type climate for the first time in 1965, with frequent recurrence thereafter until the end of 1980. The increasing trend in aridity index and decreasing trend in moisture index are statistically significant. The negative trend shown by the monsoon rainfall is not statistically significant.

Hingane *et al.*[12] studied the long-term trends of surface air temperature over India and reported a pronounced warming in the mean annual temperature over the northcentral and northeast Indian regions. The Mahanadi basin is in close proximity of the region of significant warming observed by them. Further, we have noticed significant decreasing trends in the pre-monsoon and post-monsoon seasonal rainfall (details not presented here). These factors can contribute to the observed significant trends in the yearly aridity and moisture indices.

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RESEARCH COMMUNICATIONS

A mathematical prey–predator model for tree and man

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A prey-predator model can be developed to preserve the environment with two interdependent variables, labelled as man and tree. The parameter tree is measured in terms of the land covered by vegetation. The system of simultaneous differential equations representing the interaction is nonlinear. The equilibrium condition provides the critical values for their populations. The nonequilibrium behaviour of this interacting system is studied as a perturbation from the equilibrium state. When a portion of the land is continuously made unusable the population of man irreversibly goes to zero.

DEFORESTATION and the subsequent loss of environmental protection is a matter of serious concern in the study of ecology. One has reason to believe that under desirable conditions there should be an upper limit for the population of man and a lower limit for trees[1,2]. We were motivated to attempt this problem on the World Environmental Day. In this article we present our mathematical analysis of this problem. Our approach to this problem is similar to the original work by Volterra applied to the case of foxes and rabbits[3–8].

There are several factors which contribute to the growth and decay of the two competing variables, man and tree. These factors make the problem of man–tree coexistence a very complex one. Nevertheless we can make a beginning by starting with a simple model that qualitatively describes the system. We need to make reasonable assumptions and approximations in the first stage of the problem. To begin with, we simplify the many-variable problem into a two-variable problem by making the following assumption — the variable man is actually a function of many factors which result in the destruction of trees and the variable tree is a function of many variables which are resourceful to man. In simple words we assume that man always benefits from trees either directly or indirectly at the cost of the trees. In this way we build the variable tree as a representative of agricultural farms, forests, etc. which are of biological origin, enabling the survival of mankind. The variable man represents many factors such as human beings, pests and other creatures which thrive on green food.

Accordingly we have made a crude approximation based on the fact that human beings form the single most important factor affecting the vegetation.

Let us understand the variables more carefully. Can the tree be a sole representative of different trophic levels like forests and croplands? The ecosystem is schematically shown in Figure 1*a*. This figure is a representation of the energy flow through the system[9]. There is a source for an unlimited supply of energy. The total cultivable land L_0 restricts the inflow of energy. This land consists of different autotrophs—the forests (L_a), croplands (L_b) and others (L_c). The produce from these autotrophs gives rise to the biomasses (B1, B2, B3)[10]. The biomass is, in turn, converted to usable energy for man by obtaining the product between the biomass, its calorific value and the efficiency[11]. The man converts the usable energy into unusable form. In this ecosystem the two extreme variables are the land and the man. Therefore we use these two variables in our analysis. From the ecologists point of view it is necessary to investigate the specific interactions between man and the biomasses to obtain quantitative results, but that is the second stage of the problem. We attempt to set up the first stage of the problem which is more directly addressed to general readers and environmentalists.

Formulation of the problem

We begin with a representation of how the trees grow. In the absence of man the rate of growth of trees can be written as,

$$\frac{dL}{dt} = \alpha (L_0 - L), \tag{1a}$$

where L_0 is the total cultivable land, L the land occupied by trees at a time t; t, the time in years; α the growth rate constant.

The equation (1a) is a representation of a self-limiting growth. While the earlier analyses involving foxes and rabbits or lynx and hares etc.[3] have assumed unlimited growth of the prey in the absence of the predator, equation (1a) stands for a self-limiting growth. This is more realistic from a physical consideration given the constraints of resources[12, 13]. It is seen from equation (1a) that the growth rate of trees becomes zero once the land is full of trees. Figure 1*b* schematically shows the self-limiting and unlimited growths. The self-limiting growth is not a density-dependent growth[14]. While the density-dependent growth is an intrinsic property of the growing species, the self-limiting growth is due to the limited resources available for growth.

