

UNSTEADY MIXED CONVECTION LAMINAR BOUNDARY-LAYER FLOW OVER A VERTICAL PLATE IN MICROPOLAR FLUIDS

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(Received 9 February 1987)

The combined effect of forced and free convection on the unsteady laminar incompressible boundary-layer flow of a thermo-micropolar fluid over a semi-infinite vertical plate has been studied when the free-stream velocity, surface mass transfer and wall temperature vary arbitrarily with time. The partial differential equations with three independent variables governing the flow have been solved using quasilinearization in combination with an implicit finite-difference scheme. The results indicate that the buoyancy parameter, coupling parameter, mass transfer and unsteadiness in the free-stream velocity strongly affect the skin friction, microrotation gradient and heat transfer whereas the effect of microrotation parameter on the skin friction and heat transfer is rather weak, but microrotation gradient is strongly affected by it. The heat transfer is strongly dependent on the Prandtl number, the dissipation parameter and the variation of the wall temperature with time whereas the skin friction and microrotation gradient are weakly dependent on it. The buoyancy parameter causes an overshoot in the velocity profile. The magnitude of the velocity overshoot increases as the buoyancy parameter increases and it decreases as time increases.

1. INTRODUCTION

When the velocity of the fluid is small and the temperature difference between the surface and ambient fluid is large than the buoyancy effects on forced convective heat-transfer become important. The combined effect of forced and free convection over a heated vertical plate for Newtonian fluid has been studied by several investigators¹⁻⁴. The flow and heat transfer behaviour of Polymeric fluids, colloidal fluids, real fluid with suspensions, liquid crystals and animal blood cannot be explained on the basis of Newtonian and non-Newtonian fluid theory. The theory of micropolar and thermomicropolar fluids was introduced by Eringen⁵⁻⁷. An excellent review of the micropolar theory is given by Ariman *et al.*⁸⁻⁹. Several investigators¹⁰⁻¹⁴ have studied the steady forced or free convection boundary-layer flow for micropolar fluids. Recently, Jena and Mathur¹⁵ have studied the steady mixed convection laminar boundary-layer flow of a micropolar fluid from a vertical plate without dissipation effects. It may be noted that the unsteady mixed convection flow over a vertical plate in a micropolar fluid has not been studied so far.

We have investigated the unsteady laminar incompressible boundary-layer flow of a micropolar fluid over a vertical flat plate when the free-stream velocity, mass transfer and the wall temperature vary arbitrarily with time. The effects of the surface mass transfer which varies arbitrarily with time, viscous dissipation and the Prandtl number have also been taken into account. The partial differential equations with three independent variables governing the flow have been solved numerically using a quasilinear finite-difference scheme. The results have been compared with Oosthuizen and Hart³, Gryzagoridis⁴ and Jena and Mathur¹³.

2. GOVERNING EQUATIONS

We consider the unsteady laminar incompressible boundary-layer flow of a thermomicropolar fluid past a vertical plate under the combined effect of forced and free convection. It has been assumed that the free-stream fluid temperature remains constant and the free-stream velocity, surface mass transfer and the wall temperature vary with time. Under the foregoing assumptions, the equations governing the flow can be written as^{5,7,10-15}

$$u_x + v_y = 0 \quad \dots(1a)$$

$$u_t + uu_x + vv_y = (u_e)_t + [(\mu + k_1)/\rho] u_{yy} + (k_1/\rho) N_y + g\beta (T - T_\infty) \quad \dots(1b)$$

$$N_t + uN_x + vN_y = (\gamma/\rho j) N_{yy} - (k_1/\rho j) [2N + u_y] \quad \dots(1c)$$

$$T_t + uT_x + vT_y = Pr^{-1} (\mu/\rho) T_{yy} + (\alpha_1/\rho c_p) [T_x N_y - T_y N_x] \\ + (1/\rho c_p) \left[(\mu + k_1/2) u_y^2 + 2k_1 (N + u_y/2)^2 \right. \\ \left. + \gamma N_y^2 \right]. \quad \dots(1d)$$

