# Sonic velocity measurements in gas–liquid mixture with low void fractions

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**We present results of experiments on sonic velocity measurements in a gas–liquid mixture with low void fractions. A vertical shock-tube-type test rig was developed for this purpose. It had two test sections in series; one was a brass tube and the other a Perspex tube. The travel time of the leading edge of an expansion wave between two stations in each test section was measured using piezoelectric pressure transducers and a digital storage oscilloscope. For generation of the two-phase mixture with low void fraction an 'electrolysis bubble generator' was developed. A mixture of water and finely dispersed 0.2-mm-diameter hydrogen bubbles was created using the device. Sonic velocity measurements were made in the two sections, with void fraction varying from $0.9 \times 10^{-4}$ to $10^{-3}$. The results clearly show that sonic velocity in this void-fraction range is a very sensitive function of tube elasticity and void fraction. Comparison with earlier measurements is good in the overlapping void-fraction range. In addition, the measured sonic velocities are in excellent agreement with theory developed earlier.**

THE speed of sound in a given medium is a fundamental quantity that characterizes the rates at which some disturbances in the medium, such as pressure waves, travel from one point to another. Here we are interested in the speed of sound in a mixture of gas bubbles and a liquid, this combination being of great interest in safety-related problems in the nuclear and chemical industries. What is of relevance and somewhat surprising is the fact that the speed of sound in a gas–liquid mixture can be considerably lower than either of the speeds in the homogeneous media constituting the mixture. The presence of even a small amount of free gas-like air in the form of bubbles in water can result in substantial reduction in the speed of sound. A comprehensive review of the topic can be found in Jeevanashankara *et al.*[1] From this and more recent studies[2], it is apparent that there is a definite lack of measurement of speed of sound in gas–liquid mixtures with very low void fractions. Experimentally, it is possible to attain a low-void-fraction mixture by having a small number of large bubbles or a large number of smaller bubbles dispersed in the liquid. However, in terms of homogeneity of the mixture, there would be a difference: finer bubbles are likely to result in more homogeneous conditions. In the past, this has been one of the difficulties in work with low-void-fraction mixtures. In fact, Mori *et al.*[3] added a surfactant to reduce the size of the bubbles to improve the conditions of homogeneity. In the present work, the primary aim was to measure the sonic velocities in gas–liquid mixtures at void fractions approaching zero. Instead of using standard techniques of bubbling gas through a porous medium to attain bubbly-flow regime, a new device called electrolysis bubble generator was used and very good results have been obtained.

The analytical background to the problem under consideration is in Mori *et al.*[3] They considered the problem of shock-wave propagation in a bubbly mixture contained in an elastic tube. Since our interest was in the measurement of sonic velocities, the propagation of the leading edge of an expansion wave in a bubbly mixture contained in an elastic tube was examined. The leading edge of an expansion wave can be considered to be a 'small' disturbance and hence its propagation speed can be taken to be the sonic velocity in the medium. On the contrary, the propagation speed of a shock wave would approach the sonic velocity only in the limit of pressure ratio across the shock tending to zero. Considering a small disturbance moving in an elastic pipe, the following expression for the isothermal speed of sound can be derived[3]:

$$C^2 \simeq \left\{ \left[ \frac{\alpha}{P} + \frac{1}{\rho_l a_l^2} + \frac{2\gamma r}{tE} \right] \rho_l \right\}^{-1}. \tag{1}$$

Here, $C$ is the speed of sound in an elastic tube containing a two-phase mixture of void fraction $\alpha$, $P$ is the pressure, $\rho_l$ is the liquid density, $a_l$ is the isothermal speed of sound in the liquid, $r$ is the tube radius, $t$ is the tube thickness, $E$ is the modulus of elasticity of the tube material, and $\gamma$ is a constant related to Poisson's ratio $\mu$ by $\gamma = 1 - \mu/2$ or $\gamma = 1 - \mu^2$ depending on the tube-end conditions. Further, if $C_\infty$ is the speed in a rigid tube ($E \to \infty$), then, in terms of $C$,

$$C_\infty^2 = C^2 \left\{ 1 + \left[ \frac{2\gamma r}{tE} \right] \Big/ \left[ \frac{\alpha}{P} + \frac{1}{\rho_l a_l^2} \right] \right\}. \tag{2}$$