The density independent equation does not give a realistic behaviour at extreme values of L. A slightly better equation is

$$\frac{dL}{dt} = \alpha L (L_0 - L), \tag{1b}$$

which follows the curve for unlimited growth at low densities but switches over to self-limiting growth when land becomes a constraining factor. For nominal values of L the behaviour of both functions is the same and so, for the time being, we will retain equation (1a).

Now we introduce the variable man in equation (1a) and get,

$$\frac{dL}{dt} = (\alpha - \beta m)(L_0 - L) \tag{2}$$

The parameter β represents the contribution to the rate of growth of trees due to a reduction in the land of trees. One can integrate equation (2) and find that for a fixed m at a given time the amount of trees in the

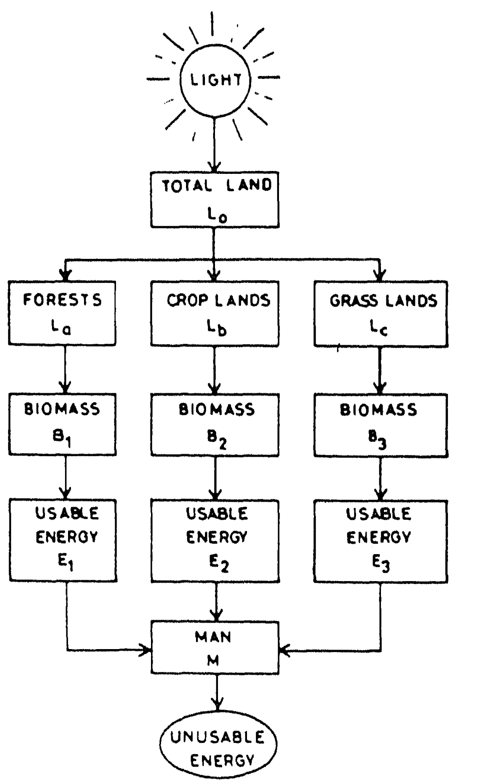

Figure 1*a*. Energy flow diagram representing the ecosystem of man and tree.

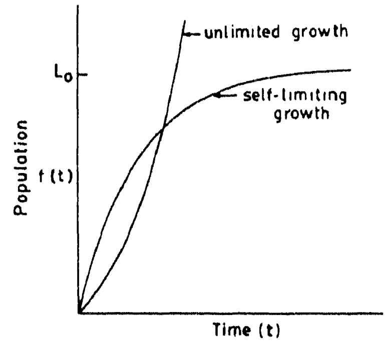

Figure 1*b*. In unlimited growth the population goes to infinity as time increases. In self-limiting growth the population approaches a finite value (e.g. L_0) as time increases.

presence of man is less than the amount of trees which would have been in the absence of man.

The rate of growth of man is taken as,

$$\frac{dm}{dt} = -(\Gamma - \sigma L)m. \qquad (3)$$

The population of man will decay exponentially when $L=0$ (i.e. when there are no trees on earth). The parameter σ actually represents rate of harvest of the land of trees for the sustenance of man.

We have formulated two equations (2, 3) which will form the basis of our analysis. The coefficients $\alpha, \beta, \Gamma, \sigma$ are assumed to be positive real values which implies that the interaction between man and tree is symbiotic in nature. Symbiotic relationships as in the case of algae and fungi (lichens) are well known. While the existence of tree is beneficial to man, in what way does the presence of man offer a 'protection' to the tree? In order to enable his own survival, man protects and cultivates certain specific species of vegetation. For example, in the absence of protection to paddy, weeds may take over the land.

One can derive the equilibrium condition from equations (2) and (3) by separating the variables and integration. The equilibrium condition is given by

$$e^{\sigma(L_0 - L)} \cdot (L_0 - L)^{(\Gamma - \sigma L_0)} = \mathrm{K} m^{\alpha} e^{(-\beta m)}. \qquad (4)$$

Equation (4) is a special case where the parameters m and L can maintain their values indefinitely. At this equilibrium condition the value for m is α/β and the value for L is Γ/σ. Therefore the critical values for the populations are given as ratios of the constants involved in the equations. At equilibrium there is no addition to the population of tree or man.

Are the various parameters α, β, Γ and σ totally independent of each other? The equilibrium value of L given by Γ/σ is a fraction of the total land L_0. Therefore in our analysis L_0 is the normalizing quantity. The parameter β is related to σ by some function since both carry the information on reduction of vegetation. While α and Γ are independent, β is related to σ and σ is the parameter that greatly determines the evolution of the system with time. We will address the behaviour of the system for different values of these parameters in a separate work.

