$ are assumed to be related by

$$z(t) = \int_0^t h(u)du + w(t) \quad (1)$$

where $w(t)$ is a standard Wiener process, $h(t, \omega)$ is jointly measurable

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and adapted to $\mathcal{G}_t = \sigma\{x(s), w(s): 0 \leq s \leq t\}$ and

$$\int_0^T E|h(t)|^2 dt < \infty. \quad (2)$$

For every $t \in [0, T]$, $\mathcal{G}_t$ and $\sigma\{w(v) - w(u): t \leq u < v \leq T\}$ are independent. By (2), there exists a jointly measurable modification of $E[h(t)|\mathcal{G}_t]$ where $\mathcal{G}_t = \sigma\{z(s): 0 \leq s \leq t\}$. (Similar observation applies to other such situations to be encountered below.) The innovation process defined by

$$v(t) = z(t) - \int_0^t E[h(u)|\mathcal{G}_u] du \quad (3)$$

is an $(\mathcal{G}_t, P)$ standard Wiener process and for every $t \in [0, T]$, $\mathcal{G}_t$ and $\sigma\{v(v) - v(u): t \leq u < v \leq T\}$ are independent.

Theorem 1 [2]: Let (1)–(3) hold. Then, every separable square-integrable martingale $(Y(t), \mathcal{G}_t, P)$ is sample continuous and

$$Y(t) - E[Y(0)] = \int_0^t \psi(s) dv(s)$$

where $\int_0^T E|\psi(t)|^2 dt < \infty$ and $\psi(t)$ is jointly measurable and adapted to $\mathcal{G}_t$.

Let $f: S \rightarrow \mathbb{R}$ be measurable. f is said to belong to the space $D(\bar{A})$ if the following hold.

- 1) $E|f(x(t))|^2 < \infty$ for every $t \in [0, T]$.
- 2) There exists a jointly measurable real function $\bar{A}(f; t, \omega)$ adapted to $\sigma\{x(s), z(s): 0 \leq s \leq t\}$ such that $\int_0^T E|\bar{A}(f; t, \omega)|^2 dt < \infty$.
- 3) $M(f; t) = f(x(t)) - E[f(x(0))|\mathcal{G}_0] - \int_0^t \bar{A}(f; s) ds$ is a $(\mathcal{G}_t, P)$ martingale.

If $D(\bar{A}) \neq \emptyset$, then it is a vector space and $\bar{A}(\cdot; t): D(\bar{A}) \rightarrow L^2(\Omega)$ is a linear map for a.e. $t \in [0, T]$. Let us set $E^t f(x(t)) = E[f(x(t))|\mathcal{G}_t]$ for brevity.

Theorem 2 [2]: Let (1)–(3) hold. If $f \in D(\bar{A})$ satisfies

$$\int_0^T E|f(x(t))h(t)|^2 dt < \infty \quad (5)$$

then $E^t f(x(t))$ is given by

$$E^t f(x(t)) = E^0 f(x(0)) + \int_0^t E^s \bar{A}(f; s) ds + \int_0^t [E^s(f(x(s)) \cdot h(s)) - E^s f(x(s)) \cdot E^s h(s) + E^s \bar{D}(f; s)] dv(s) \quad (6)$$

where $\bar{D}(f; s)$ is defined by

$$\langle M(f), w \rangle_t = \int_0^t \bar{D}(f; s) ds \quad \text{a.s.}$$

Here $\langle M(f), w \rangle_t$ is the process of bounded variation associated with the martingales $M(f; t)$ and $w(t)$ and $\int_0^T E|\bar{D}(f; s)|^2 ds < \infty$. If $x(t)$ and $w(t)$ are independent, then

$$\langle M(f), w \rangle_t = 0 \quad \text{a.s.}$$

Explicit expressions for $\bar{A}(f; t)$ and $\bar{D}(f; t)$ are given for special cases in [2].