## Methods

### *Experimental facility*

The set-up is basically a vertical column of overall height about 9 m. A transparent Perspex tube of

50 mm i.d. and 60 mm o.d. makes up 4.57 m. The remaining is a brass tube of 50 mm i.d. and 58 mm o.d. Two test sections in series have been provided to study the influence of tube material on sonic velocity. The top of the column is connected to a section which is used to generate expansion or compression waves through the column. The bottom of the column is connected to the electrolysis bubble generator. In the initial experiments, it was found that generation of stable bubbly-flow regime with a mixing chamber and independent air and water supply was quite a difficult task. In view of this, a new technique was evolved to generate low-void-fraction bubbly-flow regime using the electrolysis bubble generator.

The principle of the electrolysis bubble generator is quite simple. It involves generation of gas bubbles by electrolysis between two electrodes immersed in the liquid column. The important consideration in the present application is to generate a substantial volume of gas at low enough voltages. Hence the electrode configuration had to be evolved accordingly. In essence it consists (Figure 1) of a brass base plate carrying 76 hypodermic needles (no. 23, 30 mm). The plate and the combined needles were connected to a DC source, with the plate as the negative terminal. The specific advantage of this configuration is that the local distance between a 'needle' and the plate is minimized, resulting in high current densities for relatively low voltages. This would not be the case if, for example, only two electrodes are used. To obviate the possibility of a short circuit, the underside of the 6-mm brass plate was machined to take a Perspex sheet, which was secured with epoxy. The brass plate and Perspex sheet were drilled through to take the needles. However, each needle had to be insulated from the plate and this was accomplished by gluing tight-fitting PVC sleeves on to the needles. The needles were firmly fixed to the plate with epoxy cement, which also eliminated any leakage through the holes. Further constructional details of the bubble generator may be found in Arakeri *et al.*[4]

