On the holographic simulation of non-circular zone plates

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Abstract. New methods involving the manipulation of fundamental wavefronts (e.g., plane and spherical) with simple optical components such as pinholes and spherical lenses have been developed for the fabrication of elliptic, hyperbolic and conical holographic zone plates. Also parabolic zone plates by holographic techniques have been obtained for the first time. The performance behaviour of these zone plates has been studied. Further a phenomenological explanation is offered for the observed improved fringe contrast obtained with a spherical reference wave.

1. Introduction

Zone plate is a fascinating optical element and its potential applications are many and varied, since it combines the action of a lens as well as a diffraction grating. The early work about the optical behaviour of zone plates has been carried out about a century ago (Soret 1875; Cornu 1875). Since then a good amount of work on zone plates has been done and a fair amount of literature has been accumulated (Wood 1898; Myers 1951; Miyamoto 1961; Kock *et al* 1966; Hart *et al* 1966; Waldman 1966; Horman and Chau 1967; Stigliani *et al* 1967; Champagne 1968; Chau 1969; Dammann 1970; Rudolph and Schmahl 1970; Chang and George 1970; Moran 1971; Young 1971; Markus 1972; Barrett and Horrigan 1973; Kirz 1974).

A zone plate in its basic form consists of a series of alternate transparent and opaque zones on a suitable substrate. For example, the simplest way to make a zone plate is as follows. Concentric circles, with radii proportional to the square roots of whole numbers, are drawn on white paper. Alternate zones are then blackened and its photograph is taken. The transparency, when illuminated with a parallel beam of light, produces a large intensity at a point on its axis at a distance corresponding to the size of the zones and the wavelength of the light used. The photographic type zone plates could also be produced by taking advantage of the phenomenon of interference of light waves and zone plates thus obtained could be classified as regular interferometric zone plates (IZPs) or holographic zone plates (HZPs), depending upon the spatial resolution of the medium used for recording the interference fringes.

Of course, there is the mechanical way, analogous to the conventional way for

making ruled diffraction gratings, for the fabrication of zone plates and they are designated as geometric zone plates (GZPs).

A perfect sinusoidal variation of intensity for the zones could be recorded in the case of HZPs, in view of the fact that in holography, unlike in conventional inter-ferometry, one uses a photographic emulsion (or other suitable recording medium) of required high resolution and dynamic range (i.e., required linear portion of the *E-t* curve) in order to faithfully record all the modulations of intensity available in the interfering wavefronts. Hence an HZP will not produce multiple diffraction orders; whereas a GZP or IZP would produce multiple orders in view of the fact that their zones possess square wave periodicity. The square wave periodicity in an IZP is a consequence of the use of low spatial resolution emulsion. In other words, the modu-lations available between I_{max} and I_{min} are recorded more or less in a binary fashion. Further an HZP will not produce ghost images as a GZP would and in view of all these advantages HZPs are becoming increasingly popular optical elements in a wide variety of optical systems. As a matter of fact, a hologram is nothing but a zone plate in action and this fact was demonstrated by Rogers (1950) on the heels of the discovery of holography by Gabor.

It is a common practice to designate a zone plate as circular or linear depending upon whether the zones are circular or straight. We can easily conceive of non-circular zone plates by following this conventional nomenclature. Thus zone plates containing elliptic zones will have to be designated as elliptic zone plates etc. Circular and linear zone plates are relatively easy to fabricate and hence their proper-ties have been studied rather extensively. In recent years, it has been discovered that non-circular zone plates (e.g., elliptic, hyperbolic, etc.) do find applications in optical computing systems, for example, and hence some attempts have been made in the past to fabricate non-circular zone plates by employing holographic methods (Fredricks 1969, 1970; Iwasaki 1976; Aggarwal *et al* 1978). The major drawback of these methods is that the required object wavefront is derived by a complicated optical set-up involving the use of cylindrical/conical lenses.