Lemma 1: Assume (1)–(3). Let $\tau, 0 \leq \tau \leq t \leq T$ be fixed and $g(\omega) \in \sigma\{x(s): 0 \leq s \leq \tau\}$ be such that

$$E|g(\omega)|^2 < \infty \quad (7)$$

and

$$\int_0^T E|g(\omega)h(t)|^2 dt < \infty. \quad (8)$$

Then

$$E^t g(\omega) = E^{\tau} g(\omega) + \int_{\tau}^t \text{cov}^u(g(\omega), h(u)) dv(u) \quad (9)$$

where $\text{cov}^u(X, Y) = E^u XY - E^u X \cdot E^u Y$.

Proof: Let $\bar{h}(s) = h(s) - E^s h(s)$; $N_{\tau} = g(\omega) - E^{\tau} g(\omega)$

$$\bar{N}_{\tau}(t) = E^t N_{\tau} = E^t g(\omega) - E^{\tau} g(\omega) \quad \text{and} \quad N_{\tau}^*(t) = \int_{\tau}^t \text{cov}^u(g(\omega), h(u)) dv(u).$$

Since $(\bar{N}_{\tau}(s), \mathcal{G}_s, P)$, $s \in [\tau, t]$, is a square-integrable martingale, to establish (9) it suffices to show that $E[\bar{N}_{\tau}(t)Y_{\tau}(t)] = E[N_{\tau}^*(t)Y_{\tau}(t)]$ for every $Y_{\tau}(t)$ such that

$$Y_{\tau}(t) = \int_{\tau}^t \psi(s) dv(s): \psi(s) \text{ bounded}$$

because such $Y_{\tau}(t)$ are dense in $L^2(\mathcal{G}_t, P)$ (up to additive constants) by Theorem 1.

Since

$$E[(E^{\tau} g(\omega))Y_{\tau}(t)] = E[E^{\tau} g(\omega)E^{\tau} Y_{\tau}(t)] = 0,$$

we have

$$E[\bar{N}_{\tau}(t)Y_{\tau}(t)] = E[E^t(N_{\tau}Y_{\tau}(t))] = E[g(\omega)Y_{\tau}(t)].$$

Hence,

$$\begin{aligned} E[\bar{N}_{\tau}(t)Y_{\tau}(t)] &= E\left[g(\omega)\left(\int_{\tau}^t \psi(s) dw(s) + \int_{\tau}^t \psi(s) \bar{h}(s) ds\right)\right] \\ &= E\left[g(\omega)E\left[\int_{\tau}^t \psi(s) dw(s) | \mathcal{G}_{\tau}\right]\right] \\ &\quad + E\left[\int_{\tau}^t E^s(g(\omega)\psi(s)\bar{h}(s)) ds\right] \\ &= E\left[\int_{\tau}^t \psi(s) E^s(g(\omega)\bar{h}(s)) ds\right] \\ &= E\left[\left(\int_{\tau}^t E^s(g(\omega)\bar{h}(s)) dv(s)\right)Y_{\tau}(t)\right] \\ &= E[N_{\tau}^*(t)Y_{\tau}(t)]. \end{aligned}$$

Corollary 1: Let $g(\omega)$ be any r.v. independent of $\sigma\{w(s): \tau \leq s \leq t\}$ satisfying (7) and (8). Then (9) holds.

Proof: Observe that

$$\begin{aligned} E\left[g(\omega)\int_{\tau}^t \psi(s) dw(s)\right] \\ = E\left[g(\omega)E\left[\int_{\tau}^t \psi(s) dw(s) | \sigma\{\mathcal{G}_{\tau} \cup \sigma\{g(\omega)\}\}\right]\right] = 0. \end{aligned}$$

Remark 1: The following easily verified observation will be used in Theorem 3. If X, Y , and Z are jointly Gaussian with means zero, then $E(XYZ) = 0$.

III. LINEAR SMOOTHING

Theorem 3: Consider the linear model in which the system and observation processes are described by

$$\begin{aligned} dx(t) &= F(t)x(t)dt + G(t)dv(t) \\ dz(t) &= H(t)x(t)dt + dw(t), \quad 0 \leq t \leq T \end{aligned} \quad (10)$$

where

- 1) $v(t)$ and $w(t)$ are independent standard Wiener processes.
- 2) $x(0) = x_0$ is a Gaussian random variable with mean 0 and variance σ_0^2 , independent of the Wiener processes $w(t)$ and $v(t)$, and $y(0) = y_0$ is a constant.
- 3) $H(t)$ is continuous in t , $F(t)$ and $G(t)$ are Borel measurable functions, and $G(t)$ is bounded and nonnegative.