The relevant initial and boundary conditions are

$$\left. \begin{aligned} u(x, y, 0) = u_t(x, y), \quad v(x, y, 0) = v_t(x, y) \\ N(x, y, 0) = N_t(x, y), \quad T(x, y, 0) = T_t(x, y) \end{aligned} \right\} \quad \dots(2a)$$

$$\left. \begin{aligned} u(x, 0, t) = 0, \quad v(x, 0, t) = v_w(t) \\ N(x, 0, t) = 0, \quad T(x, 0, t) = T_w(t) \\ u(x, \infty, t) = u_e(x, t), \quad N(x, \infty, t) = 0, \quad T(x, \infty, t) = T_\infty \end{aligned} \right\} \quad \dots(2b)$$

Here x and y are the distances along and perpendicular to the surface, respectively; t is the time; u and v are the velocity components in the x and y directions, respectively; N is the component of microrotation whose direction of rotations is in the $x - y$ plane; g is the acceleration due to gravity; ρ and T are the density and temperature of the fluid; μ , k_1 and γ are the viscosity, vortex viscosity and spin

gradient viscosity, respectively; β is the coefficient of thermal expansion; α_1 is the micropolar heat conduction coefficient; c_p is the specific heat of the fluid at constant pressure; j is the micro-inertia density; the subscripts t, x and y denote derivatives with respect to t, x and y , respectively; and the subscripts e and w denote conditions at the edge of the boundary layer and on the surface, respectively. The subscript i denotes values at the initial time $t = 0$ and T_∞ is a constant.

It may be remarked that we have assumed that the microrotation N is equal to zero on the boundary. The justification for using such a boundary condition is given in detail by Kirwan Jr¹⁶. Here the free-stream velocity u_e which vary with time can be expressed in the form

$$u_e = u_\infty \varphi(t^*), \quad t^* = u_\infty t/L. \tag{3}$$

φ is an arbitrary function of the time t^* representing the nature of unsteadiness in the external stream and has a continuous first derivative for $t^* \geq 0$.

On applying the transformations

$$\left. \begin{aligned} \eta &= (u_\infty/2\nu\xi L)^{1/2} y, \quad \xi = \bar{x}, \quad \bar{x} = x/L \\ \psi &= (2\nu u_\infty \xi L)^{1/2} f(\xi, \eta, t^*) \varphi(t^*) \end{aligned} \right\} \tag{4a}$$

$$\left. \begin{aligned} u &= u_e F, \quad f' = F, \quad u_e = u_\infty \varphi(t^*) \\ v &= -(\nu u_\infty/2\xi L)^{1/2} \varphi [f + 2\xi f_\xi - \eta F] \\ N &= [u_\infty^2 2\xi/(\nu L)]^{1/2} s \\ (T - T_\infty)/(T_{w0} - T_\infty) &= G \\ (T_w - T_\infty)/(T_{w0} - T_\infty) &= G \varphi_1(t^*) \end{aligned} \right\} \tag{4b}$$

$$\left. \begin{aligned} f &= \int_0^\eta F d\eta + f_w, \quad f_w = A \xi^{1/2}/\varphi \\ A &= -(\nu_w/u_\infty) (\text{Re}_L/2)^{1/2}, \quad \text{Re}_L = u_\infty L/\nu \end{aligned} \right\} \tag{4c}$$

to (1), we find that (1a) is satisfied identically and (1b—d) reduce to

$$\begin{aligned} (1 + N_1) F'' + \varphi f F' + 2\xi [\varphi^{-1} \varphi_i^* (1 - F) - F_i^*] + 2\xi \varphi^{-1} [N_1 s' + \lambda G] \\ = 2\xi \varphi [FF_\xi - f_\xi F'] \end{aligned} \tag{5a}$$

$$\begin{aligned} N_3 s'' + N_2 \varphi [f s' - s F'] - N_1 [4\xi s + \varphi F'] - 2\xi N_2 s_i^* \\ = 2\xi \varphi N_2 [F s_\xi - f_\xi s'] \end{aligned} \tag{5b}$$

$$\begin{aligned} Pr^{-1} G'' + \varphi f G' + 2\xi \alpha [s' G_\xi - s_\xi G'] - \alpha s G' + Br [1 + N_1/2] \varphi^2 F'^2 \\ + 2N_1 Br [2\xi s + \varphi F'/2]^2 + 2\xi N_2 Br s_i^* - 2\xi G_i^* \\ = 2\xi \varphi [FG_\xi - f_\xi G'] \end{aligned} \tag{5c}$$