It is found that the non-circular HZPs (viz., elliptic, hyperbolic and conical) could be produced relatively effortlessly by using a simple optical system as shown in figure 1, wherein only the manipulation of fundamental wavefronts (viz., plane and spheri-cal) by simple optical elements such as pinholes or simple spherical lenses is involved. As a matter of fact these fundamental wavefronts are referred to by Stroke (1969) as the building blocks of holography. Recently, we have utilised these fundamental wavefronts for the holographic simulation of a wide variety of mirrors (Ananda Rao and Pappu) and now we have used them for the generation of non-circular HZPs.

The purpose of this paper is to describe new and simple methods for the fabrication of several non-circular HZPs and discuss their performance. Further, an explanation is offered for the difference in fringe contrast observed between the utilisation of a spherical wave and a plane wave as a reference wave.

2. Experimental

The desired object and reference wavefronts are obtained by using the set-up shown in figure 1. We have used in our experiments a 0·5 mW Spectra Physics Model 155 He-Ne laser as the source and Agfa 10E75 photographic plates as the recording media.
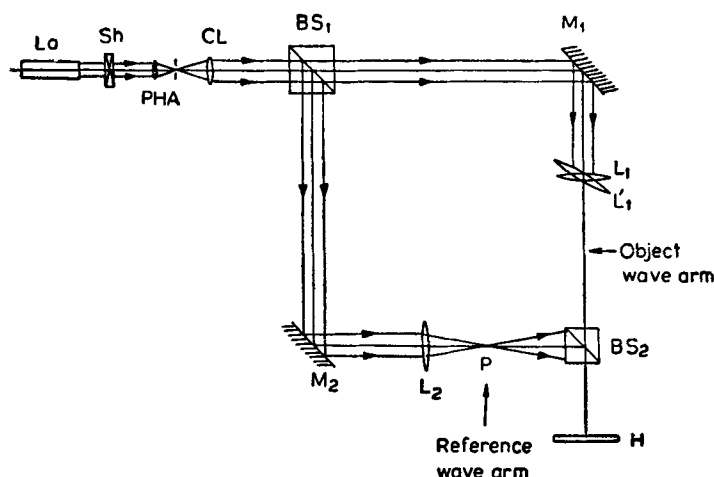
Figure 1. Schematic of the set-up used for the construction of various types of holographic zone plates (HZPs). La=laser, Sh=shutter, PHA=pinhole assembly, CL=collimating lens, BS_1 and BS_2=beam splitters, M_1 and M_2=mirrors, L_1=simple spherical lens in the object wave arm, L_1'=tilted position for the lens L_1, L_3'=simple spherical lens in the reference wave arm, P=point source derived with lens L_2, H= holographic plate.

The salient features about the utilisation of the set-up shown in figure 1 are as follows. We could derive a plane or a spherical wave as reference wave just by removing or keeping in place the lens L_2. Wtih lens L_1 in place, we get the facility to mix a spherical object wave with either a plane or a spherical reference wave. Under these circumstances, circular fringes will be formed, which when recorded on plate *H*, will yield a circular zone plate.

The desired non-circular (e.g., elliptic and hyperbolic) object wavefronts could be generated effortlessly just by tilting the lens L_1 through a suitable angle as shown schematically by L_1' in figure 1. These non-circular object wave surfaces could then be mixed with either a plane or a spherical reference wave and the resulting fringe system could be recorded on the photographic plate *H*. Upon processing, the plate will function as a particular type of zone plate (e.g., elliptic, hyperbolic, etc.) depending upon the nature of the contours of the fringes (viz., elliptic, hyperbolic, etc.) recorded on it.

The HZPs in our experiments have been realised by using the on-axis method of recording. The performance of the zone plates is checked under plane wave illumination. During this reconstruction schedule, however, a simple innovation of illuminating the HZP away from its central region has been incorporated and this has given the advantage of separating out the various images from the DC region. In other words, the benefits that would have accrued had we recorded the HZPs by the conventional Leith and Upatnieks off-axis method have been realised effortlessly by our simple trick.

