Prof. R. Rajaraman of the Indian Institute of Science knew J. S. Bell, the distinguished 'philosopher' of quantum physics and had also collaborated with him. We publish below the invited article he wrote for us in memory of this outstanding savant.

—Ed.

John Stuart Bell—The man and his physics

R. Rajaraman

With the shocking and untimely death of Professor John Stuart Bell on 1 October 1990, the world has lost not just a very distinguished physicist but an exceptional human being. For nearly three decades he was a moral presence in the world of physics, maintaining high standards of intellectual clarity and professional integrity, while making a series of important contributions to his field. His path-breaking work on the foundations of quantum theory, acclaimed of course by the physics community, had also made him famous in a larger world—a celebrity status that he handled with quiet dignity and gentle amusement. As a person, he was kind and soft-spoken, yet commanded much respect, sometimes bordering on awe.

I cannot claim the privilege of having known John very intimately, or for long. I met him for the first time in early 1983 when he visited our Centre for Theoretical Studies in Bangalore, and I never saw him again after bidding goodbye to him at CERN, Geneva, in late 1985. During those three years, however, we did have a fair bit of contact at both the professional and social levels. We also collaborated on and co-authored a couple of research papers. In the process I, like many others before me, grew to admire and respect him. I was also a recipient of his kindness in several ways.

In this brief homage to John Bell, I shall first refer to his physics, and give an introduction to two of his major contributions. I will then hazard a few personal impressions of John Bell the man, recalled with affection and respect.

Over the years, John worked on a wide range of subjects in physics. Not many physicists of the current generation may know, for instance, that one of the first review articles on the theory of nuclear matter was written by him, co-authored with E. J. Squires in 1961 when that field was still in its infancy. He made important contributions to such widely different areas as accelerator physics and neutrino scattering from nuclei. My own work with him in the mid-eighties was on the mysteries of fractional charge in polymers and one-dimensional field theories. But of his numerous contributions, perhaps the two that are most famous are what have come to be known as Bell's theorem (on the foundations of quantum theory), and the Adler–Bell–Jackiw anomaly in quantum field theory.

Bell's inequality

Bell's theorem or Bell's inequality, deals with the foundations of quantum theory. In order to appreciate why this work evoked so much interest even beyond the world of physics, I must first say a few words about quantum theory itself. Quantum theory is more than just the theoretical basis of modern physical science. It is one of the most profound constructs of the human mind, whose significance transcends the scientific discoveries galore that it has led to, spectacular though these are. This is because, underlying its working rules, quantum theory carries a conceptual structure radically different from that of all the 'classical' science that

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preceded it for centuries. It has demanded fundamental changes in our ideas of scientific predictability, of determinism and indeed of the whole nature of physical reality. These aspects of quantum theory, to which Bell's theorem was addressed, have fascinated not just physicists but the larger intelligentsia, including philosophers, theologians and even litterateurs.

To begin with, the predictions of quantum theory are probabilistic. But unlike the use of probability and statistics in classical physics or in the social sciences, the probabilistic feature of quantum theory is meant to be intrinsic, not due to limitations of available data or our calculational stamina. Quantum theory demands an unavoidable influence of the very act of measurement on its result. If we measure the position of a particle, knowledge of its momentum becomes totally uncertain, and vice versa. Similar statements are true for many pairs of 'simultaneously incommensurate' observables. Quantum theory also forces us to accept situations in which a system consists of, say, two spatially well-separated components where, while the results of measurement cannot be precisely predicted in either component, yet, given any specific result in one of the components the result in the other is fully determined! These are examples of the famous EPR paradox, to which we shall return shortly.

Is the real world actually so bizarre? Or are these vagaries of a very successful but nevertheless incomplete description called quantum theory, while 'actually there is an objective reality out there', with simultaneous and precise values for positions, momenta, etc.? Is it even meaningful to ask such questions about the nature of 'true reality' within the purview of science, unless one can identify measurable criteria which can answer them objectively?

Such issues have bothered people ever since the inception of quantum theory. The great Albert Einstein had serious reservations about quantum theory because of its conceptual features and in 1935 he wrote (with B. Podolsky and N. Rosen) a seminal paper constructing the EPR paradox mentioned earlier, to give focus to what worried him. The debates between Niels Bohr and Einstein ('God does not play dice'—this from Einstein) on these questions are legendary. Inspired by Einstein, several people tried to construct a more fundamental theory which is deterministic and consistent with classical ideas of objective reality. Constructing such theories in a responsible manner is not at all easy. It must not only reproduce all the experimentally confirmed predictions of quantum theory, but also suggest other concrete measurable consequences that could distinguish it from quantum theory.

Not surprisingly then, this field of study progressed slowly and inconclusively, with occasional carefully thought out papers by very serious thinkers mixed in with relatively superficial hidden-variable alternatives which did not carry conviction, not to mention missives from a variety of nuts, cranks, and malcontents.

