Heat transfer to an Ellis model fluid flowing in a circular pipe

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The heat transfer problems in non-Newtonian fluids are of considerable importance in processing for polymer solutions and melts. Since the shear stress encountered in the polymer solutions and melts is very small, the major non-Newtonian characteristics of the polymer solutions cannot be adequately described by power-law model. However, Ellis model has been found to be adequate for polymer solutions [1], and solutions for thermally fully developed flows in parallel plates and circular pipes under constant wall temperature and constant heat flux conditions have been obtained in Refs. 12-31.

In the present work, the heat transfer in the thermal entrance region has been considered for an Ellis model fluid in a circular pipe with constant wall temperature and constant heat flux conditions. The thermal entrance problem arises in many industrial equipments. For example, cooling of electrical machines like high capacity A.C. generator, and heating and cooling of polymer solutions in a heat exchanger behave as thermal entrance problems. The fluid is assumed to obey Ellis model law given by the relations

$$dV_z/dy = -[k_1 \tau_{yz} + k_2 (\tau_{yz})^n] \quad (1)$$

where V_z is the velocity along the axis of the pipe ; k_1 , k_2 and n are parameters; and τ_{yz} is the shear stress. For constant thermal and physical properties of the fluid flowing in a circular pipe neglecting viscous dissipation and heat conduction along the axis of the pipe, the equation of energy in dimensionless form is where r] and y respectively are dimensionless radial and axial distances, θ is dimensionless temperature, and θ_0 is dimensionless constant. The boundary conditions can be expressed as

$$[(1 - \eta^2) + (1 - \eta^{n+1})\Phi] \frac{\partial \Theta}{\partial \psi} = \frac{\partial^2 \Theta}{\partial \eta^2} + \frac{1}{\eta} \frac{\partial \Theta}{\partial \eta} \quad (2)$$

where η and ψ respectively are dimensionless radial and axial distances, Θ is dimensionless temperature, and Θ is dimensionless constant. The boundary conditions can be expressed as

$$\Theta = 1 \text{ at } \psi = 0 \text{ (for constant wall temperature)} \quad (3a)$$

$$\Theta = 0 \text{ at } \psi = 0 \text{ (for constant heat flux conditions)} \quad (3b)$$

$$\Theta = 0 \text{ at } \eta = 1 \text{ (for constant wall conditions)} \quad (4a)$$

$$\Theta = -1 \text{ at } \eta = 1 \text{ (for constant heat flux conditions)} \quad (4b)$$

$$\partial \Theta / \partial \eta = 0 \text{ at } \eta = 0 \quad (5)$$

Eqn. (2) under the boundary conditions (3) to (5) has been solved numerically using Crank-Nicolson implicit finite difference scheme and the resulting simultaneous algebraic equations have been solved by Thomas algorithm. The average temperature Θ_b is defined as

$$\Theta_b = \frac{\int_0^1 \Theta(\psi, \eta) V_z(\eta) \eta d\eta}{\int_0^1 V_z(\eta) \eta d\eta} \quad (6)$$

The local Nusselt number Nu for constant wall temperature can be expressed as

$$Nu = 1 - \Theta_b / [(V_z)_{\max} \psi / (V_z)_b] \quad (7a)$$

and for constant heat flux,

$$Nu = 2 / (\Theta_w - \Theta_b) \quad (7b)$$

where subscripts b and w respectively denote average and wall values.

Fig. 1. Nusselt number at $\psi = 1$ for constant temperature and constant heat flux.

The present results are in excellent agreement with those of Lyche and Bird [4] which indicates the accuracy of the method. The bulk temperature Θ_b and the local Nusselt number Nu for various values of ψ , Θ and n have been obtained. For constant wall temperature when Θ is large, the temperature profile is not fully developed even at $\psi = 1$ indicating that very long tube is required. The local Nusselt number Nu at $\psi = 1$ for constant wall temperature and constant heat flux conditions is shown in Fig. 1.

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