3. Results and discussion

Before we proceed with the actual realisation of the desired non-circular HZPs, it is necessary to know the nature of the object wave surface emerging from the tilted lens

in figure 1.   It is discovered that the foolproof way of finding out the nature of the
wave surface existing in space is to mix it with a plane reference wave and then make
a record of the resulting fringe system.   By this procedure, the output wave surfaces
emerging from the tilted lens at various points of their propagation path have been
identified to follow the sequence: ellipsoidal waves—vertical line focus—hyperboloidal
waves—horizontal line focus—ellipsoidal waves again, as shown in figure 2.

With a spherical reference wave, however, it was not possible to ascertain unambi-
guously the nature of the wave surface existing in space, because the fringe system
obtained in this case is found to be dependent upon the location of the point souree
P in figure 2 and thus the fringe system observed is sometimes different from those
presented in the sequence shown in figure 2.

In the light of our observations, we conclude that a plane reference wave is ideally
suited for an unambiguous identification of a wave surface existing in space.   Hence
a plane reference wave could be called as, to coin a phrase, an Analyser wave.

3.1   *Elliptic and hyperbolic HZPs*

The principles of operation of elliptic and hyperbolic zone plates are essentially the
same, in that both give rise to a vertical and a horizontal focal lines.   But there can
be situations where an elliptic zone plate is to be preferred to a hyperbolic zone plate
and vice-versa.   Hence we have fabricated both elliptic and hyperbolic HZPs sepa-
rately and checked their performance.

The elliptic HZPs are obtained by recording the elliptic fringes shown in figure 2
on a suitably located holographic plate.   Similarly, hyperbolic HZPs are realised
by recording the hyperbolic fringes shown in figure 2.   We have chosen to show, as
an example, the results obtained with a hyperbolic HZP when it is illuminated with
a plane wave and the results are shown in figure 3.   The important features to note
are the evolutions of the vertical line focus and the horizontal line focus through the
intermediate stages of elliptic/hyperbolic waves.   The noteworthy features of our
results, in contrast to the results of earlier workers who have used the off-axis method
for the realisation of zone plates (Aggarwal *et al* 1978), is the unambiguous revelation
of the character of the wave surfaces that exist in space as a function of propagation
distance during reconstruction.   This revelation has been possible because of the
interference effects between the DC component and the output wave.

It should be mentioned here that there is still simpler method for the realisation of
elliptic HZPs.   This involves the use of the set-up of figure 1 in the configuration for
the realisation of a circular zone plate and then tilting the holographic plate *H*
suitably in order to record the elliptical fringes.   The principle that the geometrical
projection of a circle onto an inclined plane is an ellipse is exploited here.   It should
be obvious that hyperbolic HZPs cannot be produced with this method.

The imaging performance of a hyperbolic HZP has been studied using a trans-
parency containing a triangle as the object.   The images obtained are displayed in
figure 4.   Similar results are naturally to be expected from elliptic HZPs also and
they have been obtained.