We are also aware that a system like the above can be taken far from its equilibrium. Therefore we have to work out a more general solution away from this equilibrium point. We apply the following transformations to equations (2) and (3)

$$m = \frac{\alpha}{\beta} + m_1; \quad L = \frac{\Gamma}{\sigma} + L_1. \qquad (5)$$

In equation (5) the parameters m_1 and L_1 are deviations from the equilibrium point. The transformation moves the equilibrium point in the (m, L) plane to the origin. After applying the transformation we get,

$$\frac{dL_1}{dt} = -(\beta m_1)\left(L_0 - \frac{\Gamma}{\sigma} - L_1\right) \qquad (6)$$

$$\frac{dm_1}{dt} = \left(\frac{\alpha}{\beta} + m_1\right)(\sigma L_1) \qquad (7)$$

Equations (6) and (7) represent a situation where the system in equilibrium, as determined by the constants $\alpha, \beta, \Gamma, \sigma$, has been perturbed by certain amounts given by m_1 and L_1. The time evolution of this perturbed system is of interest. Basically this means we want to find out how the functions $L_1(t)$ and $m_1(t)$ look like.

Unfortunately the nonlinear system of equations (6), (7) cannot be solved analytically. Nevertheless we can numerically evaluate the functions $L_1(t)$ and $m_1(t)$ using the Runge–Kutta method. Here we have to consider two cases while numerically evaluating these functions—one in which the initial deviations L_1 and m_1 are small and the other in which they are large.

Figure 2 shows the functions $L_1(t)$ and $m_1(t)$ for small initial deviations. Here we have made some reasonable assumptions for the values of constants as given in the figure captions. The time axis is scaled as $(1/\alpha)$ in order to make it dimensionless. Observe that the functions are sinusoidal with a phase difference. The functions differ by a phase $\pi/2$. The result can also be obtained from analytically evaluating equations (6), (7) after linearizing them. The solutions analytically obtained for small initial deviations are:

$$L_1(t) = (1/\sqrt{2})(\sin(2\pi f t + \pi/4)) \qquad (8)$$

$$m_1(t) = (1/\sqrt{2})(\alpha\sigma/2\pi f \beta)(\sin(2\pi f t - \pi/4)), \qquad (9)$$

where the frequency $f = \sqrt{(\alpha\sigma(L_0 - \Gamma/\sigma))}$.

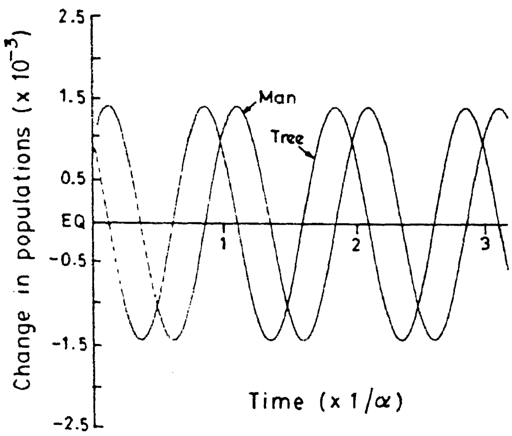

Figure 2. The oscillating solution obtained numerically for small deviations from the equilibrium values. The various parameters have the following values: $(L_0 - \Gamma/\sigma) = 1.0$; $\alpha/\beta = 1.0$; $\sigma = 1.0$; $L_1(0) = 0.001$; $m_1(0) = 0.001$.

The time independent equation of state obtained after linearizing is

$$\alpha\sigma L_1^2 + \beta^2 (L_0 - \Gamma/\sigma) m_1^2 = C, \qquad (10)$$

where C is a constant. Equation (10) represents a family of ellipses which describes an oscillating solution to the perturbed man–tree problem in terms of elementary functions.

