Comment on evidence for new interference phenomena in the decay $D^+ \to K^-\pi^+\mu^+\nu$

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Abstract

The experimental determination of low energy πK scattering phase shifts would assist in determining scattering lengths as well as low energy constants of chiral perturbation theory for which sum rules have been constructed. The FOCUS collaboration has presented evidence for interference pheomena from their analysis of D_{l4} decays based on decay amplitudes suitable for a cascade decay $D \to K^* \to K\pi$. We point out that if the well-known full five body kinematics are taken into account, πK scattering phases may be extracted. We also point out that other distributions considered in the context of K_{l4} decays can be applied to charm meson decays to provide constraints on violation of $|\Delta I| = 1/2$ rule and T-violation.

Keywords: Chiral perturbation theory, Semileptonic decay of charm mesons, πK scattering phase shifts

1. Chiral perturbation theory [1] as the low energy effective theory of the standard model is now in a remarkably mature phase. Several processes have been computed to two-loop accuracy and remarkable predictions exist for low energy processes. One of the important processes that has been studied is that of $\pi\pi$ scattering, for a recent comprehensive review, see ref. [2]. It has been traditionally difficult to study this experimentally in the low-energy regime due to the absence of pion targets. One important source of information comes from the rare kaon decay K_{l4}. Using well-known techniques [3, 4] one can extract the phase difference for pion scattering of the iso-scalar S-wave and iso-triplet P-wave phase shifts $\delta_0^0 - \delta_1^1$ from an analysis of the angular distributions, where the final state or Watson theorem relates the phase of the decay form factors to the scattering phase shifts. Recently the E865 collaboration [5] at Brookhaven National Laboratory has carried out the analysis of data from a high statistics experiments which has brought about a remarkable marriage between experiment and theory. There are preliminary measurements also from NA48 for the semi-leptonic decays, ref. [6]. Scattering lengths will also be measured at high precision by the CERN experiment DIRAC from the lifetime of the pionium atom, and from the enormous statistics gathered by the NA48 collaboration by employing the recent proposal of Cabibbo, see refs. [7, 8], of analyzing the cusp structure of the invariant mass of the dipion system produced in the reaction $K^+ \to \pi^+\pi^+\pi^-$.

2. Chiral perturbation theory that involves the s-quark degree of freedom is yet to be tested at a corresponding degree of precision. One sensitive laboratory is the pion-Kaon scattering amplitude [9]. For recent studies on the comparison between the amplitudes evaluated in chiral perturbation theory, and phenomenologcal determination, see refs. [10, 11]. It has been pointed in these that it is desirable to have high precision phase shift determinations so that accurate predictions for scattering lengths can be made. The search for an experimental system where these phase shifts can be measured, leads us naturally to an analog of the K_{l4} decay in the charm-meson system, which is the decay D_{l4}[1]. It is clear that one might be able to extract information on the πK scattering amplitude as well due to the final state or Watson theorem. What is required is an analog for the technique used in the case of K_{l4} decay for the D_{l4} decay. Note that in the D_{l4} case, the dimeson pair in

[1]Indirect sources of information include pion production from scattering of kaons off nuclei, e.g. [12].

the final state is composed of unequal mass particles and that the iso-spin of the system is different from that in the corresponding $\pi\pi$ system. At leading order in the weak interaction, one obtains only $|\Delta I| = 1/2$ amplitudes and the system yields information on the phase shift difference $\delta_0^{1/2} - \delta_1^{1/2}$. A comprehensive and self-contained account of this is to be found in ref. [13]. Note that for the moment, the analog of the pionium system for the πK atom is only in the planning stage, and determination of the πK scattering lengths from an analog of the proposal of Cabibbo from, say $D^+ \to K^-\pi^+\pi^+$ or $\bar{K}^0\pi^+\pi^0$ would not be feasible due to limited statistics. As a result, it is imperative that the D_{l4} decay be exploited to determine the phase shifts of interest. The data so obtained could be in conjuction with the recent accurate solutions to the Roy-Steiner equations [14].

3. In this paragraph, we briefly recall the main features of the formalism of ref. [13]. The process considered is

$$D(p_1) \to K(p_2) + \pi(p_3) + l(k) + \nu(k'). \tag{1}$$

The authors give an explicit form for the 5-fold differential width

$$\frac{d^5\Gamma}{dq^2 ds_{23} d\cos\theta d\chi d\cos\theta^*} = \frac{G_F^2 |V_{cs}|^2 q^2 \sqrt{a_2} X}{96(2\pi)^6 m_1^3} \sum_i l_i H_i, \tag{2}$$

where $q = k + k'$, m_1 is the mass of the D meson, $s_{23} = (p_2 + p_3)^2$, $a_2 = 4|\mathbf{p_2}|^2/s_{23}$, $X = \sqrt{s_{23}}|\mathbf{p_1}|$, θ is the angle between the charged lepton and the D meson in the dilepton center of mass frame, θ^* is the angle between the K meson and the D meson in the dimeson center of mass frame, χ is the angle between the lepton and meson decay planes, and the sum over i runs over the symbols U, L, T, V, P, F, I, N, A, with the H_i being the helicity structure functions, and the l_i given as follows for the case of massless charged leptons:

$$l_U = \frac{3}{8}(1 + \cos^2\theta),\ l_L = \frac{3}{4}\sin^2\theta,\ l_T = \frac{3}{4}\sin^2\theta\cos(2\chi),$$
$$l_V = -\frac{3}{4}\sin^2\theta\sin(2\chi),\ l_P = \frac{3}{4}\cos\theta,\ l_F = \frac{3}{2\sqrt{2}}\sin(2\theta)\sin\chi,$$
$$l_I = -\frac{3}{2\sqrt{2}}\sin(2\theta)\cos\chi,\ l_N = \frac{3}{\sqrt{2}}\sin\theta\sin\chi,\ l_A = -\frac{3}{\sqrt{2}}\sin\theta\cos\chi.$$

We do not explicitly list all the H_i except for a few for purposes of illustration (see below).

Writing the hadronic matrix element as

$$\langle p_2, p_3 | A_\mu + V_\mu | p_1 \rangle =$$
$$\frac{1}{m_1}\left[f(p_2+p_3)_\mu + g(p_2-p_3)_\mu + r q_\mu + \frac{ih}{m_1^2}\epsilon_{\mu\nu\alpha\beta} q^\nu (p_2+p_3)^\alpha (p_2-p_3)^\beta \right],$$

where the form factors f, g, r, and h are in general functions of s_{23} ,q^2 and θ^* (r makes no contribution in the case of massless charged leptons). The H_i can now be expressed in terms of the form factors. For instance,

$$H_{F(A)} = \frac{X}{m_1^2}\frac{\sqrt{a_2 s_{23}}}{\sqrt{2q^2 m_1^2}} \mathrm{Im}\,(\mathrm{Re}) \left(h^* \left[Xf + gX\frac{m_2^2 - m_3^2}{s_{23}} \right.\right.$$
$$\left.\left. + g\sqrt{a_2}\frac{m_1^2 - s_{23} - q^2}{2}\cos\theta^* \right] \right) \sin\theta^*,$$
$$H_V = -\frac{X a_2 s_{23}}{m_1^4}\mathrm{Im}(h^* g)\sin^2\theta^*.$$

It was shown first by Pais and Treiman that the choice of variables made by Cabibbo and Maksymowicz leads to the simple decomposition, eq. (2) of the 5-fold differential width and thus makes the determination of physical observables amenable. Furthermore, by parametrizing the functions f, g, h and identifying their phases with πK phase shifts (a consequence of Watson's theorem), the partial wave expansion of f, g, and h read

$$f = \tilde{f}_s e^{i\delta_0^{1/2}} + \tilde{f}_p e^{i\delta_1^{1/2}}\cos\theta^* + \ldots,$$
$$g = \tilde{g}_p e^{i\delta_1^{1/2}} + \ldots,$$
$$h = \tilde{h}_p e^{i\delta_1^{1/2}} + \ldots.$$

It may, therefore be seen from the above that an analysis of the decay distribution would yield information on the phase shifs of interest.

3. The FOCUS Collaboration has recently published "evidence for new interference phenomena in the decay $D^+ \to K^-\pi^+\mu^+\nu$" [15]. By including an S-wave in a straightforward manner into the decay amplitude that is dominated by the P-wave K^* resonance, and finding a superior fit to certain distributions, this result has been established. The decay amplitude has been adopted from ref. [16] which considers the three body final state kinematics. In particular, the process considered in [16] is the reaction of the type

$$D(p_1) \to K^*(p^*) + l(k) + \nu(k') \qquad (3)$$

for which the hadronic part of the amplitude is written down in terms of the matrix element

$$\langle K^*(p^*)|A_\mu + V_\mu|D(p_1)\rangle = \epsilon_2^{*\alpha} T_{\mu\alpha}, \tag{4}$$

where

$$T_{\mu\alpha} = F_1^A g_{\mu\alpha} + F_2^A p_{1\mu} p_{1\alpha} + F_3^A q_\mu p_{1\alpha} + iF^V \epsilon_{\mu\alpha\rho\sigma} p_1^\rho p^{*\sigma}, \tag{5}$$

and $q_\mu = (p_1 - p^*)_\mu$ is the momentum transfer. Note that F_3^A contributes only in the case of decays with massive charged leptons. The differential decay rates are expressed in terms of helicity amplitudes which evaluate to

$$H_0 = \frac{1}{2M_1\sqrt{q^2}} \left((M_1^2 - M^{*2} - q^2) F_1^A + 2M_1^2 p^2 F_2^A \right),$$
$$H_\pm = F_1^A \pm M_1 p F^V,$$

where p is the momentum of the K^* in the D rest system, M_1 and M^* are the masses of the D and the K^* respectively. In ref. [15][2], the massless lepton case alone is considered, in which case, the differential decay rate is written down as

