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Allocation of advertising space by a web service provider using combinatorial auctions

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Abstract. Advertising is a critical process for promoting both products and services in global trade. Internet has emerged as a powerful medium for trade and commerce. Online advertising over the internet has increased more than hundredfold since 2001. In the present work, we address problems faced by online advertisement service providers. In this paper, we propose a multi-slot and multi-site combinatorial auction for allocating scarce advertisement slots available on multiple sites. We observe that combinatorial auctions serve as effective mechanisms for allocating advertising slots over the internet. We resort to "ant" systems (ant – social insect/intelligent agent) to solve the above $\mathcal{NP}$-hard combinatorial optimization problem which involves winner-determination in multi-item and multi-unit combinatorial auctions.

Keywords. Web page service provider (WSP); winner-determination problem (WDP); multi-dimensional knapsack problem (MDKP); combinatorial auction problem (CAP); ant-colony optimization (ACO); online advertisements.

1. Introduction

1.1 *Motivation*

Advertising is realized to be the key for penetration of products and services. Consumer spending on paid internet content was estimated as USD2 billion in 2003[1]. According to a recent study from Jupiter Research [1], the paid content market will grow at 30 % compared to 2002. The company forecasts that the paid content market will grow at an annual rate of more than 20 % until 2007, when it will be worth USD 5·4 billion. Jupiter Research further predicts that syndication revenues for consumer content will reach USD 1·4 billion in 2007. The research company forecasts that online advertising will continue to be the main money-spinner for online media outlets with internet ad spending set to reach USD 14 billion in 2007, up from USD 6·2 billion in 2003. It has also been determined by several surveys that

[1]Source: Internet News [1]
A glossary of terms used is given at the end of the paper in appendix D
Numbers in square brackets refer to websites references

Table 1. Offline vs. online advertisements.

Offline advertisements	Online advertisements
Static mode of advertising	Dynamic mode of advertising
Once hosted the content cannot be changed	Content can be modified easily
Content searching is cumbersome	Content searching is easy
Content development is low, printing charges are high. Each additional print copy adds to the cost	Initial development costs are high, later hosting charges are low
Reach of offline advertisements is limited	Reach is unlimited, any one on the internet can watch the advertisement.
Immediate buying (and/or) selling is not possible	Online advertisements are linked with online buying (and/or) selling, thus making transactions easy
Customization of advertisements is difficult	Advertisements can be customized based on the user preferences (age-group, geography etc.)

the impact of advertising over the internet is more than that of television advertisements[2]. The above factors motivated us to investigate the problem of allocating advertising slots over the websites using combinatorial auctions. To the best of our knowledge, ours is one of the first to adopt the auction framework in the internet advertisement setting.

The internet has emerged as a powerful medium of communication and information exchange. With the tremendous increase in the usage of internet over the past decade, there is a concomitant increase in advertising over the internet. We note that for Yahoo!, listings and fees revenue reached $ 83 million in the third quarter, up 124 % or equivalently 33 % of total revenue in the year 2001 [3]. Also in the year 2001, online advertising spending over the internet was projected to be about $ 6 billion (Prasada *et al* 2003). A given web-page has finite space for hosting potential advertisements and this brings in the revenue for the web-page service provider (WSP).

Online advertisments provides increased scope of bargaining in both B2B and B2C markets. For example, please see SmartBargains [4], which provides advertisements for products ranging from toys to computer peripherals. Further an electronic catalogue is developed by the company for customized products and services across various age-groups. It is also evident that with increase in eBusiness, there will be clear increase in the spending on on-line advertising.

1.2 *Offline vs online advertisements*

Advertising over the internet (dynamic) provides several advantages over offline (static) advertising, some of which are listed below.

The problem of allocating advertisements of different clients over the limited web-page space happens to be an $\mathcal{NP}$-hard allocation problem. The complexity of this problem is enriched by the clients' preference over a bundle of advertising slots instead of a single slot,

[2]Reported on AdAge.com, Oct. 13, 2003 [2]. For example, 17 % of 700 US consumers surveyed in the past six months said television advertisements influenced their car-buying decisions. In contrast, advertisements on internet search engines influenced 26 % of consumers

in addition to which there can be quantity restrictions on slots. We notice that there is no work in the literature addressing the winner-determination problem in the context of advertising over the internet. In the current work, we address the problem faced by the web-page service provider on "how to optimally allocate the advertising space available on her web-page so as to maximize the overall revenue of the system." We refer to this problem as the (advertising) "Dilemma of the Web Service Provider." We use the terms *web-page service provider* and *web service provider* interchangeably.

The problem faced by WSP can be cast as a multi-site and multi-slot combinatorial auction problem (MSMS-CAP) which is the most complex of all auction types[3]. Combinatorial auctions are increasingly gaining the attention of academic researchers and industry. The main bottleneck in adapting combinatorial auction framework for allocation of resources is the winner-determination problem. The computation time for the winner-determination problem grows exponentially with the number of items, number of clients etc. Commercially available solvers like ILOG-CPLEX, LINGO etc., cannot tackle this situation in limited time. This in-turn motivates us to focus on efficient heuristics for solving the winner-determination problems. For the first time, we make use of Ant-Colony Optimization for solving the Combinatorial Auction Problems. CAP problems applied to internet advertisements can be broadly classified into four types, based on the number of slots and sites: (1) Single slot and single site (SSSS) CAP; (2) single slot and multi site (SSMS) CAP; (3) multi slot and single site (MSSS) CAP and (4) multi slot and multi site (MSMS) CAP. Of all these CAPs, SSSS happens to be the simplest one, MSMS is the most complex one, and SSMS and MSSS fall between these two extremes.