Into this somewhat confused scenario with a heterogenous literature came John Bell's work, cutting through it like a beacon of crisp cold light. Given a class of EPR type of experiments, Bell constructed explicit measurable criteria which could distinguish between the quantum and classical pictures of reality. His criteria were in the form of simple mathematical inequalities. To paraphrase (a potentially dangerous step in this subject), his inequality in such an experiment would involve a combination (let us call it C) of quantities that can be objectively measured by these experiments. If the experimental results were fully in accord with the standard predictions of quantum theory, then the value of C, suitably normalized, would have to be less than one. On the other hand, if the system were governed by some deeper 'classical' type of theory, (where all particles did simultaneously 'possess' specific values for all their physical attributes, such as their positions, momenta, all spin-projections, etc., governed in turn by some deterministic rules) then the value of C would have to be greater than one! This is regardless of the specific mechanisms and the details of the underlying classical candidate theory. The important feature of Bell's ingenious criterion was that it was based solely on objectively measurable experimental numbers. It elevated the forty-year-old debate over the quantum versus the classical nature of reality from being a perennially inconclusive controversy involving metaphysical or subjective preferences, to something that could be objectively decided.

Subsequently, Alain Aspect and collaborators at Paris conducted a practical version of such thought-experiments. On applying Bell's inequality to the data, quantum theory was vindicated. More importantly, the possibility of some deeper classical explanation of the data was ruled out. Of course all this does not diminish the bizarre nature of the quantum view of reality, which continues to violate our intuitive notions based on day-to-day experience. But, as Bell's work has established, it nevertheless seems to be unavoidably true, and we just have to live with it.

The ABJ anomaly

In 1969, John Bell and Roman Jackiw, another distinguished theoretical physicist now at MIT, discovered the phenomenon of 'anomalies' in four-dimensions. Stephen Adler at Princeton had also discovered the same thing around the same time, independently and by different methods. Anomalies refer to the violation, upon quantization, of some symmetry of a system (and the associated conservation law) present at its classical level. Generally speaking, the symmetries and conservation laws of a dynamical system can be preserved upon quantization. For instance, classical mechanics tells us that the total momentum, angular momentum and energy of a pair of bodies bound to one another by any central force will be conserved. This is one of the most important and useful results of classical physics, and is related to the fact that such a system is symmetrical with respect to overall displacements in space and time as well as with respect to rotations. When such a system is quantized, i.e. the dynamics of the system obtained using the rules of quantum mechanics, all these conservations continue to hold. The non-relativistic quantum theory of the hydrogen atom is a well-known example. This is true even in a careful relativistic treatment of the electron and its radiation field, as is done in quantum electrodynamics (QED). In fact, in QED, besides total energy, momentum and angular momentum, total electric charge is also conserved. This is indicated by the continuity equation, $\partial_\mu j^\mu = 0$ where j^μ refers to the electric current of the electron–positron system.

What Adler, Bell and Jackiw (ABJ) discovered was that such preservation of classical conservation laws need not hold in every instance, even in QED. The culprit they uncovered was the axial vector current (the pseudovector counterpart of the electric current) denoted by j^μ_5. If electrons are taken to be massless then QED enjoys at its classical level, an additional symmetry called chiral symmetry, with the associated conservation of this axial current. ABJ found that when the quantization of this theory is carried out carefully, this axial current is in fact not conserved. Instead one gets

$$\partial_\mu j^\mu_5 = \frac{e^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

where, $F_{\mu\nu}$ is the electromagnetic field tensor. This is the ABJ anomaly.

[It should be mentioned that the first example of such an anomaly was actually found way back in 1962 by Julian Schwinger, one of the architects of modern quantum field theory, in a two-dimensional toy version of electrodynamics. But Schwinger, a man of few words and many long formulae, took this result in his stride and did not especially emphasize it. Most physicists, including most particle-theorists either did not know about this finding of Schwinger, or took it to be an artefact of two dimensions. When Adler, Bell and Jackiw discovered a similar effect in realistic four space–time dimensional QED, it was a great surprise since, by then, QED had already been studied extensively by thousands of theorists for decades.]

That the mass of electrons in the real world, though small, is not actually zero does not diminish the importance of the ABJ anomaly. True, the axial current is then not conserved even classically, but the extent of its non-conservation in the quantized theory is substantially altered by the ABJ anomaly. Hence the ABJ anomaly is not just some theoretical sophistry. It affects the behaviour of real electrons, quarks, etc. and has experimental consequences such as in the decay of the π^0 meson. Subsequent to the ABJ papers, similar anomalies have been unearthed in other contexts. The subject of anomalies has grown into a sub-field of particle theory, yielding among other things, an important principle restricting the class of permissible models that can be entertained in particle physics. It also provided a principal motivation for superstring theory. At a deeper level anomalies also have a geometrical significance, and have been instrumental in introducing modern mathematical ideas of cohomology into particle theory.