3.2. *Conical HZPs*

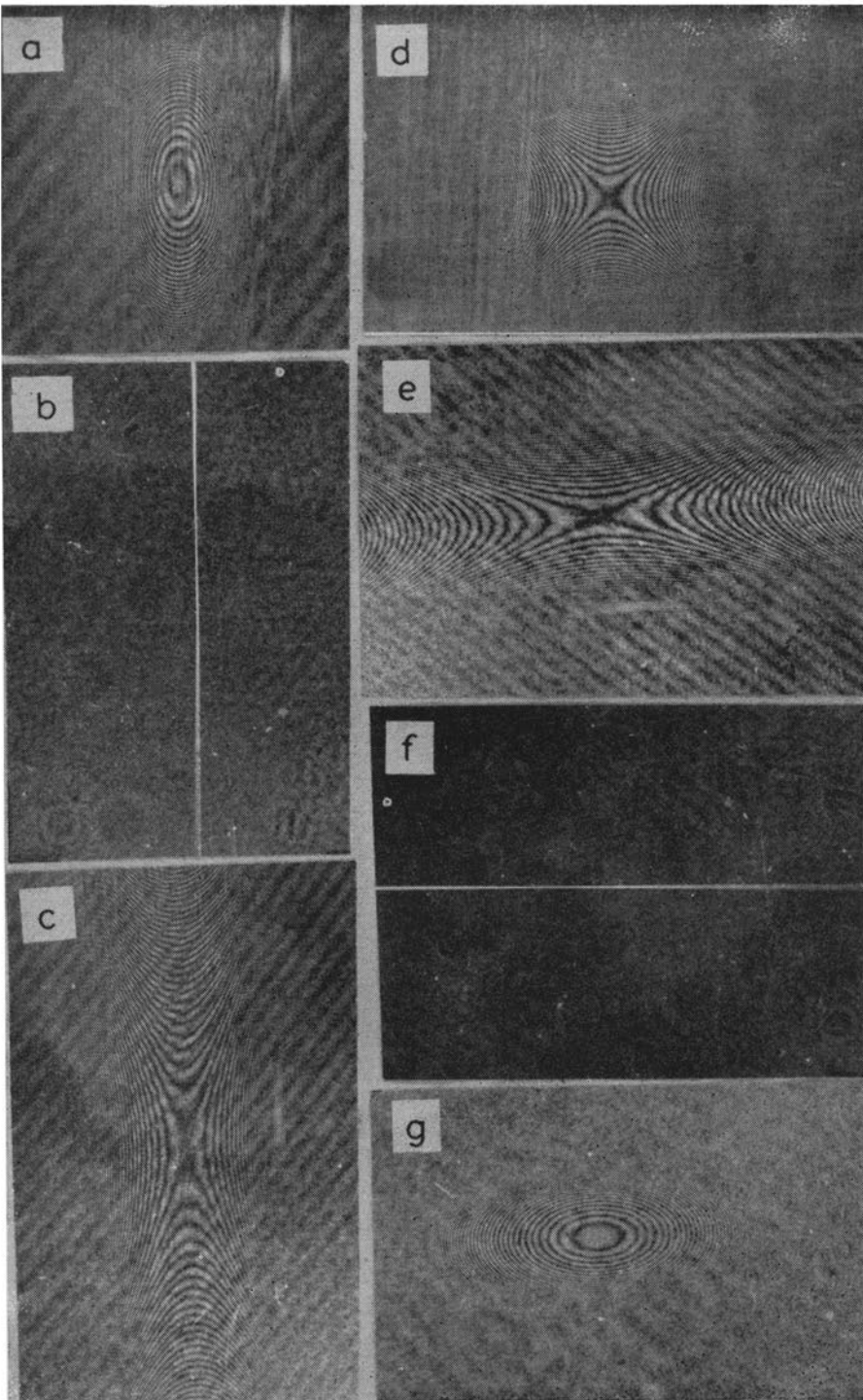A conical HZP is realised by the following method.   The lens L_1 in its normal

