

Synthesis of Equally Excited Linear Arrays

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Abstract-Synthesis of linear arrays employing a perturbation technique to achieve sidelobe reduction by varying element positions alone or phases of antenna currents alone, keeping magnitudes of antenna currents equal, is discussed. Both the mean squared and mini-max error criteria are employed. The technique may also be used to yield patterns with all sidelobes equal.

I. INTRODUCTION

Synthesis of linear antenna arrays to yield certain prescribed characteristics has been studied to a great depth [1], [2]. A case of practical utility may be the one in which magnitudes of the antenna currents are constrained to be equal, synthesis being carried out by varying only the element positions or phases of the antenna currents. Sidelobe reduction has been effected by Hamngton [3] and Ishimaru [4] by employing unequal spacings and by Harrington [5] and Ma [6] by using nonuniform progressive phase shift. Baklanov *et al.* [7] have described a method for synthesizing equally excited and unequally spaced arrays having patterns with all sidelobes of equal magnitudes.

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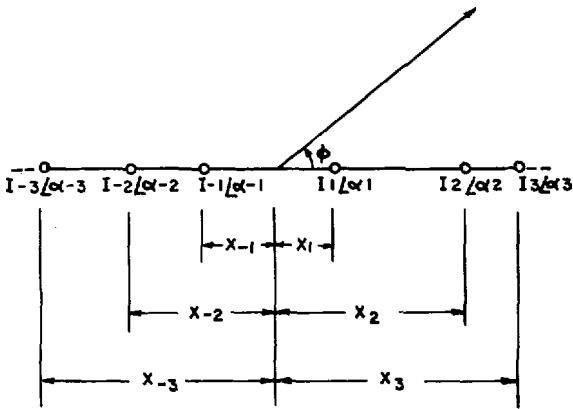


Fig. 1. Geometry of linear array.

II. THEORY

Theories of iterative perturbation techniques employed for the synthesis of arrays for the case when only the element positions are varied and also for the case when only the phases of antenna currents are perturbed are given below.

A. Unequally Spaced Arrays

The array factor of an N (say, even)-element, center-symmetric, equally excited linear array (Fig. 1) at the start of the $(1+1)$ th iteration is given by

$$f_s^l(\bar{X}, \phi) = \frac{2}{N} \sum_{k=1}^{N/2} \cos \psi_k^l, \quad \text{where } \psi_k^l = \beta x_k^l \cos \phi + \alpha_k^l.$$

In order to obtain a new set of element positions which yields a closer approximation to the desired pattern, the element positions are perturbed. That is, $x_k^l \rightarrow x_k^l + \delta_k^l$. Correspondingly the array factor changes as given below:

$$\begin{aligned} f_s^l &\rightarrow \frac{2}{N} \sum_{k=1}^{N/2} [\cos \psi_k^l \cos (\beta \delta_k^l \cos \phi) \\ &\quad - \sin \psi_k^l \sin (\beta \delta_k^l \cos \phi)]. \end{aligned} \quad (2)$$

Assuming δ_k^l to be small to the first order of approximation, the expression for the array factor becomes

$$f_s^{l+1} = \frac{2}{N} \left[\sum_{k=1}^{N/2} \cos \psi_k^l - \sum_{k=1}^{N/2} \delta_k^l V_{kx}^l \right] \quad (3)$$

where

$$V_{kx}^l = \beta \cos \phi \sin (\beta x_k^l \cos \phi + \alpha_k^l). \quad (4)$$

The error $\sigma_p^{(l+1)}$, employing the L_p -norm, between the desired pattern f_d and the array factor $f_s^{(l+1)}$ is given by

$$\begin{aligned} \sigma_p^{(l+1)} &= \left\| f_s^{(l+1)} - f_d \right\|_p \\ &= \left\| (f_s^l - f_d) - \frac{2}{N} \sum_{k=1}^{N/2} \delta_k^l V_{kx}^l \right\|_p. \end{aligned} \quad (5)$$

The δ_k^l are now so chosen that

$$\frac{2}{N} \sum_{k=1}^{N/2} \delta_k^l V_{kx}^l$$

is the best approximation to $(f_s^l - f_d)$ which is nothing but the error function of the previous iteration. The minimization of $\sigma_p^{(l+1)}$ for the cases of $p = 2$ and $p = \infty$ is dealt with in [8]. The iterative process is carried on until no further reduction in the error becomes possible.