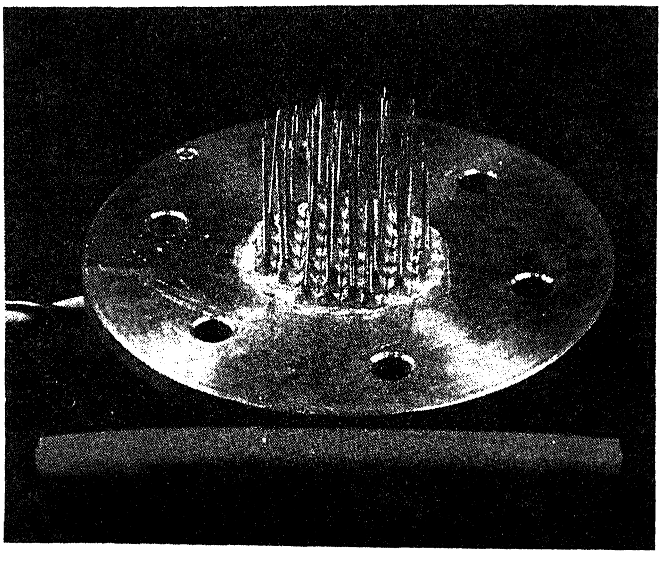

**Figure 1.** Electrolysis bubble generator in final configuration.

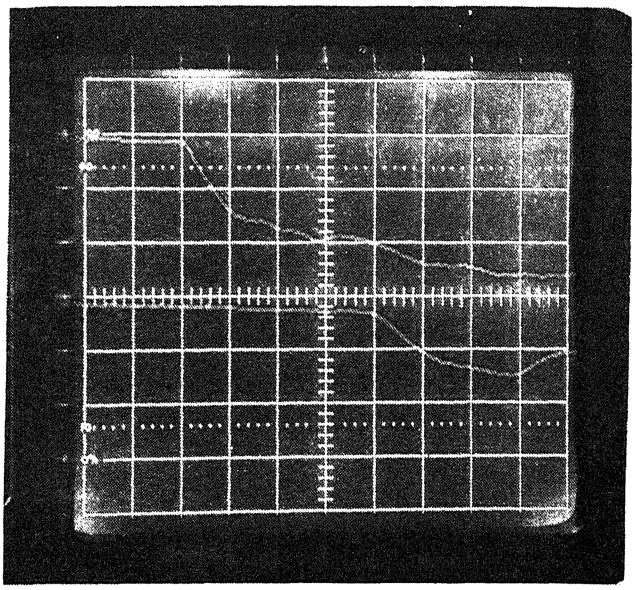

**Figure 2.** A typical stored signal on the digital storage oscilloscope screen. Perspex tube, $\alpha=0$, vertical scale = 0.1 V/div., sweep = 1 ms/div.

*Instrumentation*

Signals from piezoelectric pressure transducers were used to obtain the sonic velocity data presented here. Two transducers were mounted on the Perspex section of the column 2.403 m apart. The signals were stored on two channels of a digital storage oscilloscope. The oscilloscope used had the facility to examine the signal characteristics prior to and after trigger point. This is a very useful feature in the present application. A typical stored signal is shown in Figure 2; the points at which the pressure levels start to fall can be clearly seen. In the photograph shown, the time delay between the points where pressure starts to fall is 4 ms and its accuracy can be claimed to be within ± 1.25%. This directly translates to an accuracy of ± 1.2% in sonic velocity inferences in the Perspex section. The accuracies in the measurement in the brass section are also of the same order.

The value of the void fraction, $\alpha$, was obtained by measuring the static pressure difference over a distance of 2.403 m in the Perspex section. With certain approximations (like $\rho_g \ll \rho_l$), it is possible to show that $\alpha$ is given by

$$\alpha=\frac{\Delta p/\gamma_l}{h}. \tag{3}$$

Here, $\Delta p$ is the pressure difference between the two points in the column due to the presence of bubbles, $\gamma_l$ the weight density of the liquid, and $h$ the distance between the two points of measurement. A sensitive inductive pressure transducer along with its electronics was used to measure the values of $\Delta p$. The pressure-measuring system was appropriately calibrated and was able to measure $\Delta p$ values corresponding to fraction of a millimetre of water column. One of the problems associated with the accuracy was drift in the reading; but by studying its characteristics carefully it was possible to obtain results that were very reasonable, in the sense that for sufficiently low values of current there was linear relationship between current and void fraction.

## Theory of two-phase flows

It is interesting and useful to analyse the performance of the electrolysis bubble generator on the basis of drift flux theory for two-phase flows. The basis of this theory is contained in Wallis[5]. The drift flux, $j_{21}$, is defined as

$$j_{21}=U_{21}\alpha(1-\alpha), \tag{4}$$

where $U_{21}$ is the relative velocity between the phases and $\alpha$ is the void fraction. The drift flux $j_{21}$ is related to the individual fluxes $j_1=Q_1/A_c$ and $j_2=Q_g/A_c$ by

$$j_{21}=(1-\alpha)j_2-\alpha j_1, \tag{5}$$

where $A_c$ is the cross-section of the tube used and $Q_l$ and $Q_g$ are the volumetric flow rates of the liquid and gas respectively. In the present case $Q_l=0$ (i.e., bubbles are rising through a stagnant liquid column) and hence $j_1=0$, and also we are interested in the conditions where $\alpha\ll 1$. Hence

$$j_{21}\simeq j_2. \tag{6}$$

Empirically $j_{21}$ is known to have the behaviour

$$\frac{j_{21}}{U_\infty}=\alpha(1-\alpha)^n, \tag{7}$$

where $U_\infty$ is the terminal velocity of a single bubble in infinite fluid medium and $n$ is a constant. Again restricting ourselves to small $\alpha$ values, we get

$$\frac{j_{21}}{U_\infty}\simeq\alpha. \tag{8}$$

The terminal velocity $U_\infty$ can be obtained by considering a force balance on the bubble. Assuming the drag coefficient of a bubble to be the same as for a spherical solid particle (this assumption is valid if $Re_\infty=U_\infty d_b/\nu$ is less than about 30), the following equation results[5]:

$$24\,Re_\infty(1+0.15\,Re_\infty^{0.687})=\frac{4}{3}\frac{d_b^3 g}{\nu^2}. \tag{9}$$