Then,

- 1) The joint conditional distribution of $x(s_1), \dots, x(s_k)$, $s_j \in [0, t]$, $j = 1, \dots, k$ given $\mathcal{G}_t = \sigma\{z(s): 0 \leq s \leq t\}$ is Gaussian with conditional means $E^t x(s_j)$ and conditional covariances $\text{cov}^t(x(s_i), x(s_j))$, $i, j = 1, \dots, k$.
- 2) The filtered estimate $\hat{x}(t) = E^t x(t)$ and the conditional variance $P(t) = E^t x^2(t) - (\hat{x}(t))^2$ satisfy

$$d\hat{x}(t) = F(t)\hat{x}(t)dt + P(t)H(t)dv(t) \quad (11)$$

$$dP(t)/dt = 2F(t)P(t) + G^2(t) - P^2(t)H^2(t) \quad (12)$$

where

$$\begin{aligned} dv(t) &= dz(t) - H(t)\hat{x}(t)dt, \\ \hat{x}(0) &= E^0x(0) = 0 \quad \text{and} \quad P(0) = \sigma_0^2. \end{aligned}$$

3) Let τ , $0 \leq \tau \leq t \leq T$ be fixed. The *fixed point smoothed estimate* $E^t x(\tau)$, the conditional covariance $\text{cov}^t(x(\tau), x(t))$ and the conditional variance $\text{var}^t(x(\tau))$ satisfy

$$dE^t x(\tau) = H(t) \text{cov}^t(x(\tau), x(t)) dv(t) \quad (13)$$

$$d \text{cov}^t(x(\tau), x(t)) / dt = (F(t) - H^2(t)P(t)) \text{cov}^t(x(\tau), x(t)) \quad (14)$$

$$d \text{var}^t(x(\tau)) / dt = -H^2(t)(\text{cov}^t(x(\tau), x(t)))^2 \quad (15)$$

with the initial conditions

$$E^T x(\tau) = \hat{x}(\tau) \quad \text{and} \quad \text{cov}^T(x(\tau), x(\tau)) = \text{var}^T(x(\tau)) = P(\tau).$$

4) The *fixed interval smoothed estimate* $E^T x(t)$ and the conditional variance $\text{var}^T(x(t))$ satisfy

$$dE^T x(t) / dt = F(t)E^T x(t) + (G^2(t)/P(t))(E^T x(t) - \hat{x}(t)) \quad (16)$$

$$d \text{var}^T x(t) / dt = 2(F(t) + G^2(t)/P(t)) \text{var}^T x(t) - G^2(t) \quad (17)$$

with the initial conditions

$$E^T x(T) = \hat{x}(T) \quad \text{and} \quad \text{var}^T(x(T)) = P(T).$$

5) Let $a \in [0, T]$ be a fixed delay. The *fixed lag smoothed estimate* $E^t x(t-a)$, the conditional covariance $\text{cov}^t(x(t-a), x(t))$, and the conditional variance $\text{var}^t(x(t-a))$ satisfy

$$\begin{aligned} E^t x(t-a) &= E^0 x(0) + \int_0^{t-a} (F(u) + G^2(u)/P(u)) E^t x(u) du \\ &\quad - \int_0^{t-a} (G^2(u)/P(u)) \hat{x}(u) du \\ &\quad + \int_0^t H(u) \text{cov}^u(x(0), x(u)) dv(u) \end{aligned} \quad (18)$$

$$d \text{cov}^t(x(t-a), x(t)) / dt = (F(t) + F(t-a) + G^2(t-a)/P(t-a) - H^2(t)P(t)) \text{cov}^t(x(t-a), x(t)) \quad (19)$$

$$\begin{aligned} d \text{var}^t(x(t-a)) / dt &= 2(F(t-a) + G^2(t-a)/P(t-a)) \text{var}^t(x(t-a)) \\ &\quad - H^2(t)(\text{cov}^t(x(t-a), x(t)))^2 - G^2(t-a) \end{aligned} \quad (20)$$

with the initial condition $\text{cov}^a(x(0), x(a))$ and $\text{var}^a(x(0))$ obtained from (14) and (15) (taking $\tau=0$ and $t=a$).