1.3 *Contribution of the paper*

Extant work on combinatorial auctions have progressed from simple linear programming models to complex dynamic programming models for exact analysis, whereas for approximate analysis, simple greedy search to stochastic local search mechanisms are employed.

Any approach for solving the winner-determination problem (WDP) in CAP should satisfy the constraints of resources in bundles. Ant-colony optimization (ACO) as a method for solving WDP in CAP is proposed in the current paper. It is our firm belief that applying ACO to CAP can be useful in terms of quality of solution obtained for larger instances of CAP.

Organization of the paper is as follows: In § 2 we present classification of relevant literature on combinatorial auctions and also give an overview of the techniques for solving the combinatorial optimization problem with emphasis on ant-colony optimization. In § 3 we present an overview of the problem we address in the current research. In the following §4, we present the MSMS-CAP model which is supplemented with a detailed overview on ACO heuristic in § 5. We present the details of computational experiments in § 6 and conclude with results and discussion in § 7.

2. Relevant literature

We integrate two different strands of literature, namely combinatorial auctions and intelligent evolutionary optimization techniques. We present the literature survey of auctions, followed by the ant-colony optimization literature.

[3]Here we focus our attention on single attribute auctions. However, our model is readily extensible to the multi-attribute case also

There is a rich source of literature available on resource allocation using auction mechanisms. We classify the literature into two parts based on the type of the resource auctioned. single item auction and multi-item auction.

2.1 *Auctions*

The simplest of all auctioning mechanisms is SSSS. The winner-determination problem is simplified due to single units. Some of the popular auction mechanisms related to this are English auction and Dutch auction. These auctions can be considered as single round or multiple round (de Vries & Vohra 2003). Multiple-unit auctions are difficult to solve compared to single-unit auctions due to the complexity of winner-determination. Here, the agents can have choices over a set of items whose combined valuation need not be the same as the sum of the valuations of the individual items. This gives rise to the concept of bundle valuations or combinatorial auctions. The resulting problem of winner-determination happens to be $\mathcal{NP}$-hard (Papadimitrou & Steiglitz 2001). There are several papers in this area (Sandholm 2000; Tennenholtz 2002). Tennenholtz (2002) shows that quantity restricted multi-object combinatorial auctions, multi-item combinatorial auctions with binary bids or triplet bids or near additive bids are computationally tractable. The author uses graph theoretic arguments and provides an intuitive proof for the computational tractability. Jones & Koehler (2002) propose a heuristic for solving the integer programming formulations of combinatorial auctions with rule based bids. Gavish (2003) gives an overview of combinatorial auctions with a mention of the open issues for research.

2.1a *Combinatorial auctions:* Combinatorial auctions are the most common auctions seen in digital markets where users have the choice to bid on combinations of items, referred as “bundles”. User valuation on items need not be same as the sum of the valuations of the items in the bundle, making the decision maker’s problem of winner-determination more complex (Sandholm 2000). Procurement auctions with discounts are discussed by Hohner *et al* (2003). Narahari & Dayama (2005) give a detailed survey of combinatorial auctions in eBusiness. There are other surveys on combinatorial auctions by de Vries & Vohra (2003) and Kalagnanam & Parkes (2003). The common problem observed in combinatorial auctions is the WDP. We note that WDP is $\mathcal{NP}$-hard for the case of MSSS (Sandholm 2000; Gavish 2003). Combinatorial auctions have been applied in a wide variety of industrial settings. Gavish (2003) gives an overview of a variety of applications where combinatorial auctions were successfully employed. Further he gives the case of spectrum auctions considering bundles across different geographical locations–which is a particular case of MSSS CAP. Mars–IBM (Hohner *et al* 2003) is an industrial application of combinatorial auctions. The auction mechanism in Mars comes under the general category of MSMS CAP.

2.2 *Ant-colony optimization*

Ant-colony optimization (ACO) is a recent meta-heuristic advance in intelligent optimization. It was first proposed by Dorigo (1992) in his thesis. The basic idea of ACO is to mimic the ability of a colony of ants to find the shortest path to a food source and route around obstacles. For a detailed overview on ACO including the communication behaviour of ants the reader is referred to the work by Dorigo *et al* (2000). For a qualitative overview on ACO the reader is referred to Bonabeau *et al* (2000), Abraham (2004) and Silver (2004). Wright (2003) provides a detailed overview of working mechanisms of meta-heuristics, and gives the following seven stages necessary to carry out intelligent search, namely, (i) problem definition,

(ii) search space definition, (iii) neighbourhood definition, (iv) generation of initial solution, (v) neighbourhood generation, (vi) acceptance criterion, and (vii) stopping rule.