B. Nonuniformly Phased Arrays

A closer approximation to the desired pattern may also be obtained by perturbing the phases α_k^l of the antenna currents iteratively. After the l th iteration, let the phases α_k^l be perturbed as

$$\alpha_k^l \rightarrow \alpha_k^l + \nu_k^l. \quad (6)$$

Applying (6) to (1) and, as before, assuming the perturbations ν_k^l to be small to the first order of approximation, give

$$f_s^{(l+1)} = \frac{2}{N} \left[\sum_{k=1}^{N/2} \cos \psi_k^l - \sum_{k=1}^{N/2} \nu_k^l V_{k\alpha}^l \right],$$

where

$$V_{k\alpha}^l = \sin \psi_k^l, \quad (7)$$

and

$$\sigma_p^{(l+1)} = \left\| (f_s^l - f_d) - \frac{2}{N} \sum_{k=1}^{N/2} \nu_k^l V_{k\alpha}^l \right\|_p \quad (8)$$

Note that the form of (8) is similar to that of (5) and hence ν_k^l may be determined in a similar manner.

C. Patterns with all Sidelobes Equal

Note that if the desired pattern $f_d(\phi)$ is a continuous function having nonzero values over a part of the ϕ plane and zero outside it and if this pattern is synthesized employing the L_∞ -norm, then, in view of the equal ripple property of the mini-max approximation [8], the synthesized pattern may be expected to have sidelobes of equal magnitudes in the regions where $f_d(\phi)$ is zero. Hence, arrays having patterns with all sidelobes of equal magnitudes may be synthesized by using the techniques described in the preceding sections.

III. NUMERICAL EXAMPLES

Some examples are presented in this section to illustrate the techniques described above. In order to keep the computations simple: the number of elements is chosen to be six.

A. Illustration I

The following Gaussian pattern is synthesized employing the L_2 -norm by a six-element center-symmetric array:

$$f_d(\phi) = \exp \left[-15 \left(\phi - \frac{\pi}{2} \right)^2 \right]. \quad (9)$$

Since $f_d(\phi)$ is a broadside pattern, $\alpha = 0$. The weight function is chosen to be constant equal to $2/\pi$ so that

$$\int_0^{\pi/2} w(\phi) d\phi = 1.$$

The iteration is initiated from the following two sets of ele-

TABLE I
SYNTHESIS USING THE L_∞ -NORM

NO.	Initial Element Positions (in wavelengths)			Final Element Positions (in wavelengths)			No. of Iterations	Side-lobe Level (dB)	Beam-width (∞ -dB down)
	x_1'	x_2'	x_3'	x_1	x_2	x_3			
1.	0.25	0.75	1.25				5		
2.	0.2	0.6	1.0	0.252	0.503	1.122	3	-20.21	50.8"
3.	0.175	0.525	0.875				4		
4.	0.15	0.45	0.75				—Iteration process diverges—		

ment positions (expressed in wavelengths):

$$1) \quad x_1' = 0.25, \quad x_2' = 0.75, \quad \text{and} \quad x_3' = 1.25$$

and

$$2) \quad x_1' = 0.15, \quad x_2' = 0.45, \quad \text{and} \quad x_3' = 0.75. \quad (10)$$

In each case, the iterative process has converged to the final element-positions given by $x_1 = 0.260$, $x_2 = 0.480$, and $x_3 = 1.083$. The desired and synthesized patterns are shown in Fig. 2. Note that the sidelobe level has been reduced to -20.4 dB.

The Gaussian pattern given by (9) has also been synthesized employing the L_∞ -norm making use of Lawson's algorithm [9]. The iteration process is initiated from four different sets of initial element positions. The results are tabulated in Table I. The desired and synthesized patterns are given in Fig. 2. Note that the sidelobes formed in the region where the desired pattern is close to zero, are of equal level (see Section II-C).