where

$$\left. \begin{aligned} N_1 &= k_1/\mu, N_2 = (u_\infty/L) (\rho j/\mu) \\ N_3 &= (u_\infty/L) (\gamma\rho/\mu^2), \alpha = (\alpha_1/\mu c_p) (u_\infty/L) \\ \lambda &= Gr/\text{Re}_L^2, Gr = g\beta (T_{w0} - T_\infty) L^3 \rho^2/\mu^2 \\ Br &= u_\infty^2/[c_p (T_{w0} - T_\infty)]. \end{aligned} \right\} \dots(6)$$

Here ξ and η are transformed coordinates; t^* is the dimensionless time; u_∞ is the free-stream velocity at $t^* = 0$; L is the length of the plate; ν is the kinematic viscosity; ψ and f are the dimensional and dimensionless stream functions, respectively; $F(f')$, s and G are the dimensionless velocity, microrotation and temperature, respectively. The parameters N_1, N_2, N_3, α and Br are the coupling parameter, micro-inertia density parameter, microrotation parameter, micropolar heat conduction parameter and dissipation parameter, respectively; λ is the buoyancy parameter; Gr is the Grashof number; and Pr is the Prandtl number. f_w is the surface mass transfer parameter. If the normal velocity at the wall v_w is selected in such a manner that $(v_w/u_\infty) (\text{Re}_L/2)^{1/2}$ is a constant then the parameter A will be a constant. Hence the mass transfer parameter f_w will vary according to (4c). $A \geq 0$ according to whether there is a suction or injection. The subscripts ξ and t^* denote derivatives with respect to ξ and t^* , respectively. The prime denotes derivatives with respect to η .

The transformed boundary conditions are given by

$$\left. \begin{aligned} F = 0, s = 0, G = \varphi_1(t^*) \text{ at } \eta = 0 \\ F \rightarrow 1, s \rightarrow 0, G \rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \right\} \text{ for } t^* \geq 0. \dots(7)$$

We assume that the flow is initially steady and then becomes unsteady for $t^* > 0$. Hence the initial conditions for F, s and G at $t^* = 0$ are given by steady flow equations obtained by putting

$$\varphi(t^*) = \varphi_1(t^*) = 1, \varphi_{t^*} = F_{t^*} = s_{t^*} = G_{t^*} = 0 \dots(8)$$

in (5) and also in (7) (for boundary conditions) and they are

$$(1 + N_1) F'' + fF' + 2\xi [N_1 s' + \lambda G] = 2\xi [FF_\xi - f_\xi F'] \dots(9a)$$

$$N_3 s'' + N_2 [fs' - sF] - N_1 [4\xi s + F'] = 2\xi N_2 [Fs_\xi - f_\xi s'] \dots(9b)$$

$$\begin{aligned} Pr^{-1} G'' + fG' + 2\xi \alpha [s'G_\xi - s_\xi G'] - \alpha sG' + Br [1 + N_1/2] F'^2 \\ + 2N_1 Br [2\xi s + F'/2]^2 + 2\xi N_3 Brs'^2 = 2\xi [FG_\xi - f_\xi G'] \end{aligned} \dots(9c)$$

It may be remarked that the steady-state equations (9 (a)–(c)) reduce to that of Jena and Mathur¹⁵ if we replace $2\xi N_1 s'$ by $N_1 s'$ in equation 9(a), $-N_2 sF$ by $N_2 sF$ and

$-N_1 [4\xi s + F']$ by $-2\xi N_1 [2s + F']$ in equation 9(b) and put $\alpha = Br = 0$ in equation 9(c). The equations 9(a)—(c) also reduce to those of mixed convection for Newtonian fluids which have been studied by Oosthuizen and Hart³ and Gryzagoridis⁴ if we put $N_1 = 0$ and replace $2\xi\lambda G$ by λG in 9(a), consequently, $s = 0$ and equation 9(b) becomes superfluous.