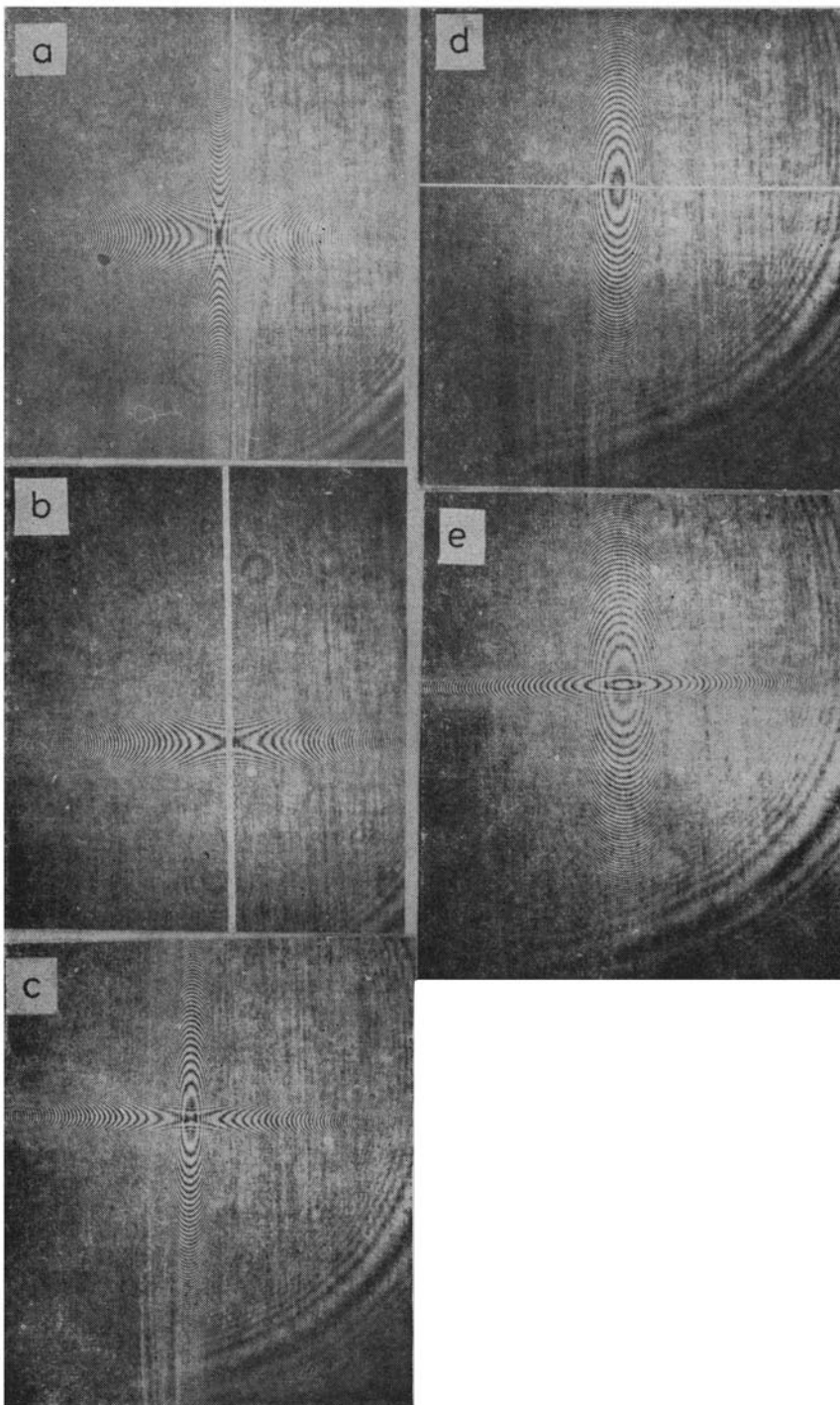**Figure 2.** (*See caption in p.* 366)