Here, $Re_\infty$ is the bubble Reynolds number defined above with $d_b$ the bubble diameter, $g$ the acceleration due to gravity, and $\nu$ the kinematic viscosity of the liquid. For a given bubble size $d_b$, the above can be used to obtain $U_\infty$. Since $j_{21}\simeq j_2$, we obtain, from our earlier expression for $j_{21}$ and definition of $j_2$,

$$\alpha=\frac{Q_g}{A_c}\frac{1}{U_\infty}. \tag{10}$$

In the present case $Q_g$ refers to the volume of hydrogen gas liberated and can be estimated by applying Faraday's law[6] to the electrolysis process. By doing this, the following expression for $\alpha$ results:

$$\alpha=\frac{3.22\times10^{-3}\times I\,(\text{amps})}{U_\infty\,(\text{cm/s})}, \tag{11}$$

with $A_c$ in the present experiments being 19.64 cm$^2$. From the above, for different bubble sizes a linear relationship between $\alpha$ and $I$ is predicted and is quantitatively shown in Figure 3. Also shown are results of measured $\alpha$ versus the current $I$ and it can be seen that prediction agrees well with the measured behaviour for $d_b=0.2$ mm. In fact, magnified photographs of the bubbles did show a size very close to this value. Therefore, here we have a device whose performance can be predicted quite accurately from basic considerations. It is worth noting that for $d_b$ of 0.2 mm the $Re_\infty$ value is 3.4, and hence our assumption of drag of a bubble of this size being equal to the drag of a solid particle of the same size is quite accurate.

## Sonic velocity measurements

As noted earlier, the sonic velocity measurements were based on storing the pressure signal from a pair of transducers in the relevant test section with the help of a digital storage oscilloscope. The variation of measured sonic velocity in the brass and Perspex test sections as a function of void function is shown in Figure 4. The scatter in the data for the case of $\alpha=0$ with the brass