The skin-friction coefficient at the wall is given by

$$C_f = 2\tau_w / [\rho (u_x^2)_{x=0}] = (2 \text{Re}_x^{-1})^{1/2} (1 + N_1) \phi F'_w \quad \dots(10a)$$

where

$$\tau_w = [(\mu + k_1) u_y + k_1 N]_{y=0}. \quad \dots(10b)$$

The heat-transfer coefficient in terms of Nusselt number is given by

$$Nu = 2xq_w / [k_c (T_{w0} - T_\infty)] = (2 \text{Re}_x)^{1/2} G'_w \quad \dots(11a)$$

where

$$q_w = [k_c T_y + \beta_c N_x]_{y=0}. \quad \dots(11b)$$

The couple stress coefficient is expressed in the form

$$M = m_w / [\rho (u_x^2)_{x=0} x] = (\text{Re}_x)^{-1} N_3 s'_w \quad \dots(12a)$$

where

$$m_w = \gamma (N_y)_{y=0}. \quad \dots(12b)$$

Here C_f , Nu and M are, respectively, the skin-friction coefficient, Nusselt number and couple stress coefficient; τ_w , q_w and m_w are, respectively, the shear stress, heat-transfer rate and couple stress at the wall; β_c is the heat conduction parameter and k_c is the thermal conductivity.

3. RESULTS AND DISCUSSION

The equations (5a)—(5c) under boundary conditions (7) and initial conditions (9) have been solved numerically using an implicit finite-difference scheme with a quasi-linearization technique. Since the detailed description of the method is given in Bellman and Kalaba¹⁷ and Inouye and Tate¹⁸, its description is not repeated here. Computations have been carried out for various values of the parameters λ ($-0.25 \leq \lambda \leq 20$), N_1 ($0.5 \leq N_1 \leq 13.5$), N_2 ($0.5 \leq N_2 \leq 4.5$), A ($-0.5 \leq A \leq 0.5$), Pr ($0.7 \leq Pr \leq 7.0$) and Br ($-0.2 \leq Br \leq 0.2$) with $N_3 = 1.0$, $\alpha = 1.0$, $\epsilon = 0.2$, $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$ and $\omega^* = 5.6$. The unsteady free-stream velocity and wall temperature distributions considered here are given by

$$\phi(t^*) = 1 + \epsilon t^{*2}, \quad \phi(t^*) = 1 + \epsilon_1 \sin^2(\omega^* t^*), \quad \phi_1(t^*) = 1 + \epsilon_2 t^*$$

where ϵ , ϵ_1 and ϵ_2 are constants and ω^* is the frequency parameter. The effect of step sizes in η , ξ and t^* directions and the edge of the boundary layer η_∞ on the solution have been studied with a view to optimize them. Finally, the computations were carried out with $\Delta\eta = 0.02$, $\Delta\xi = 0.05$, $\Delta t^* = 0.1$ and η_∞ has been taken between 4 and 8 depending upon the values of the parameters. The results presented here found to be independent of the step size $\Delta\eta$, $\Delta\xi$, Δt^* and the edge of the boundary layer η_∞ at least up to 4th decimal place.

In order to assess the accuracy of method, we have compared our Nusselt number results for Newtonian fluids ($N_1 = 0$) with those of Oosthuizen and Hart³ and Gryzagoridis⁴. We have also compared our skin-friction and heat-transfer results for micropolar fluids ($N_1 > 0$) for steady flow ($t^* = 0$) with those of Jena and Mathur¹⁵. In both the cases, the results are found to be in good agreement and the comparison is shown in Figs. 1 and 2.