Figure 3. (a) to (e) represent the sequence of reconstructed wavesurfaces, as a function of propagation distance, obtained from a hyperbolic HZP upon its illumination with a plane wavefront. Since the HZPs are derived by employing the on-axis methods, the observed fringe system is to be expected because of the interference of the DC component with the output wave. Note the expected formation of the vertical and the horizontal focal lines respectively in figures **3(b)** and **3(d)**.
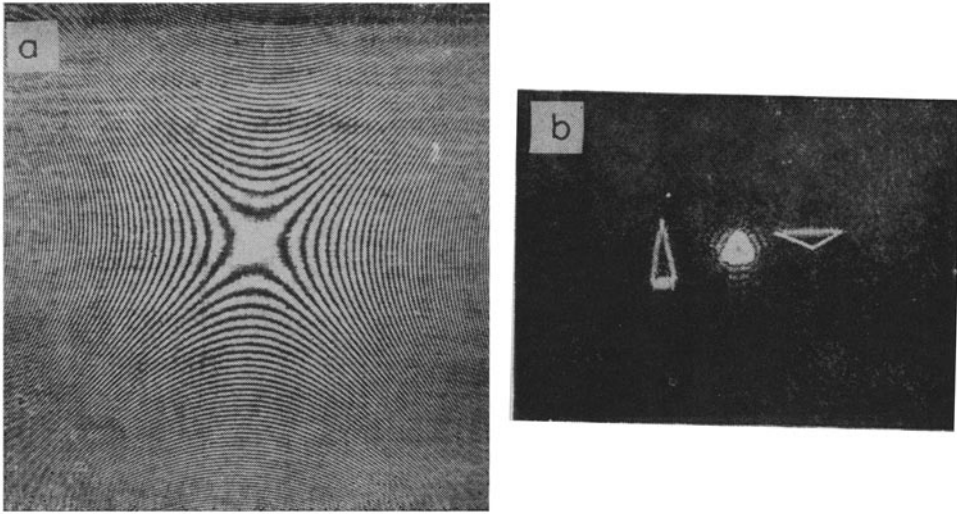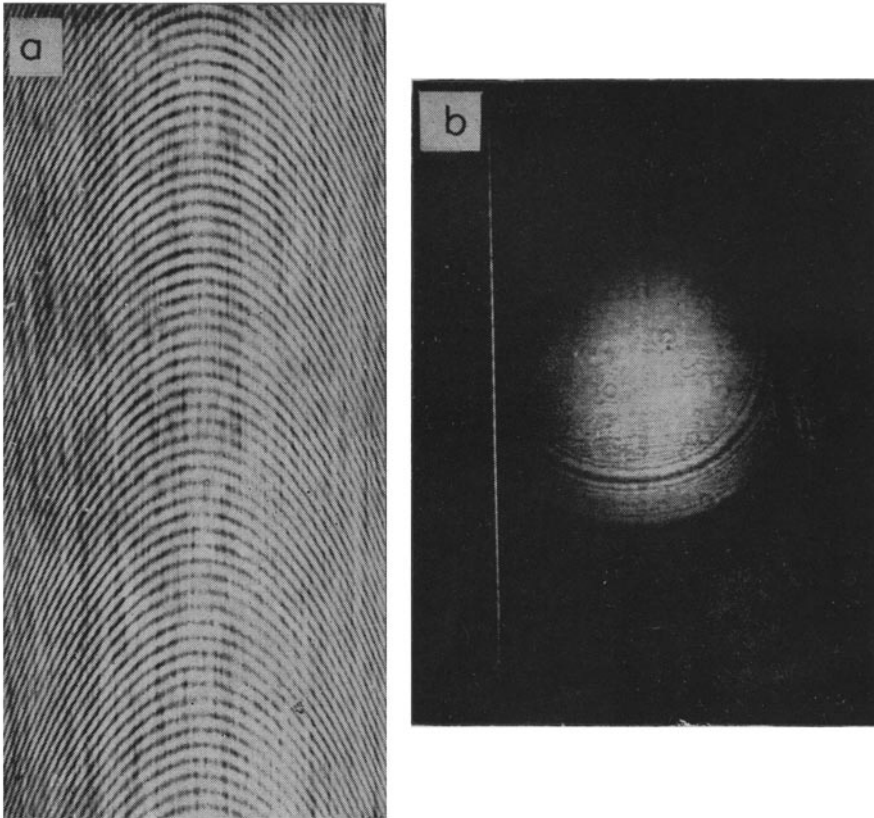
Figure 4,