$$\frac{d^4\Gamma(D \to K^* \to K\pi)}{dq^2 d\cos\theta d\chi d\cos\theta^*} \propto B(K^* \to K\pi)$$
$$\Big(\frac{9}{32}(1+\cos^2\theta)\sin^2\theta^*(|H_+|^2 + |H_-|^2) + \frac{9}{8}\sin^2\theta\cos^2\theta^*|H_0|^2 -$$
$$\frac{9}{16}\sin^2\theta\cos 2\chi\sin^2\theta^*\mathrm{Re}(H_+H_-^*) - \frac{9}{32}\sin 2\theta\cos\chi\sin 2\theta^*\mathrm{Re}(H_+H_0^* + H_-H_0^*)$$
$$\pm\frac{9}{16}\cos\theta\sin^2\theta^*(|H_+|^2 - |H_-|^2) \mp \frac{9}{16}\sin\theta\cos\chi\sin 2\theta^*\mathrm{Re}(H_+H_0^* - H_-H_0^*)\Big)$$

where the upper and lower signs in the last two (partity violating) terms refer to the two cases $l^-\bar{\nu}_l$ and $l^+\nu_l$ respectively.

Note that in the treatment above, there will be no contributions of the type $i = F, N, V$. The FOCUS collaboration in the analysis of its data, finds that a simple analysis based on a 1^{--} does not fit the data well. They make an *ad hoc* assumption and introduce an amplitude with the properties of an S- wave $A\exp i\delta$. Introducing this generates interference terms which would correspond to terms that appear as $i = F$, N and a term of the $i = L$

[2] In ref. [16] a discussion is provided on the multipole behaviour that is expected of the functions F_1^A, F_2^A, F_V; the FOCUS collaboration assumes all of them to have monopole behaviour in their analysis, ref. [15].

type. This assumption cannot generate a term of the type $i = V^3$. In [15] the narrow-width approximation for the K^* is replaced by a Breit-Wigner and a full 5-fold distribution is written down.

4. It is our main comment here that the FOCUS collaboration must account for the dynamics in its entireity by using the formalism of ref. [13]. In this manner, they would also be able to determine the phase shift difference which would allow us to pin down low energy strong interactions observables to better precision. Note also that a complete description of the four body final state with lepton mass effects included is presented in ref. [13][4]. By binning the data in the variable s_{23} and carrying out integrations in the variables θ^* and χ and fitting the resulting distribution to experimental data, it would be possible to determine the phase shifts and the form factors themselves. We note here that unlike in the K_{l4} decay where the dimeson system is composed of equal mass particles, in the present case a ratio of e.g., $\langle H_F \rangle / \langle H_A \rangle$ cannot directly yield information on $\delta_0^{1/2} - \delta_1^{1/2}$. Only a comprehensive fit to all the $\langle H_i \rangle$ can be used to extract this quantity. In this regard, it would be useful to follow the procedure described at length for the case of K_{l4} decays in ref. [18].

5. We recall here that in the context of K_{l4} decays, the original 5 body decay kinematics were discussed in ref. [3], where the authors discussed only 1-dimensional distributions. In ref. [4] 2-dimensional distributions were considered, and also analyzed in the context of limited statistics. [The latter was the basis of the analysis of the events from the well-known experiment, ref. [18].] Subsequently Berends, Donnachie and Oades (BDO) [19], again considered 1-dimensional distributions, but with limited statistics. They also discussed $|\Delta I| = 3/2, 5/2$ transitions, and also looked at tests of T-invariance. Recently the NA48 collaboration [6] has observed some evidence for the violation of the $|\Delta I| = 1/2$ rule consistent with standard model expectations in K_{l4} decays using the the technique of BDO.

BDO in the context of K_{l4} decays consider the 2-fold distribution given

[3] Note that this is consistent with H_V of the previous paragraph vanishing in the S- and P- wave approximation

[4] In this regard, the FOCUS collaboration has analyzed data with charged lepton mass effects with their modified formalism of ref. [16] in ref. [17].

by

$$\frac{d^2\Gamma}{d\cos\theta^* d\chi}$$

which could receive contributions from T-violating interactions assuming that higher wave contributions are absent. Here we point out that such a distribution for D-meson decays could receive additional contributions from T-violation in the decays. Also considered in BDO are the distributions

$$\frac{d\Gamma}{d\chi}, \frac{d\Gamma}{d\cos\theta^*}$$

which could be used to fit the form factors in an analysis independent of the Pais-Treiman type distributions. The work of BDO can be readily extended to D-meson decays to search for the violation of the $|\Delta I| = 1/2$ rule if there is a sizable number events for other reactions including $D^+ \to \bar{K}^0 + \pi^0 + l + \nu_l$, but this need be pursued after a compelling analysis of presently available data for the determination of phase shifts of interest. For a recent discussion on $|\Delta I| = 3/2$ amplitudes, see ref. [20].

6. In summary, we point out that the FOCUS collaboration with its large sample of D_{l4} decays can carry out a determination of much sought after πK phase shifts by adopting the methods of Pais and Treiman, and those of Cabibbo and Maksymowicz, and Berends, Donnachie and Oades, and go beyond establishing an interference phenomenon. This would be a valuable source of information for important low energy observables such as pion-Kaon scattering lengths.

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