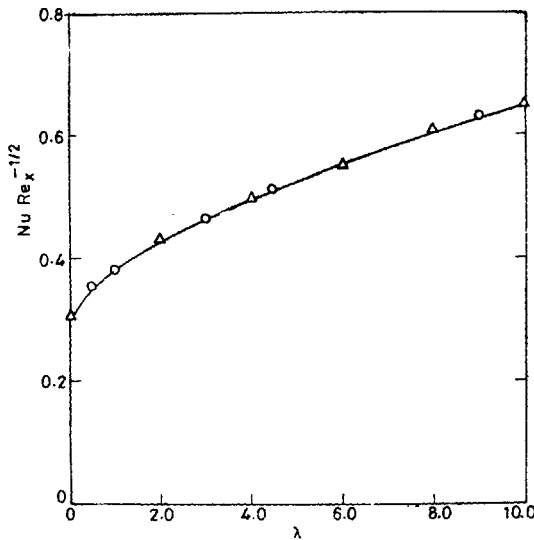


FIG. 1. Comparison of heat-transfer coefficient $Nu Re_x^{-1/2}$ for $\varphi(t^*) = 1.0$, $\varphi_1(t^*) = 1.0$, $N_1 = N_2 = N_3 = \beta = A = Br = 0$. ———, present method; \circ , Oosthuizen and Hart; Δ , Gryzagoridis.

The results for the case $\varphi(t^*) = 1 + \epsilon t^{*2}$, $\epsilon > 0$ (accelerating flow) are given in Figs. 3–6 and those for the case $\varphi(t^*) = 1 + \epsilon_1 \sin^2(\omega^* t^*)$ (fluctuating flow) in Fig. 7.

The effect of buoyancy parameter λ , mass transfer parameter A , Prandtl number Pr , coupling parameter N_1 , microrotation parameter N_3 , variation of wall temperature with time $\varphi_1(t^*)$, distance ξ and dissipation parameter Br on the skin-friction, microrotation-gradient and heat-transfer parameters (F'_w , $-s'_w$, $-G'_w$) are shown in

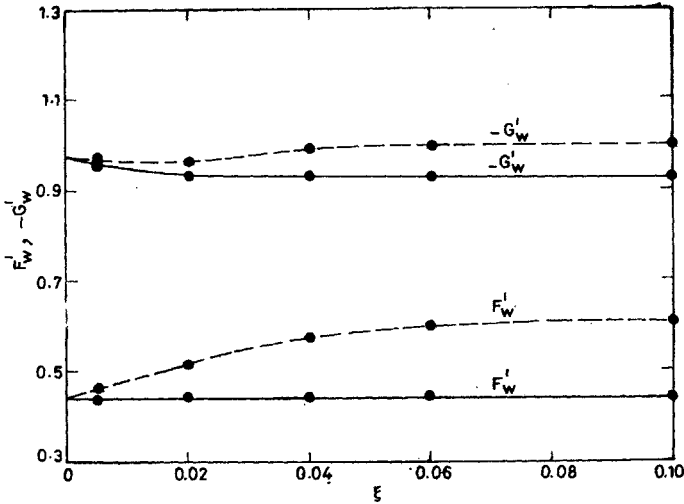


FIG. 2. Comparison of skin-friction parameter F'_w and heat-transfer parameter $-G'_w$ for $\varphi(t^*) = 1.0$, $\varphi_1(t^*) = 1.0$, $N_1 = 0.1$, $N_3 = 0.02$, $Pr = 9.0$, $A = 0.0$, $\alpha = 0.0$, $Br = 0.0$. —, $\lambda = 0$; ---, $\lambda = 4$; ●, Jena and Mathur.

Figs. 3-5. Figures 6(a)-(c) depict the effect of buoyancy parameter λ on velocity, microrotation and temperature profiles.

The results indicate that the skin-friction parameter F'_w increases with time t^* for $\lambda \leq 0$ and decreases with it for $\lambda > 0$ whereas the microrotation-gradient parameter $-s'_w$ increases with t^* for values of λ [see Figs. 3(a)-(b)]. For $\lambda \leq 0$, the heat-transfer parameter $-G'_w$ increases with t^* but for $\lambda > 0$ it increases only after certain value of t^* [Fig. 3(c)]. It has also been observed that for all t^* , the parameters F'_w , $-s'_w$ and $-G'_w$ increase as λ increases. Similar behaviour has also been observed by Jena and Mathur¹⁵ for steady case ($t^* = 0$). This is due to the fact that the buoyancy force ($\lambda > 0$) gives rise to favourable pressure gradient which accelerates the fluid in the boundary layer and thereby increases the skin-friction, microrotation-gradient and heat-transfer parameters. The skin-friction, microrotation-gradient and heat-transfer parameters (F'_w , $-s'_w$, $-G'_w$) are reduced due to injection ($A < 0$) whatever may be the value of t^* and the effect of suction ($A > 0$) is just the opposite. This is because injection increases the momentum, microrotation and thermal boundary layer thicknesses which cause deceleration in the fluid and suction does the opposite. Figures 3(a)-(b) also show that the skin-friction parameter F'_w and microrotation-gradient