ACO is an evolving method of optimization which has been used to solve various $\mathcal{NP}$-hard applications like job-shop scheduling (Rajendran & Ziegler 2004), batch scheduling (Abraham 2004), project scheduling (Merkle *et al* 2002), vehicle-routing problem (Gambardella *et al* 1999) quadratic assignment problem (Gambardella *et al* 1997), sequential ordering problem (Gambardella & Dorigo 1997), flow shop scheduling (Stutzle 1998), multi-dimensional knapsack problem (Fidanova 2002) etc. In the current paper, we make use of the ACO algorithm for MDKP from Fidanova (2002). We present a detailed description of this ACO in § 5.

3. Problem description

We elicit various problems faced by the WSP. Further we discuss the problem that we address in the current paper. Advertising for products (and/or) services is shown to have a major impact on sales. With the increase in the penetration of the internet there is a concomitant increase in the eBusiness trading volume as well as price.

We consider a typical WSP who has control over some websites with multiple advertisement slots on them. The WSP uses the auction mechanism for allocating these slots. The process of allocation is described below.

3.1 *Auction-based slot allocation*

The WSP announces the list of websites along with the number of slots on each of these sites available for advertisement and, importantly, the WSP also announces the period for which the slots will be available. The WSP discloses the rules of allocation, such as, if more than one bid of equal value on the same site along with the same slots are present, then the bid received earlier will be preferred for allocation. The bidders bid on multiple-slots which are not necessarily on the same site. The WSP collects all bids and then runs the auction model to determine the winners. Winning bids are notified to all bidders and the auction is terminated for the period. This ends the first round of auction. The next round of auction is after the first round of auction, in a multi-period setting.

3.2 *Some scenarios*

The plethora of problems encountered by a typical WSP can be broadly decomposed into two types - *Slot allocation problem* and *Slot scheduling problem*.

(1) *Slot allocation problem*: Here the WSP is interested in allocation of multiple slots available on one or more web-pages.
(2) *Slot scheduling problem*: This involves sequencing of allocating slots. In other words, scheduling involves time-phasing of the slots allocated to the winning bidders.

Slot allocation problem and scheduling problem can be modelled for a single period or for a multi-period setting. There can be several variants of these above problems based on desired objectives (and/or) associated constraints.

3.3 *Scope of current problem*

In the current research, we concentrate on the slot allocation problem in a single period setting. We ignore the temporal value of a slot on a site throughout the period. In other words, we assume the bid value on a bundle remains the same throughout the model.

We address the following problem faced by a web-page service provider (WSP). The WSP has several web-pages, each of which contains slots for hosting advertisements. We focus on a single web-page and show the mechanism for optimally allocating the web page space. Later we extend our analysis to a multiple web-page allocation problem. The clients are free to bid on any number of slots with a choice of quantities for each slot. This makes the model more generic compared to existing models with exceptions of multi-item and multi-unit combinatorial auctions presented by Bikhchandani & Ostroy (2002). We cast the current multi-slot and multi-site (msms) cap as a multi-dimensional knapsack problem (MDKP) by considering the following natural mapping between the CAP and the MDKP.

The WSP has "$\mathcal{M}$" advertising slots and each slot can be hosted on a number of mirror sites. Thus advertisers can specify the quantity per slot. It is interesting to note that the quantity per slot is not fixed and depends on various factors such as geography, age group etc. For example, consider Yahoo's website "www.yahoo.com". The hosting of advertisements on its pages depends on the geography. For instance, the hosting on its Indian site "www.yahoo.co.in" varies from that of the hosting of its UK site "www.yahoo.co.uk". However, there can be some common hosting on all (or some) of the sites.

The clients submit their bids for various bundles. There are no restrictions on the number of bids and the quantity. This preserves the economic incentiveness of the mechanism.

Our models can be easily adapted to many industrial settings such as:

(1) Wireless spectrum allocation (Gavish 2003)
(2) Carrier lanes allocation (freight auctions) (Song & Regan 2003).
(3) Hosting the websites by an internet service provider (ISP).
(4) Scheduling (Kutanoglu & Wu 1999).

We first present the mathematical programming formulation of multi slot and single site (MSSS) CAP and later extend the same to multi slot and multi site (MSMS) CAP. It is easy to relax these two generic models to the corresponding models with single slot. It may be noted that the above models can be extended to multi-round auction setting by considering the revenue of bids accumulated in each round. This can be taken care of by using an index "t" for indicating the round. The objective(s) and constraint(s) will have the round index (t) as their subscript in addition to bundle index. In addition to this there will be extra summation over this index. Our current model is specified for a single round auction.

4. Models

Our modelling assumptions include the following.

Assumption 1. In the case of bids on identical bundles, we account the bundle with maximum bid value into the model(s).

Assumption 2. The clients bid for the advertising slots. Each of them have their own preferences for a particular slot. Further, they can bid for a combination of slots and quantity of each slot. We will be referring to the combination of slots along with the quantity as a "bundle".

Table 2. Notation for auction models.

$\mathbf{N}$	Number of bidders (agents or clients)
$\mathcal{M}$	Number of bundles
$\mathcal{C}_{k,j}$	Bid value for bundle j submitted by agent k
$\mathcal{C}_j$	Maximum bid value for bundle $j = \text{Max}\, \mathcal{C}_{k,j} \ \ \forall k \in \mathbf{N}$
$\mathbf{X}_j$	Binary decision variable $= \{0, 1\}$, 1 if bundle j is selected, else 0
q_{ij}	Number of slots of type i in bundle j
a_{ij}	Index for slot of type i present in bundle j (= 1 if ith slot is present in jth bundle, 0 else)
$\mathcal{K}_i$	Number of advertising slots of type i available for bidding.
i	Index for advertising slot of type $i \in \mathcal{K}_i$
j	Index for bundle type $j \in J = \{1, 2, \ldots, \mathcal{M}\}$. Each bundle is a vector $\langle i, q_{ij}, \mathcal{C}_j \rangle$
k	Index for client ($k \in \mathbf{N}$).