B. Illustration 2

Let the desired pattern coincide with the major lobe of the pattern of a sixelement, 0.2λ spaced, Hansen-Woodyard [10] endfire array and let it be zero everywhere else. That is,

$$f_d(\phi) = \begin{cases} \frac{\sin N -}{N \sin \frac{\psi}{2}}, & -2N \leq \psi < N - 1 \\ 0, & -2\beta d - \frac{\pi}{N - 1} < \psi < \frac{\pi}{2N} \end{cases} \quad (11)$$

where $\psi = \beta d \cos \phi - \beta d - \pi/(N - 1)$.

Let a sixelement, 0.2λ spaced array be synthesized by iteratively perturbing the phases of the antenna currents until the best match is obtained employing the L_2 -norm. Starting from the initial values of phases given by

$$\alpha_k^1 = -\alpha_{-k}^1 = (k - \frac{1}{2}) \left(-\beta d \right) \quad (12)$$

the iteration process has converged to the following final values (in radians) in only four iterations:

$$\alpha_1 = -\alpha_{-1} = -0.894, \quad \alpha_2 = -\alpha_{-2} = -2.582,$$

and

$$\alpha_3 = -\alpha_{-3} = -4.866.$$

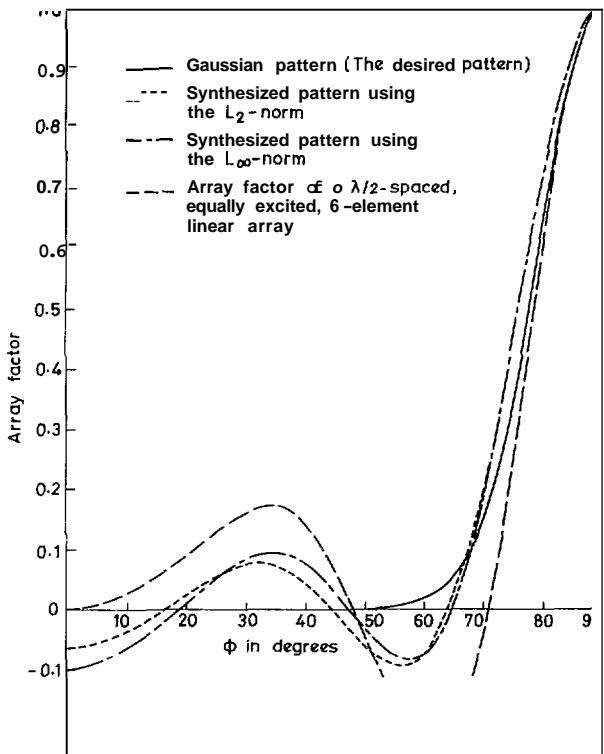


Fig. 2. Synthesis of Gaussian patterns by unequally spaced equally excited linear array.

The desired and the synthesized patterns are shown in Fig. 3. Note that the sidelobe level is decreased from -6.6 dB to -11.3 dB. However, the main beam radiation efficiency [11], which is a measure of the super-directivity, increases only slightly from 26.3 to 27.0 percent.

The pattern has also been synthesized employing the L_∞ -norm. In this case the iteration process converges to the following final values (in radians) in only three iterations:

$$\alpha_1 = -\alpha_{-1} = -0.914, \quad \alpha_2 = -\alpha_{-2} = -2.450,$$

$$\alpha_3 = -\alpha_{-3} = -4.733.$$

The synthesized pattern is shown in Fig. 3. Note that a considerable reduction in sidelobe level is effected though there is an increase in the beamwidth (56.8°). Further, the main beam radiation efficiency is increased from 26.4 to 35.3 percent thus making the array less super-directive. The directivity of the synthesized array is 11.8 and is, therefore, significantly higher than that of the Hansen-Woodyard array which is only 8.2. That is, there is a simultaneous increase in both the main beam radiation efficiency and the directivity.