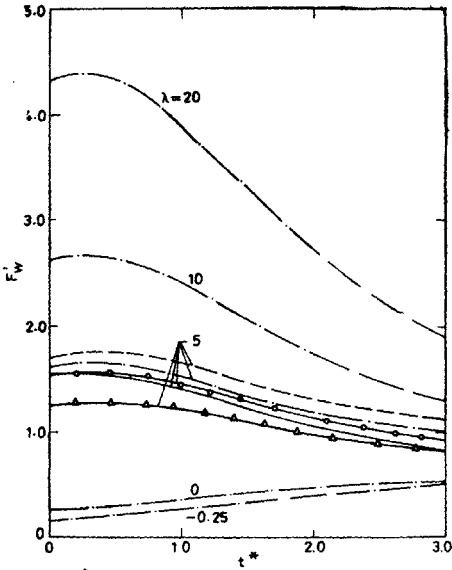


FIG. 3 (a) Skin-friction parameter F'_w

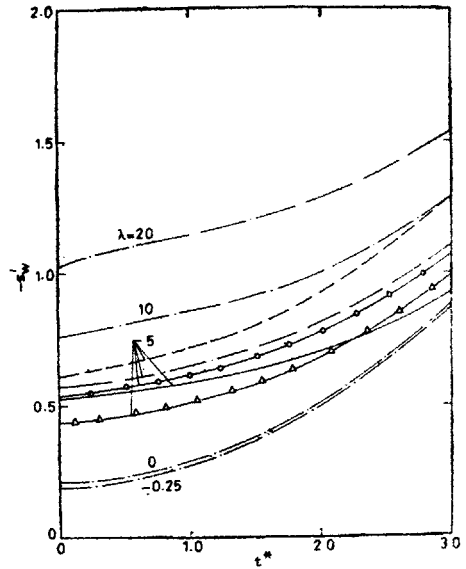


FIG. 3. (b) microrotation-gradient parameter $-s'_w$

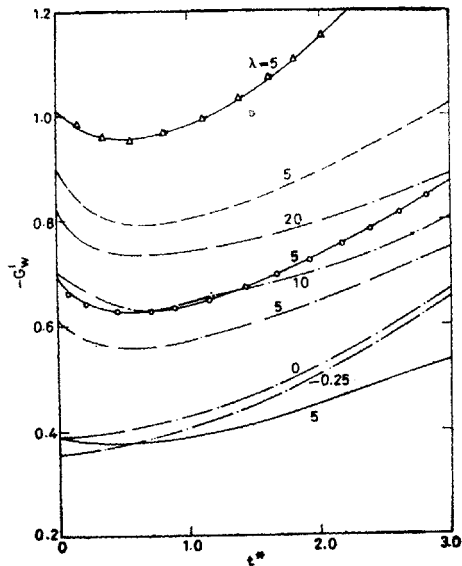


FIG. 3 (c) Heat-transfer parameter $-G'_w$ for $\varphi(t^*) = 1 + \epsilon t^{*2}$, $\varphi_1(t^*) = 1.0$, $N_1 = 1.5$, $N_3 = 1.5$, $Br = 0.0$, $\xi = 0.5$.
 - - - - - , $A = -0.5$, $Pr = 0.7$;
 - · - · - · , $A = 0.0$, $Pr = 0.7$;
 ——— , $A = 0.5$, $Pr = 0.7$;
 —○— , $A = 0.0$, $Pr = 3.0$
 —△— , $A = 0.0$, $Pr = 7.0$.

parameter $-s'_w$ decrease as Prandtl number Pr increases. Similar behaviour has also been observed by Wilks². On the other hand, the heat-transfer parameter $-G'_w$ is found to increase with Pr (see Fig. 3(c)), because a large Prandtl number results in a thinner boundary layer with a corresponding large temperature at the wall and hence a large surface heat transfer. It has also been observed that the effect of Pr on the parameters F'_w and $-s'_w$ is less as compared to the parameter $-G'_w$.