Assumption 3. The client has choice to place bids on any of the bundles. This does not prevent the possibility of no bidding for a particular bundle(s). We substitute bundles on which no bids are submitted with dummy bids carrying a value zero.

We formulate the WDP as a mixed integer linear programming (MILP) model for the case of multi-slots–single-site (MSSS) and extend it to multi-sites (MSMS) combinatorial auctions.

4.1 *Multi-slot and single-site combinatorial auction problem*

$$\text{Max} \quad \mathbf{Z_{MSSS}} = \sum\nolimits_{j \in J} \mathcal{C}_j \mathbf{X}_j, \tag{1}$$

subject to following constraints

$$\sum\nolimits_{j \in J} a_{ij} \mathbf{X}_j \leq 1 \quad \forall i \in \mathcal{M}, \tag{2}$$

$$\mathbf{X}_j \in [0, 1], \quad \forall j \in J. \tag{3}$$

The above MILP formulation maximizes the revenue of WSP. Constraint 2 ensures that an item is present at the most once in the winning bids. Constraint 3 indicates the binary nature of decision variable, $\mathbf{X_j} = 1$ if jth bundle is present in the winning bids and 0 otherwise. This formulation can be extended to the multi-unit case by considering the quantity of each item present in the bundle. It should be noted that the objective for the multi-slot multi-site (MSMS) case remains the same as that of the multi-slot and single-site (MSSS) case. It is easy to see that constraint 3 is also unaltered. There is a modification to constraint 2 in which we incorporate the quantity constraint that ensures the winning bids satisfy the constraint on availability of the slot types.

4.2 *Multi-slot and multi-site combinatorial auction problem*

$$\text{Max} \quad \mathbf{Z_{MSMS}} = \sum\nolimits_{j \in J} \mathcal{C}_j \mathbf{X}_j, \tag{4}$$

subject to following constraints,

$$\sum\nolimits_{j \in J} q_{ij} \mathbf{X}_j \leq Q_i \quad \forall i \in \mathcal{M}, \tag{5}$$

$$\mathbf{X}_j \in [0, 1], \quad \forall j \in J. \tag{6}$$

The objective (4) indicates the maximization of revenue of WSP. Constraint (5) ensures that the quantity allocated does not exceed the maximum available quantity across all available advertising slots. Constraint (6) indicates the binary nature of decision variable $\mathbf{X}_j$. This CAP is identical to the multi-dimensional knapsack formulation (Freville 2004) and hence $\mathcal{NP}$-hard.

Multidimensional knapsack problems have been solved approximately by various intelligent search techniques such as genetic algorithms (Sakawa & kato 2003), tabu search (Hanah & Freville 1998), lagrangean methods (Michelon & Veilleux 1996) and recently using ant-colony optimization (Fidanova 2002).

5. Ant-colony optimization (ACO)

We present a detailed overview of ACO. Ant-colony optimization is an example of a heuristic from nature. These are based on naturally observable phenomena from areas such as physics, biology and social sciences (Abraham 2004). All natural heuristics are based on the following two principles (Colorni *et al* 1996).

(1) *Selection* **-** Rewarding/penalizing according to fitness (i.e. optimality) criterion.
(2) *Mutation* **-** Randomness in the system that can lead to newer solution instances.

5.1 *History of ACO*

The ACO metaheuristic was first described by Dorigo (1992). ACO algorithms are inspired from the behaviour of real ant colonies. Ants are social insects and the high levels of structures that are formed in their colonies have attracted the attention of social scientists.

The specific interest of ant systems is the *foraging behaviour* of ants. This is a behaviour noticed in ant-colony systems, wherein ants always choose the shortest path to a source even in presence of some external obstacles. For a detalied discussion on foraging behaviour interested readers are referred to (Donjo 1999; Bonabeau 2000; Abraham 2004).

Biological reasoning for foraging behaviour in ant systems: Each of the ants deposits a pheromone on the ground along the path it traverses. The pheromone laid on the traversed path is referred to as pheromone trail. These pheromone trails evaporate over time. The trail of pheromone so formed attracts (guides) the later ants to keep to a particular path. It can be intuitively seen that in case of the shorter path from the ant nest to food source, the pheromone concentration will be higher simply due to smaller turnaround times for the ants, thus helping the ants to follow the shortest path.

5.2 *Ant-colony framework for multi-slot and multi-site CAP*

We now customize the MDKP formulation of ACO for the multi-slot and multi-site CAP. This framework can be easily customized for the multi-slot and single-site CAP and also for single-slot and multi-site CAP.