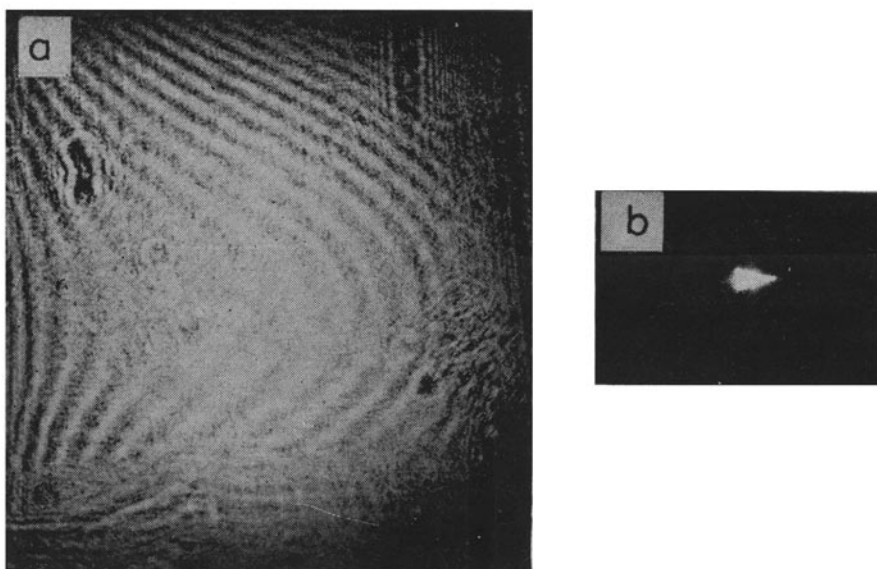Figures 4 & 5. *(See captions in p.* 366)

Figure 6. Performance of a Parabolic HZP. (a) Photograph of a parabolic HZP. (b) Line focus obtained in reconstruction from it which again is illuminated (to conform to the off-axis mode of reconstruction) with a plane wavefront. Here the DC is also shown.

Table 1. Summary of the fringe system observed and the zone plate realised

Object Wave	Reference Wave	Fringe System Observed	Zone plate Realised
Elliptic	Spherical	Elliptic or hyperbolic	Elliptic or hyperbolic
Elliptic	Plane	Elliptic	Elliptic
Hyperbolic	Spherical	Hyperbolic or elliptic	Hyperbolic or elliptic
Hyperbolic	Plane	Hyperbolic	Hyperbolic
Conical	Spherical	Conical	Conical
Cylindrical	Spherical	Parabolic	Parabolic

position is translated off-axis suitably such that a conical focus representing the state of maximal coma and minimal astigmatism (Kingslake 1927; Ose *et al* 1971) is obtained as its output. The wavefront emerging from such a focus is mixed with a spherical reference wave and a record of the resultant fringe contours has been made. The actual photograph of a conical HZP and the focus obtained from it during illumination with a plane wave are shown in figure 5.

3.3. *Parabolic HZPs*

To the best of our knowledge no attempt has been made in the past to fabricate a parabolic zone plate. We have been able to realise a parabolic HZP and in order to get it we had to replace the spherical lens L_1 with a cylindrical lens. The cylindrical wavefront is then mixed with a spherical reference wavefront suitably. The photography of a parabolic HZP and the results obtained from it upon its illumination with a plane wavefront are shown in figure 6.

A point worth mentioning here is about the sharpness of the line focus obtained with these parabolic HZPs. As a matter of fact, this is to be expected since a parabolic mirror gives rise to the sharpest focal spot and in the case of a parabolic HZP the line focus is due to the juxtaposition of a number of sharpest focal spots.

A summary of the fringe system observed and the nature of the zone plates realised under the influence of spherical/plane reference waves is given in table 1.

4. Fringe contrast improvement

In all our experiments, we have observed that the fringe contrast is better with a spherical reference wave than with a plane reference wave. Plane and spherical waves have been used extensively in holographic work; but it appears no one has paid attention to notice and to explain the better fringe contrast obtained with spherical reference wavefronts. A simple analysis shows that what we have observed should be the case and the analysis is given below.

For simplicity let us consider that a spherical object wave given by

$$S_0 = (A_0/Z_0) \exp (jkZ_0)$$

is mixed with either a plane reference wave given by

$$S_r = A_r \exp j(kZ_r + \phi)$$

or a spherical reference wave given by

$$S_r = (A_r/Z_r) \exp j(kZ_r + \phi),$$

where A_0, A_r, k and ϕ have the usual meaning. Z_0 and Z_r represent the distances of the object and the reference source respectively from the holographic plate. The fringe contrast when the spherical object wave is mixed with a spherical reference wave can be expressed as (Pernick and Leib 1978)