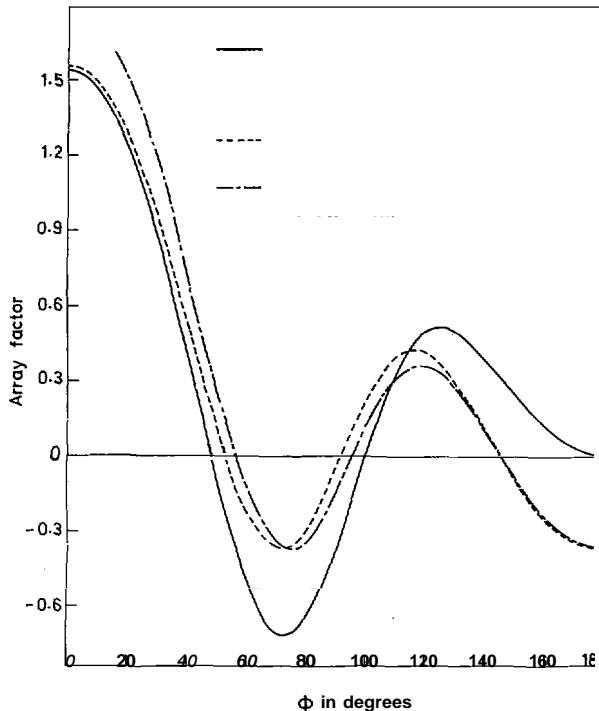


Fig. 3. Synthesis of Hansen-Woodyard pattern by nonuniformly phased equally excited linear array.

Further, note that the sidelobes are of equal magnitudes (see Section II-C).

IV. CONCLUDING REMARKS

From the theory and illustrations presented in this communication, it is clear that a considerable reduction in the sidelobe level may be obtained by allowing the interelement spacings or phases of antenna currents to vary. Note that the techniques do not involve any trial and error procedures. The convergence of the iteration to the final solution has been found to be fast, usually within ten iterations. However, when the L_∞ -norm is employed, the iteration process may not converge, if the element positions obtained at any stage of the iteration are such that V_k do not constitute a Chebyshev set [8]. Ma and Walters [12] point out that this condition may occur if the average interelement spacing is greater than $\lambda/2$. Hence techniques described here are applicable to arrays with small spacings.

Note that reduction of the sidelobe level is not because of any tradeoff relationship with beamwidth, directivity, or efficiency, but because more parameters are under the control of the designer. For instance, in illustration 2, the reduction in the sidelobe level is accompanied by an increase in the directivity and also in the main beam radiation efficiency.

The technique, described here, for the synthesis of optimum arrays is considerably simpler than that employed by Baklanov *et al.* [7]. However, it may be noted that this method ensures equality of levels of only those sidelobes that are formed in the region where the desired pattern is zero.

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A Coupled Resonant Directive Antenna Consisting of a Dipole, a Reflector, and Disks

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Abstract—An experimental study of a coupled resonant directive antenna consisting of a $\lambda/2$ dipole (h is wavelength), a reflector of about 2λ diameter, and disks of about $\lambda/2$ diameter is presented. The $\lambda/2$ spacing between the reflector and a disk and the $\lambda/2$ spacing between disks constitute leaky resonant cavities which are mutually coupled. The antenna shows a relatively high gain per element and a good suppression of sidelobes. Its simple and axially symmetric structure makes the antenna suitable for elements of an array antenna. Characteristics are mostly studied by models at 5 GHz and summarized in the form of equicontour maps. The antenna at 1.6 GHz yields a gain of 12.8 dB above a $\lambda/2$ dipole. An array of 16 elements at 400 MHz yields a gain of 24.3 dB above an isotropic source.

INTRODUCTION

An antenna consisting of a dipole, a reflector, and disks has been a subject of many studies. Ehrenspeck studied an antenna with one disk spaced $\lambda/2$ from the reflector [1]-[3], which was called a short-backfire antenna. He also showed that a rim surrounding the reflector improved the gain and the sidelobe suppression [4]. Hass showed that the use of two disks yields comparable characteristics without using rim [5]. An antenna

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