We can also investigate the other case where the initial deviations are of large magnitude. In other words we analyse the case where the populations are far from their equilibrium values. We can only obtain $L_1(t)$ and $m_1(t)$ numerically. Figure 3 shows these functions graphically. A comparison of Figure 3 with Figure 2 immediately reveals that the oscillations are no more sinusoidal. We find that within each cycle the exponential terms dominate. We also find that the period of oscillation is only marginally different in spite of the large magnitude of the applied perturbation [for the same values of constants, i.e. $(L_0 - \Gamma/\sigma) = 1.0$; $\alpha/\beta = 1.0$; $\sigma = 1.0$)]. Yet another aspect is about the amplitude of oscillations. Unlike Figure 2 we see in Figure 3 the amplitudes are unequal although the applied initial perturbations are equal in magnitude ($L_1(0) = -0.98$; $m_1(0) = 0.98$). Within each cycle there is a rapid fall in the population of man, while the recovery to the equilibrium value and above takes much longer.

Finally, one needs to know the behaviour of the system when there is a continuous fall in the total cultivable land given by L_0. There could be many reasons for a decrease in L_0. For example, the construction of concrete buildings, radiation from atomic plant failures, laying of pathways, urbanization and so on can contribute to its reduction. If L_0 decreases with time owing to the above reasons by a quantity εm, then the total arable land at any given time takes the form:

$$L_{\text{total}} = L_0 - \varepsilon \int m \, \mathrm{d}t, \qquad (11)$$

where ε is the contribution from every human being to the loss of cultivable land. Now equation (6) becomes

$$\frac{\mathrm{d}L_1}{\mathrm{d}t} = -(\beta m_1)\left(\frac{\Gamma}{\sigma} L_{\text{total}} - L_1\right). \qquad (12)$$

We have equation (7) for m,

$$\frac{\mathrm{d}m_1}{\mathrm{d}t} = \left(\frac{\alpha}{\beta} + m_1\right)(\sigma L_1). \qquad (7)$$

The three equations (11), (12) and (7) are coupled equations and they can be solved numerically. It is quite interesting to see that there are two differential equations coupled to an integral equation. Figure 4 shows the evolution of the system for a set of values of the parameters. The oscillating behaviour of the perturbed system is visible from the figure. The fall in the populations is permanent unless the wasteland is reconverted back into increasing L_0. While this point is intuitively obvious, a mathematical touch could now be given to the arguments for the preservation of the environment and the need for a green revolution.

In this work the man–tree problem has been reduced to a geometrical problem of limited space. This framework of the problem can be expanded by considering the biomass as the variable, instead of land, to obtain quantitative results. The change of variable from land to biomass involves a transfer function and results are obtainable in a straightforward manner. In our analysis we have assumed an exponential growth form for the interacting species. We have arrived at similar results as above, by assuming growth functions different from the exponential function and we find that the exponential function is a satisfactory representation for the growth function in addition to being the easiest to work with. Finally, the conclusions obtained in this work are also applicable to other systems of two

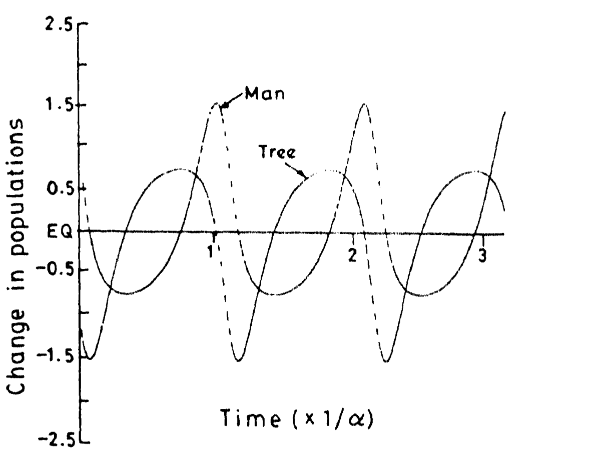

Figure 3. The behaviour of $L_1(t)$ and $m_1(t)$ for large initial deviations: $L_1(0) = -0.98$; $m_1(0) = 0.98$.

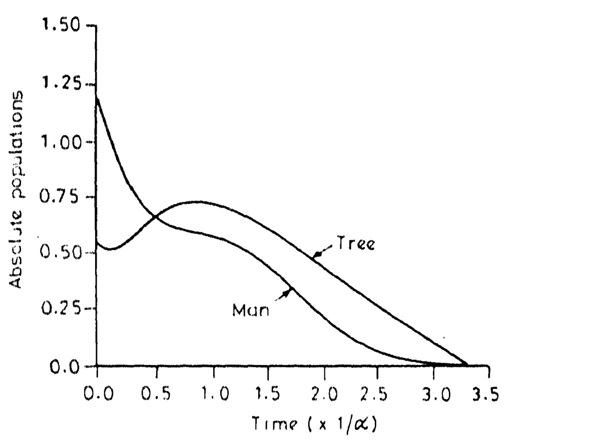

Figure 4. A permanent decrease in the populations of man and tree occurs when a fraction of the total land is permanently rendered waste: $\varepsilon = 0.1$; $m_1 = 0.2$; $L_1 = -0.2$; $\Gamma/\sigma = 0.75$; $L_0 = 1.0$.

interacting variables where one of them has a self-limiting growth.