$$C_{ss} = \frac{2 A_0 A_r Z_0 Z_r}{A_0^2 Z_r^2 + A_r^2 Z_0^2}. \tag{1}$$

The fringe contrast when the spherical object wave is made to interfere with a plane reference beam is given by,

$$C_{sp} = \frac{2 A_0 A_r Z_0}{A_0^2 + A_r^2 Z_0^2}. \tag{2}$$

Our experimental observation suggests that $C_{ss} > C_{sp}$, which means that

$$C_{ss} - C_{sp} = (Z_r - 1)(A_r^2 Z_0^2 - A_0^2 Z_r),$$

must be positive. Since Z_r and Z_0 are always greater than unity, it means that

$$(A_r^2 Z_0^2 - A_0^2 Z_r) > 0. \tag{3}$$

It is a common practice in holography to use a reference beam whose strength is greater than the object beam. This means $A_r > A_0$ and with this condition then (3) conveys that $C_{ss} - C_{sp}$ would always be positive if $Z_0 \geqslant Z_r$. This condition is invariably fulfilled in all holographic work and hence it can be concluded that a better fringe contrast should naturally be expected in the case of a spherical reference beam, as we have observed in all our experiments.

5. Conclusions

Simple methods for the fabrication of elliptic, hyperbolic, conical and parabolic holographic zone plates are described and the performance of these zone plates has been studied. The basic methodology for the simulation of the noncircular zone plates involves the manipulation of fundamental wavefronts (viz., plane and spherical) with simple optical elements such as pinholes and spherical lenses.

It is observed that the fringe contrast is always superior in the case of a spherical reference wave and a simple phenomenological explanation for this observation is offered.

Acknowledgement

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Figure 2. Sequence of output wave surfaces emerging from the lens L_1 when tilted about its vertical axis, as a function of propagation distance. The fringe system shown here is obtained by mixing a plane wavefront with the corresponding wave surface. (a) Vertical elliptic fringe system corresponding to the defocused state before the vertical line focus. (b) Vertical line focus. (c) Vertical hyperbolic fringe system corresponding to the defocused state after the vertical line focus. (d) Hyperbolic fringe system with equal eccentricities for its two branches corresponding to the midway location between the vertical and the horizontal line foci. (e) Horizontal hyperbolic fringe system corresponding to the defocused state before the horizontal line focus. (f) Horizontal line focus. (g) Horizontal elliptic fringe system corresponding to the defocused state after the horizontal focal line. It should be noted that no fringes will appear in figures 2(a), (c), (d), (e) and (g) had we not mixed the wave surfaces with a plane reference wavefront. The fringe visibility in figures 2(b) and 2(f) is poor because of the overwhelming intensity of the object wave appearing as focused energy. Another point to be noted is the fact that the sequence will go from (g) to (a) if the lens L_1 is tilted about the horizontal axis. The background fringes in all the photographs are due to noise.

Figure 4. Imaging performance of a hyperbolic HZP. The zone plate is realised in the on-axis mode. But its imaging behaviour is checked by illuminating the zone plate on its side (i.e., away from the central region) with a wavefront coming from the object transparency which is illuminated with a plane wavefront. This in effect means an off-axis mode of reconstruction from an on-axis zone plate and hence the images due to the vertical focusing and the horizontal focusing actions of the hyperbolic HZP will be separated from the DC. (a) Photograph of the hyperbolic HZP. (b) Images of an object transparency as obtained with the hyperbolic HZP. The object, namely the equilateral triangle, can be seen in the DC region, while the images due to the vertical focusing action and the horizontal focusing action of the HZP could be seen to the left and the right sides of the DC respectively.

Figure 5. Performance of a conical HZP. (a) Photograph of the conical HZP. (b) Conical focus obtained in reconstruction from the HZP upon its illumination with a plane wavefront. Here also, as in the case of hyperbolic HZP, the reconstruction is achieved by illuminating the conical HZP to correspond to an off-axis mode of reconstruction. Hence the conical focus (shown in the figure) is separated from the DC (not shown).