In order to apply ACO, first, we need to express the CAP models in ACO framework. Specify the tuple (S, f, Ω) for MSMS CAP. Construct an finite set of bundles on which bids are received and are considered. $\mathcal{M} = \{j_1, j_2, \ldots, j_M\}$. The states of the problem are defined in terms of the sequences $\overline{\mathbf{S}}_\mathbf{m} = \langle j_1, j_2, \ldots j_m \rangle$ over the elements of $\mathcal{M}$. We denote the set of all feasible bundles by $\mathcal{X}$ (please see equations (5), (2), (6) and (3)). The length of a sequence indicating the number of winning bundles is given by $\overline{\mathbf{S}}_\mathbf{m}$. The finite set of constraints Ω defines the set of feasible states $\widetilde{\mathcal{X}}$, with $\widetilde{\mathcal{X}} \subseteq \mathcal{X}$. Please see equations (1)–(6). A set S^* of

Table 3. Additional notation for ACO heuristic.

ρ	Evaporation rate for pheromone trail
$p^k_{j_m}$	Probability of kth ant selecting bundle j_m as winning bundle sequence.
$\tau_{j_l j_m}$	Pheromone trail on the arc (j_l, j_m)
t	Iteration counter.
MaxIter	Maximum number of iterations
τ_0	Initial pheromone trail on all arcs
α	Constant used in ACO $(0 < \alpha < 1)$
L_{GB}	Best value of the objective function from the beginning
$\Delta\tau_{j_l j_m}$	Difference in global update value of the pheromone level while traversing on arc (j_l, j_m)
S	Set of potential solutions (i.e., paths available for ants)
f	Objective of the ACO (4) and (1)
s	An element belonging to the solution set S, $(s \subseteq S)$
Ω	Set of constraints (equations (5), (2), (6) and (3))
$\mathcal{M}$	Set of all bundles on which bids are received , $\mathcal{M}\{j_1, j_2, \ldots, j_M\}$
j_m	Revenue associated with bundle m
$\mathcal{X}$	Set of all feasible bundles–these bundles satisfy the site and slot availability requirements
$\overrightarrow{S_m}$	Number of winning bundles in the allocation in a sequence of ant traversal is given by $\lvert S_{j_m} \rvert$
S^*	Set of all feasible solutions
$f(s, j_m)$	Reward associated with each candidate solution $s \in S$ and with bundle j_m
$\eta_{j_m}(\hat{S}_k(t))$	Heuristic information associated with bundle j_m under feasible solution set $\hat{S}_k(t)$

feasible solutions is given with $S^* \subseteq \widetilde{\mathcal{X}}$ and $S^* \subseteq S$. (Reward) $f(s, j_m)$ is associated with each candidate solution $s \in S$ and bundle j_m.

Thus we have constructed the ACO framework which is initially left for exploration by ants till a global optimum range is reached. Later ants converge to the "global" optimal value by exploiting the solution state space. The effectiveness of any ACO depends on the balancing between the exploration and exploitation features[4].

The ACO algorithm makes use of intelligent ants which iteratively construct candidate solutions. Solution construction is guided by the pheromone trails and problem-dependent heuristic information. In the current problem, there are "$\mathcal{M}$" constraints (refer to 5). Hence we refer to the CAP problem as "$\mathcal{M}$-dimensional combinatorial auction problem." In MDKP the ordering of solutions is not important and the solution length need not be fixed. We first define the graph for the MDKP with the following properties.

- Nodes of the graph correspond to the bundles of advertising slots
- The graph is fully connected
- Pheromone trails are laid by the ants on the arcs visited.

With a partial solution $\hat{S}_k = \langle j_1, j_2, \ldots, j_l \rangle$ built by ant "k", the probability $p^k_{j_m}(t)$ of selecting j_m as the next item in the solution space is given by the following equation.

[4]The balance between exploration and exploitation is achieved by choosing the ACO prameters - pheromone evaporation rates, local and global, and also the local/global pheromone updation rule.

$$p^k_{j_m}(t) = \begin{cases} \left\{\tau_{j_l j_m}\eta_{j_m}(\hat{S}_k(t))\right\} \Big/ \left\{\sum_{j_q \in allowed_k(t)} \tau_{j_l j_q}\eta_{j_q}(\hat{S}_k(t))\right\}, & \text{if } j_m \in \ allowed_k(t), \\ 0, & \text{otherwise,} \end{cases} \quad (7)$$

where $\tau_{j_l j_m}$ is the pheromone level on the arc (j_l, j_m), $\eta_{j_m}(\hat{S}_k(t))$ is the heuristic value based on the (partial solution) obtained till the point of current exploration and *allowed*$_k(t)$ is the set of remaining feasible bundles. It should be noted that higher the value of $\tau_{j_l j_m}$ and $\eta_{j_m}(\hat{S}_k(t))$ the more profitable it is to include the bundle i_m in the current solution. The pheromone level on arc (j_l, j_m) is updated by the local updating rule given as:

$$\tau_{j_l j_m} \longleftarrow (1-\rho)\tau_{j_l j_m} + \rho\tau_0, \quad (8)$$

where $0 < \rho < 1$ and τ_0 are constants. After all ants have completed their tours, global updating of the pheromone levels is updated by the following global update rule:

$$\tau_{j_l j_m} \longleftarrow (1-\alpha)\tau_{j_l j_m} + \alpha\Delta\tau_{j_l j_m}, \quad (9)$$

where

$$\Delta\tau_{j_l j_m} = \begin{cases} L_{GB}, & \text{if } (j_l, j_m) \in \text{global best,} \\ 0, & \text{otherwise.} \end{cases} \quad (10)$$