We invite from the readers, any comments, criticisms as well as suggestions towards a further progress in this important problem which is both scientific as well as social.

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On the occurrence of lava flows from Krol Formation, Narendranagar area, Garhwal Himalayas, Tehri–Garhwal district, Uttar Pradesh

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We report here the occurrence of lava flows in the Krol Formation, hitherto thought to be 'essentially a carbonate facies with evaporite beds in the middle section'[1].

THE generalized geological map of the area around Narendranagar with locations of volcanic flows is given in Figure 1. Table 1 gives generalized lithological characters of the mapped sequence.

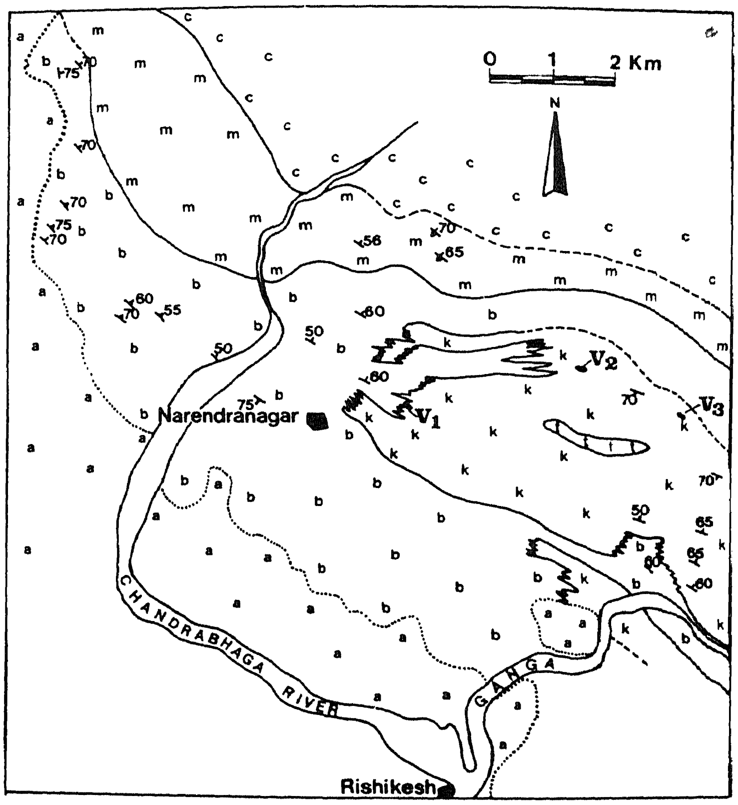

Figure 1. Geological map of the study area. c: Chandpur Formation, m: Mandhali Formation, b: Blaini Formation, k: Krol Formation, t: Tal Formation, a: Alluvium. V_1, V_2, V_3 are the locations of the lava flows.

Table 1. Lithological characters of the sequence exposed in the study area

Formation	Lithology
Tal Formation	Light grey to greenish grey splintery shale
Krol Formation	Light to dark grey and bluish grey limestone and dolomite with interbands of red, light to dark grey and occasionally black shale. Occurrences of thin lava flows and minor gypsum
Blaini Formation	Diamictite, quartzite, grey and purple shade with lenticular limestone
Mandhali Formation	Dirty white quartzite and dark grey phyllite
Chandpur Formation	Dark grey slate, phyllite and subordinate quartzite

The highly altered lava flows are brownish green to dark brown in colour with thickness varying from a fraction of a metre (about 30 cm) to about a metre. They have fine grained, aphanatic texture and sparsely amygdaloidal character.

In thin sections, the samples exhibit fine grained, merocrystalline character and have dominantly subophitic, intersertal, intergranular and occasionally inequigranular, porphyritic texture.

When porphyritic, the microphenocrysts of plagioclase ($\leqslant 5\%$ by volume) usually occur as individual grains and are seldom seen forming glomerophyric