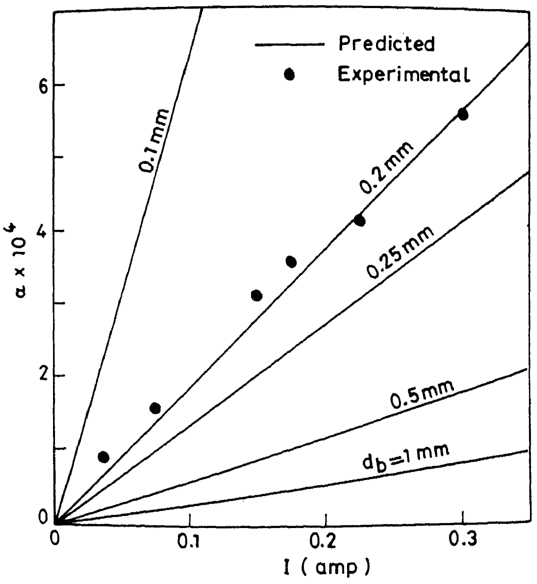

**Figure 3.** Predicted and observed performance of the electrolysis bubble generator.

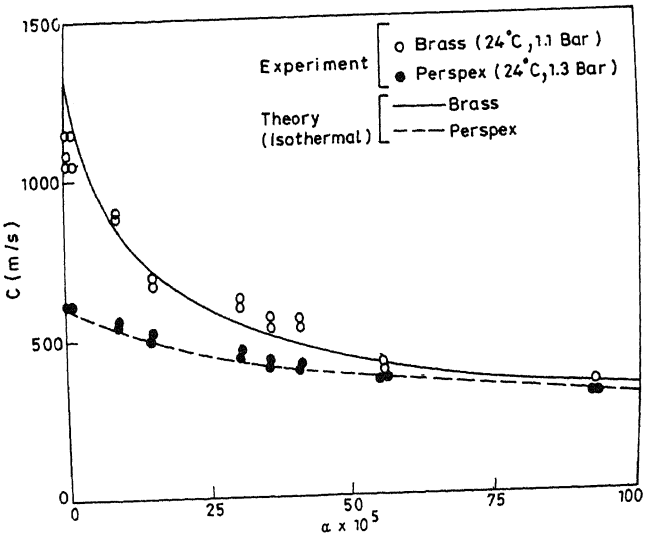

**Figure 4.** Variation of measured sonic velocity on void fraction and tube material. Also shown are theoretical results based on isothermal and homogeneous approximation.

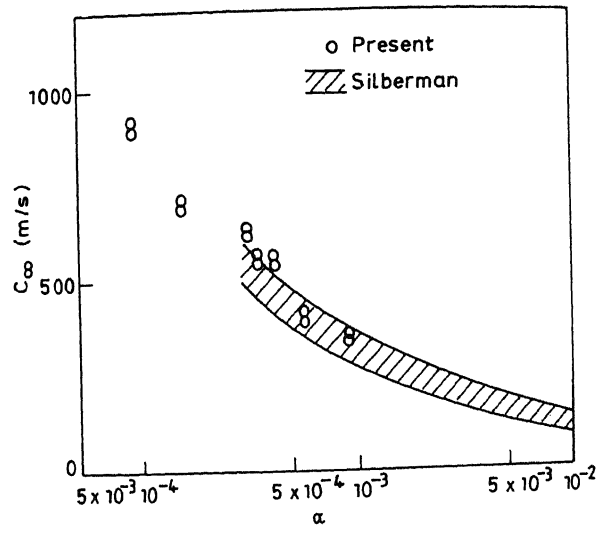

**Figure 5.** Comparison of present sonic velocity measurements with those of Silberman[7].

test section is due to the fact that it is very difficult to ensure that all the free gas has been removed in a liquid sample. The scatter is not an indication of the accuracy of the measurements; as pointed out earlier, this is better than 2%. The $\alpha$ values indicated are average values over the section of measurement. However, since the pressure decreases as we go up the column, the gas bubbles expand and hence there is a variation of $\alpha$ in the vertical direction. If it is assumed that gas expands isothermally, it is a simple matter to arrive at the change in $\alpha$ due to height variation. Over the test section height of the present experiments, if $\alpha$ is the value at the centre, then at the bottom $\alpha$ will be lower by about 6% and at the top it will be higher by the same amount. Thus the variation in $\alpha$ can be $\pm 6\%$.

The results shown in Figure 4 correspond to an average temperature of 24°C and an average pressure of 1.1 bar in the brass test section and 1.3 bar in the Perspex test section. It is clear from the results that, in the brass test section, sonic velocity is a very sensitive function of $\alpha$ at low void fraction values, whereas, in the Perspex section, it is not very sensitive. Therefore, sonic velocity at low void fractions is strongly dependent on tube material properties, as predicted by Mori *et al.*[3]

## Comparison with theory, other results

The present experimental results have been compared with the theoretical predictions on the basis of eq. (1). The assumption of isothermal expansion of gas bubbles at low void fractions can be justified since the two-phase mixture consists mostly of liquid. In theoretical computations, the value of Poisson's ratio $\mu$ was taken to be 0.36 for brass and 0.5 for Perspex, and the corresponding values for $E$ were 100 GPa and 3.3 GPa. There is some uncertainty in the value of $\gamma$ to be used, but use of extreme values showed an effect of 0.1% for brass and about 2% for Perspex. Similarly, the mean pressure $P$ also varies over the test section length and, again, use of extreme values shows an effect of about 4% for brass, whereas a significantly smaller effect is predicted for Perspex. Therefore the major uncertainty is still associated with the variation of $\alpha$ over the column height of the individual test sections. On the basis of average properties, the theoretical predictions are in excellent agreement with the measurements and this is apparent from the comparison shown in Figure 4.