With the above description of the ant-colony framework, ants build the partial solutions initially by moving on the construction graph $\mathcal{G} = (\mathcal{M}, \mathcal{L})$, where $\mathcal{L}$ fully connects the bundles (elements of $\mathcal{M}$). A certain number of ants are randomly selected and they are allowed to construct the partial solutions by traversing on the graph $\mathcal{G}$. The quality of solution constructed by ants is valued based on the pheromone trails on the path traversed by the respective ants. The level of pheromone trails on the connections is updated according to a local pheromone update rule (see 8). The local pheromone update is also referred as “useful forgetfulness property”, which is needed in order to avoid faster convergence to a non-global optimum. The best solution obtained so far (by including bundle j_m in the winning bids) is referred as “best tour”. If at least one ant is successful in finding an optimal solution, the best tour obtained so far is updated with the “Global best tour” and the pheromone levels are updated by using global pheromone update rule (see 9). The remaining ants follow this global best tour, thereby leading to the convergence of the solution (Gutjahr 2002). We can also notice a concomitant increase in the pheromone levels on the bundles in global best tour (winning bundle slots). We refer interested readers to Appendix for a detailed proof of convergence of ACO heuristic applied for CAP.

6. Computational experiments

We construct the ACO heuristic based on the above framework and implemented it using Microsoft Visual Basic 6.We consider some of the sample test cases generated using uniform distribution [5]. Fidanova (2002) reports the results of ACO applied to multi-dimensional knapsack problems. Our experimentation differs from the work of Fidanova in the following ways: First, we consider the presence of bundles with different items. Second, we take the highest bid value into account in case of identical bids for identical items. It should also be

[5]“http://www.mgmt.iisc.ernet.in/~sandeepd/Sadhana_Testdata.zip”

Table 4. Comparison of results ACO vs. LINGO.

		ACO			LINGO		
Testcase	No of bundles	Optimal revenue (in $)	Time (in s)	Iterations	Optimal revenue (in $)	Time (in s)	Iterations
Dataset 1	500	25684	9	4	25684	25	1896
Dataset 2	500	28797	9	4	28797	20	1699
Dataset 3	500	27495	8	3	27495	20	2512
Dataset 4	500	28328	9	4	28328	20	1456
Dataset 5	1000	38474	11	9	38474	50	2504
Dataset 6	1000	43814	11	9	43814	50	956
Dataset 7	1000	28051	11	9	28051	50	4533
Dataset 8	1000	29421	11	9	29421	50	6146

noted that the we cannot use the same datasets used by Fidanova which did not have any bundle constraints.

6.1 *Datasets*

We assume 5 advertising slots (top, bottom, left, right and middle). We assume that available quantity of slots is the same and equal to 20. Datasets are in MS-Excel (2002) spreadsheet format. These data can be taken as the weekly data and we solve the problem of deciding winners of advertisement slots on a weekly basis. The datasets used for computation can be obtained from the website [5].

The datasets 1 to 4 consist of 500 bundles each while datasets 5 to 8 consist of 1000 bundles each. We have used the following values for ACO parameters, $\alpha = 0{\cdot}85$, $\beta = 0{\cdot}7$, $\rho = 0{\cdot}78$ for all datasets. The code was run on a Pentium IV machine with 1 GB RAM. The run time for global convergence would vary depending on the system configuration.

From the results of table 4, we notice that there is no difference in the optimal revenue obtained by using ACO or LINGO.[6]

It is to be noted that we have compared ACO meta-heuristic performance with a commercial optimization solver LINGO[6]. The results presented are the optimal values. Further, our thesis here is to support the application of ACO for CAP, hence we did not compare the ACO results with other heuristics.

6.2 *Validation of ACO results*

In order to validate the results of our computation we use the following quick and easy method. We impose a super (sub)-additive bid structure on all $\mathcal{M}$ bundles. We generate the bundles randomly and match it with the (super/sub)-additive prices (for this we sort the prices in descending(ascending) order and match the highest(lowest) price with a bundle with maximum(minimum) number of slots and quantity of advertising slots. Now we run the ACO heuristic with the above data and observe that the bundle with maximum(minimum)

[6]ACO gives the same solution as that of LINGO due to the problem size. In an experiment conducted by second author using ACO for the traveling salesman problem (TSP), ACO produced the best results with 64 nodes (Gopal Prsad 2001).

Table 5. Data set for quick validation of ACO results.

Bundle Id	Slot 1	Slot 2	Slot 3	Price
Bid1	2	0	0	1·00
Bid2	0	2	0	1·50
Bid3	0	0	2	2·25
Bid4	2	0	3	4·75
Bid5	2	2	0	5·00
Bid6	1	1	3	5·00
Bid7	1	2	1	10·50

number of slots and quantity gets first place in the winning bids and is followed by the feasible bundles (that satisfy the bid price and quantity restrictions) in descending(ascending) order of number and quantity of slots. For an illustration of validating the results, we consider the following dataset with three slots (SLOT1(2), SLOT2(2) and SLOT3(3) – numbers in parentheses indicate the number of slots that are available). We have imposed the super-additive bid structure exogenously.

The maximum revenue ($= 12{\cdot}75$) is obtained by the WSP by including the most bulky bundle (Bid7) which is obtained from the result of ACO (and this verification is trivial). From the remaining bids, only Bid3 is eligible (others fail due to quantity constraints), which indeed can be inferred from the result of ACO.