Bell's theorem and the ABJ anomaly are topics quite different from one another not only at the technical level, but in the very nature of their preoccupations. That Bell could straddle two such disparate subjects, let alone make a major contribution in each, is testimony to his intellectual versatility.

John Bell, the man

Some characteristics of John Bell the man are already reflected in his physics. Take his work on the foundations of quantum theory, described earlier. Most physicists have been aware of the disquieting conceptual aspects of quantum theory, but few have worried about them seriously. Most have been content with using the theory at the operational level, where it was already complex enough to keep their intellects challenged, and where its predictions continued to be supported by millions of bits of experimental data. Partly, this attitude may have been based on just taste and temperament. But partly, it was also born of professional pragmatism. That Bell chose to work during the prime of his career in this field, of little utilitarian value and clouded with metaphysical overtones, speaks of his intellectual courage and individuality.

There was nothing remotely mystical or woolly in Bell's work leading to his theorem. On the contrary, it ingeniously brought a seemingly metaphysical controversy within the fold of objective science. Nevertheless because of its profound implications about the nature of reality, it had a wide impact and he was even sought after by religious and mystical sects. I have often discussed with him over lunch his experiences with such groups. Characteristically, he did not flinch from contact with them. While brooking no nonsense, he was willing to give the unconventional a fair chance.

For this was a man of deep convictions who made up his own mind about things. He was a vegetarian by choice, and, to the best of my knowledge, a teetotaller.

In his manner, John Bell was gentle and soft-spoken. But I do not think this was due to either timidity of soul or tepidity of feelings. I suspect that consistent with his flaming red beard there lurked volcanic passions, which he kept under tight control through self-discipline. I have seen glimpses of this during our scientific collaboration (especially the joint writing-up of our manuscripts whose wording entailed hard negotiations!). I mentioned all this once to John and his wife, Dr Mary Bell, when my wife and I were dining with them. If I remember rightly, Mary chuckled knowingly and John rewarded me with one of his gentle, wry smiles. So, I could not have been entirely wrong! Indeed, if I had tried here to paint John as an idealized saint rather than a man of real flesh and blood, I don't think he would have approved!

Mathematicians are somewhat reluctant to communicate the beauty of mathematics to others because its language is not so easily understood. When the Fields Medal was awarded to Prof. Vaughan Jones, we approached one of his collaborators, V. S. Sunder of the Indian Statistical Institute, Bangalore, to write about Jones and his work. We got his article and because of it we were able to persuade other young mathematicians (of TIFR and RRI) to write about the three other medallists—Prof. Vladimir Drinfeld, Prof. Shigefumi Mori and Prof. Edward Witten. We publish these four essays in this issue. Emboldened by this attempt we intend to publish hereafter papers/special issues on mathematical themes. We shall, of course, depend on our mathematicians to participate in this venture.

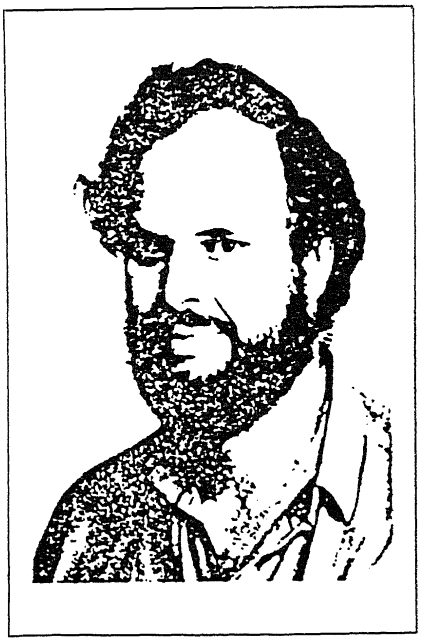
—Ed.

From von Neumann algebras to knot invariants—The work of Vaughan Jones

V. S. Sunder

VAUGHAN JONES was one of four mathematicians awarded the Fields Medal at the International Congress of Mathematicians, held at Kyoto in August 1990. (For the uninitiated reader, it may be recalled that there is no Nobel Prize for mathematics, and the Fields Medal is commonly thought of as the mathematicians' Nobel Prize, this Medal being awarded, at the International Congresses which meet once every four years, to mathematicians not yet 40 years old.)

The aim of this article is to try and give an idea, to the interested lay person, of some of the beautiful ideas that went into, and came out of, Jones' pioneering work. (The unexplained or technical terms appearing in the next paragraph will be carefully explained later in the text; the paragraph is meant to state or explain a point of view that underlies this article as well as much of Jones' research; suffice it to say that a case is being made for the operator-algebraic approach.)



To put things in a nutshell, the early eighties found Jones working on subfactors, these objects being of interest in the theory of *von Neumann Algebras*. Now the latter algebras were initially introduced by von

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