Proof: The proof of assertion 1) follows from a similar reasoning as carried out in [4]. Assertion 2) is the well-known Kalman-Bucy filter.

Following Gihman and Skorohod [3] we can show that for the solution of the system (10) and for every $n \geq 1$

$$\sup_{0 \leq t \leq T} E|x(t)|^n < \infty. \quad (21)$$

It follows that (2) holds for $h(t) = H(t)x(t)$.

An application of Itô's lemma to the system (10) shows that every $f \in C^2(\mathbb{R})$ satisfying (4) belongs to $D(\bar{A})$, and the operator

$$\bar{A}(f; t) = F(t)f_x(x(t)) + \frac{1}{2}G^2(t)f_{xx}(x(t)) \quad (22)$$

where $f_x = df/dx$ and $f_{xx} = d^2f/dx^2$.

Also $\bar{D}(f; t) = 0$ a.s. since $x(t)$ and $w(t)$ are independent.

In particular, from (21) it follows that $f(x(t)) = x(t) \cdot x(\tau)$ (τ is a fixed instant) satisfies (4) and (5) and $g(\omega) = x(\tau)$ or $x^2(\tau)$ satisfies the hypotheses of Lemma 1.

Lemma 1 with $g(\omega) = x(\tau)$ yields (13). Now let $f(x(t)) = x(t)x(\tau)$. Then from (22) and Theorem 2,

$$\begin{aligned} E^t(x(t)x(\tau)) &= E^T x^2(\tau) + \int_\tau^t F(s)E^s(x(s)x(\tau)) ds \\ &\quad + \int_\tau^t H(s) \text{cov}^s(x(s)x(\tau), x(s)) dv(s). \end{aligned} \quad (23)$$

Applying Itô's lemma to the product $E^t x(\tau) \cdot \hat{x}(t)$ [see (11) and (13)] we have

$$\begin{aligned} E^t x(\tau) \cdot \hat{x}(t) &= (E^T x(\tau))^2 + \int_\tau^t [F(s)\hat{x}(s)E^s x(\tau) \\ &\quad + H^2(s)P(s) \text{cov}^s(x(\tau), x(s))] ds \\ &\quad + \int_\tau^t H(s)(E^s x(\tau)P(s) \\ &\quad + \hat{x}(s) \text{cov}^s(x(\tau), x(s))) dv(s). \end{aligned} \quad (24)$$

Subtracting (24) from (23) yields (14) since the integrand of the resultant stochastic integral vanishes by the Gaussian property of the conditional distributions (Remark 1). Equation (15) is derived similarly. This proves 3).

From Lemma 1

$$E^T x(t) = \hat{x}(t) + \int_t^T H(u) \text{cov}^u(x(t), x(u)) dv(u). \quad (25)$$

From (14), for $t \leq u \leq T$

$$\text{cov}^u(x(t), x(u)) = P(t) \exp\left(\int_t^u (F(s) - H^2(s)P(s)) ds\right)$$

so that

$$E^T x(t) = \hat{x}(t) + P(t)Y(t, T) \quad (26)$$

where

$$Y(t, T) = \int_t^T H(u) \exp\left(\int_t^u (F(s) - H^2(s)P(s)) ds\right) dv(u)$$

satisfies

$$dY(t, T) = -(F(t) - H^2(t)P(t))Y(t, T)dt - H(t)dv(t). \quad (27)$$

Using (26), (11), (12), and (27), and Itô's lemma, (16) follows. Equation (17) is derived similarly. This proves 4).

Finally, combining the results in 3) and 4) according to the scheme

$$E^0 x(0) \xrightarrow[\text{smoothing}]{\text{fixed point}} E^t x(0) \xrightarrow[\text{smoothing}]{\text{fixed interval}} E^t x(t-a)$$

yields (18)-(20) of 5).

Note: After submitting this short paper we came to know that similar techniques have been employed to solve similar problems by Van Schuppen [7] for the continuous case and by Segall *et al.* [6] for the counting observations case. An alternate approach to the fixed point smoothing problem in linear systems is given by Bagchi [1].

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