7. Conclusions

We have presented the models for revenue maximization of WSP for auctioning the advertising space over the internet. For the first time, we have applied ACO for solving the multi-slot and single-site and multi-slot and multi-site combinatorial optimization problems which were earlier not tractable in polynomial time. The results obtained from our model show performance equivalent to the solution of commercial solvers. We also have presented the proof of convergence of the ACO heuristic for multi-slot and multi-site CAP.

Future work

Another interesting direction of modelling is to bring in game-theoretic approach for modelling competitive behaviour among the bidders, also among multiple WSP's. We list some of the challenging problems for research.

Table 6. Winning bid details for validating the ACO results.

Winning bundle Id	Slot 1	Slot 2	Slot 3	Price
Bid7	1	2	1	10·50
Bid3	0	0	2	2·25

- To model competition among the bidders – single period vs. multi period, with-out learning effects vs. with learning effects in bidding behaviour.
- To consider a framework with multiple bidders along with multiple WSPs. Model the competitive allocation via game theory etc. This problem can also be considered in single- and multi-period setting. Further enrichment can be brought in by considering learning effects of bidding - where a WSP can learn the behaviour of other WSPs and also the behaviour of bidders.
- To address the desirable properties of auction mechanism such as individual rationality, budget balance, economic incentiveness etc.
- We see a natural extension to the above work is to extend it for a multi-attribute CAP setting. Incorporating temporal value of the slot into the model is a real challenge.
- To get bounds on run-time of algorithm along with the convergence would help in the deployment of ACO heuristic in industrial setting.
- To compare the results obtained using ACO with other evolutionary algorithms like genetic algorithms, simulated annealing[7] etc.

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Appendix A

Proof of convergence of ACO

We adapt the propositions 1 and 2 from Stutzle & Dorigo (2002) for the proof of convergence of our ACO algorithm. From the proposition 1 of Stutzle & Dorigo (2002) we can infer that the pheromone trail evaporation rate (useful forgetfulness) at the tth iteration is bounded by:

$$\tau_{j_l j_m} = (1-\rho)^t.\tau_0 + \sum_{i=1}^{t} (1-\rho)^{t-i}.g(s^*),$$

where $g(s^*)$ is the global best tour value corresponding to s [8]. Since $0 < \rho < 1$, asymptotically the sum converges to: $\tau_{max} = |(1/\rho)g(s^*)|$. From this expression it is evident that pheromone levels get stabilized once the optimal solution is reached (after a certain number of iterations). We can see from the proposition 2 of Stutzle & Dorigo (2002) that once the optimal solution has been found and the globally best pheromone update rule is used, we have pheromone levels ($\tau_{j_l j_m}$) on arc connecting bundle j_l and j_m monotonically increases,

$$\forall (j_l, j_m) \in s^* \;\; \Rightarrow \underset{t\to\infty}{Lim} \left[\tau_{j_l j_m}\right] = \tau_{\max} = \left|(1/\rho)g(s^*)\right|.$$

Proposition 2 emphasises the pheromone trails once the optimal solution is reached. In other words, the optimal solution is obtained when $\tau_{\max} = |(1/\rho)g(s^*)|$.

[7] In progress

[8] We have employed the same notation as that of Stutzle & Dorigo (2002)

Theorem A1. *Let $P^*(t)$ be the probability that the ACO finds optimal solution at least once within the first "t" iterations. It can be seen that for a small $\epsilon > 0$ and for sufficiently large t, $P^*(t) \geq 1-\epsilon$ holds and as $t \rightarrow Lim_{t\rightarrow\infty} P^*(t) = 1$ (asymptotically) Stutzle & Dorigo (2002)*

Proof. We extend the proof of theorem 1. in Stutzle & Dorigo (2002) to the MSMS-CAP. We know that the pheromone trails on arc connecting bundle j_l and j_m is bounded by τ_{min} and τ_{max}. For the $\mathcal{M}$-dimensional knapsack problem we can compute the lower bound of reaching the optimal solution which can be given as:

$$p_{\min} \geq \hat{p}_{\min} = \tau^{\alpha}_{\min}/[(|\mathcal{M}| - 1)\tau^{\alpha}_{\max} + \tau^{\alpha}_{\min}] \tag{11}$$

where $|\mathcal{M}|$ is the cardinality of number of components[9](number of bundles).

*Note A*1. The implication from the above theorem is that, given any $\mathcal{M}$-dimensional knapsack problem, any generic solution s' (which also includes the optimal solution $s^* \in S^*$ can be generated with a probability $\hat{p} \geq \hat{p}^n_{\min}$, where $n < \infty$ is the optimal solution length (description of winning bids).

*Note A*2. It is enough if at least one ant finds an optimal solution, corresponding lower bound for $P^*(t)$ is given by: $\hat{P}^*(t) = 1 - (1 - \hat{p})^t$

*Note A*3. The convergence of ACO indicates that there is a positive probability of ACO heuristic for MSMS-CAP finding optimal solution. But it does not mention about the computational time required for convergence.

Appendix B

Companies providing on-line advertisement statistics

We review some of the prominent companies which provide statistics of on-line advertisements and also help in on-line advertising[10].