The corrected results for the brass section on the basis of eq. (2) are compared with earlier measurements of Silberman[7] in Figure 5. The comparison shows good agreement with the general trends of dependence of sonic velocity on void fraction. The overall good agreement is however marred by the higher sonic velocities measured by us in the void fraction range of $3 \times 10^{-4}$ to $4.5 \times 10^{-4}$. This aspect needs to be examined further since the discrepancy was seen for three void-fraction values in that range and cannot be associated with scatter in the data.

## Concluding remarks

The electrolysis bubble generator developed presently can be a powerful tool for two-phase-flow studies at low void fraction. It has the necessary flexibility for controlled experimentation; in addition, its performance can be predicted from basic considerations. In the present experimental study, the device has been utilized to measure sonic velocity in the void-fraction range of $0.9 \times 10^{-4}$ to $10^{-3}$. The results in the two test sections show that sonic velocity in this void-fraction range is a very sensitive function of tube elasticity and void fraction. The measured values agree well with theoretical predictions based on isothermal and homogeneous approximations.

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# Some characteristics of point discharge current during two pre-monsoon season thunderstorms at Pune

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**Measurements of point discharge current were made at Pune during two pre-monsoon season thunderstorms of 1987 and 1988. The monthly distribution of the number of days of thunderstorms in the pre-monsoon months of 1987 was seen to vary from that of 1988 though the total number of seasonal thunderstorm days on both occasions were equal. The daywise features of point discharge current on the days of thunderstorm during 1987 and 1988 are presented. Normalized frequency distribution of spells and diurnal time duration of point discharge current and the values of charge received by the earth are also presented.**

DURING the periods of thunderstorms when strong electric fields exist, transfer of negative/positive charge to the earth through point discharges from the surface irregularities takes place. Among conduction current, precipitation current, lightning and the point discharge current which contribute to the transfer of charge to the earth, the point discharge current plays a prominant role since its single contribution to the charge transfer is markedly higher than those of the rest[1]. Further, the studies[2-11] made of the ratio of negative to positive charge transferred to the earth by point discharges indicate that it is varying in the range 1.3 to 2.9. These studies indicate that, perhaps, this ratio is greater in tropical and subtropical regions than in the temperate regions. Also, since the total number of daily thunderstorms taking place[12] all over the earth's surface is $44 \times 10^3$, the importance of the process of point discharge current in the transfer of charge to earth as well as for the global electric budget becomes much more evident.

It has been reported[13] that the separation between charge centres of a thundercloud due to strong wind shears can extend up to 100 km. Thus the positive charge centre is out on its own ahead of the storm and discharges to ground take place directly. Asuma *et al.*[14] confirmed the earlier views that point discharges at ground level modified ground-level electric field and enhanced the mirror-image effect at the surface more than above. Studies[15] have indicated that thunderstorms often originate in preferred regions of topography and initiation sites tend to cluster into identifiable geographic locations or genesis zones. Imyanitov[16] showed that electrical properties of thunderclouds differ appreciably from those which were supposed earlier and he established the existence of thunder phenomena even in the stratiform clouds.

Several investigators have taken an account of the negative to positive charge ratio and the net balance charge received by 1 km$^2$ area of the earth by point discharges in thunderstorm situation for subtropical[2-6] and tropical[7-11] regions. Even considering the importance of the subject, we have carried out a study of the observations of point discharge current at Pune (18°32′N, 73°51′E, 559 m asl) during pre-monsoon seasons (March–June) of 1987 and 1988. The results are presented here.

The point discharging element consisted of a platinum/10% iridium needle 0.5 mm in diameter and about 2 cm long erected at a height of 14 m above the ground level. Current through the needle was carried by a coaxial cable and fed to an operational amplifier system. The output was given to an 1 mA strip chart recorder run at 1 cm per minute and a continuous record of point discharge current was obtained. The