- *Pew Internet & American Life Project*: A wing of the non-profit Pew research organization, this site regularly features extensive free primary research studies on a range of issues with Americans' use of the Internet.
- *CyberAtlas*: It is now an Internet.com property. CyberAtlas offers a comprehensive index of summaries of newly published research about the Internet.
- *Nua*: Nua has for years been providing a comprehensive round-up of Internet industry research and statistics. It is based in Ireland.
- *Online Publishing Association (OPA)*: The OPA has a frequently updated page that links to free reports about Internet media and advertising topics. The OPA also produces a bi-weekly newsletter called the "OPA Intelligence Report" that frequently cites industry research.
- *Media Post's Research Brief*: The excellent Media Post produces a round-up of Internet media and marketing-related research every day called the "Research Brief" under the organization name "Center For Media Research." The main page includes an archive (below the day's posting). Unfortunately, the archive headlines do not mention the source of the research.

[9]We consider highest the bid for a bundle and the ties are broken in favour of early bids
[10]Source: e-Marketing [7]

- *Nielsen – NetRatings*: One of the leading Internet research firms. Nielsen – NetRatings regularly reports on trends, top-ranked sites, top advertisers and other noteworthy findings.
- *comScore Network*: comScore Network has emerged a key player, arguably the market leader, in the Internet audience and online customer research sector. With a core research panel of more than 1 million U.S. Internet users, comScore also acquired one-time leader in the sector of Internet audience research Media Metrix in the spring of 2002. comScore is now NetRatings's biggest rival.
- *CMR*: CMR, a Taylor Nelson Sofres company, is a research firm specialized in advertising across many media. Their press releases routinely track trends in ad spending.
- *Forrester*: The most influential analyst group in the Internet space, Forrester cranks out its fair share of press releases about all aspects of e-business.
- *AdRelevance*: A Jupiter Media Metrix company, AdRelevance attempts to track advertisements online through proprietary technology to report on trends in online advertising.

Appendix C

Pseudocode for ACO Heuristic

```
[1] WHILE termination conditions not met DO
[2] ScheduleActivities()
[3] AntBasedSolutionConstruction()
[4] PheromoneUpdate()
    +Local
    +Global
[5] DaemonActions()
    {optional}
[6] END
[7] ScheduleActivities
[8] END WHILE
```

Appendix D

Glossary

- *Ant*: A social insect (intelligent agent) capable of exploring and exploiting the solution state space.
- *Ant-colony systems [ACS]*: A collection of ants.
- *Ant-colony optimization [ACO]*: An evolutionary heuristic based on ant-colony systems. The inspiration of heuristic being drawn from the biological systems.
- *Auto catalysis*: The process of rewarding better solutions based on better performance.
- *Bid*: The reservation price specified by customer (bidder) on a single item or a combination of items.
- *Bidder*: The client (customer) who participates in bidding.
- *Bundle*: A subset consisting of various objects represented under a common identity. In the current problem, we define a bundle as a collection of one or more than one slots associated with one or more than one site.

- *Candidate solution*: Set of feasible bundle(s) which can be attached with the prior bundles allocated. The first bundle for traversal is selected randomly.
- *Combinatorial auction problem [CAP]*: A discrete optimization problem (usually MILP) used for allocation of resources.
- *Controller actions*: Monitors behaviour of ants over time. Also effects necessary changes in pheromone levels.
- *Exploring*: The process of searching the solution state space.
- *Feasible bundle*: Any bundle that satisfies the slot and site requirements after allocation of one or more slots to winners. We make use of feasible bundles so as to reduce the state space of exploration.
- *Heuristic*: A method which, on basis of experience or judgement, seems likely to yield a reasonable solution to a problem but which cannot be guaranteed to produce the mathematically optimal solution.
- *Heuristic information*: This conveys the relative strength of current path chosen by an ant against all feasible paths including the current path. This is done by weighted average of current cost (reward) against the total cost (reward) (see 7). In the current problem, heuristic information conveys the amount of revenue gained by including the current bid in allocation in comparison with all feasible bids.
- *$\mathcal{NP}$-hard problems*: Non-deterministic polynomial problems. These problems are extremely complex and the computation time grows exponentially with the problem instance size.
- *Meta heuristic*: It is a higher level heuristic procedure designed to guide other methods and process towards achieving reasonable solutions to difficult combinatorial optimization problems [11].
- *Mixed integer linear program [MILP]*: It is a specific case of linear programming problem wherein some of the variables are constrained to take values in the integer set.
- *Pheromone*: An attracting hormone (chemical) substance left by ants along the path it follows. This helps the successor ants to trace the path followed by an earlier ant.
- *Pheromone evaporation*: Pheromone trails left by ant(s) evaporate over time. This is used to enable balance between the exploration (i. e., global search) avoids local extrema and exploration (i. e., improves local search). Pheromone levels have to be reduced at a predetermined rate and frequency. This is similar to cooling ratio in simulated annealing.
- *Site*: Any webpage hosted by webservice provider.
- *Slot*: The space available on a website for advertisement popups.
- *Solution state space*: The possible exploration space available for ants. It is a permutation of various feasible bundles.
- *Stigmetric communication*: The process of communication exchange among the biological agents.
- *Web service provider*: Agent responsible for hosting, designing and layout the webpages.

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[11]Source: Silver (2004)

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