Figures 4(a)-(c) show that any time t^* , the skin-friction, microrotation-gradient and heat-transfer parameters ($F'_w, s'_w, -G'_w$) decrease as the coupling parameter N_1 increases. Similar behaviour has also been observed by other investigators^{10,19,20}. The cause of this reduction is due to the thickening of momentum, microrotation and thermal boundary layers due to in the parameter N_1 which in turn decelerates the fluid in the boundary layer. As N_3 increases, the parameters $F'_w, -s'_w$ and $-G'_w$ decrease whatever may be the value of t^* . It has also been observed from Figs. 4(a)-(c) that for $\xi = 0$, the parameters $F'_w, -s'_w$ and $-G'_w$ increase as time t^* increases from 0 to 3.0 but for $\xi > 0$, F'_w decreases with t^* and $-s'_w$ and $-G'_w$ increase with it. We have also observed that as ξ increases, the parameters F'_w and $-G'_w$ increase for $\lambda > 0$ and they decrease for $\lambda \leq 0$. Similar trend has been observed by Jena and Mathur¹⁵ also. It is also clear from these figures that the parameters $F'_w, -s'_w$ and $-G'_w$ increase with the variation of the wall temperature with t^* . However, the heat-transfer parameter $-G'_w$ strongly affected by the variation of the wall temperature with t^* whereas its effect on F'_w and $-s'_w$ is rather weak.

The effect of dissipation parameter Br on the parameters $F'_w, -s'_w$ and $-G'_w$ is depicted in Fig. 5. This figure show that the parameters F'_w and $-s'_w$ increase as Br increases whereas the parameter $-G'_w$ decreases with it. This behaviour is independent of the value of t^* . For all values of Br , F'_w decreases with t^* (after certain t^*) whereas $-s'_w$ increases with it. As t^* increases, the parameter $-G'_w$ increases for $Br \leq 0$ and decreases for $Br > 0$. It has also been observed that the effect of Br is more pronounced on the parameter $-G'_w$ than on the parameters F'_w and $-s'_w$.

The effect of λ on the velocity, microrotation and temperature is shown in Figs. 6(a)-(c). Figure 6(a) shows that there is a velocity overshoot in F for buoyancy assisted

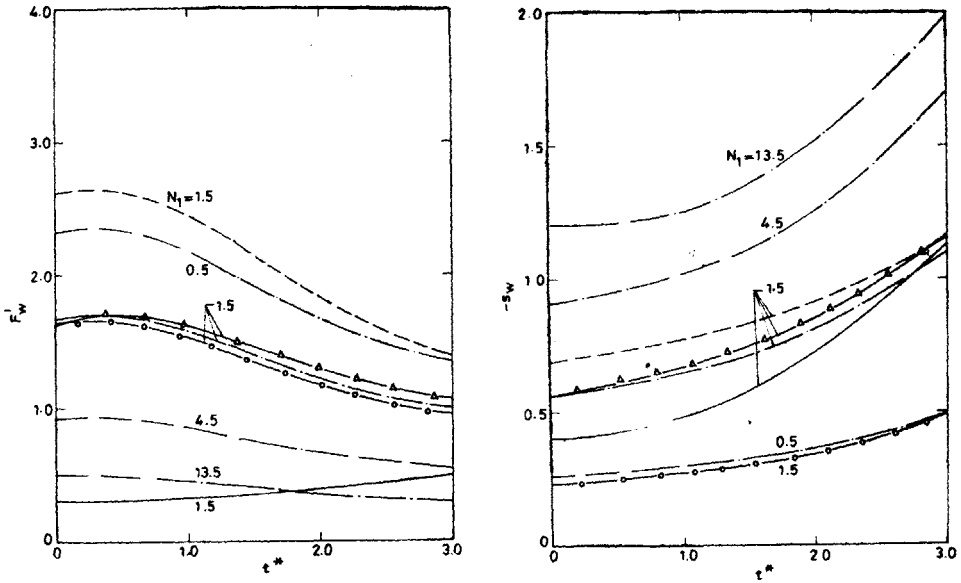


FIG. 4 (a) Skin-friction parameter F_w' , (b) microrotation-gradient parameter $-s_w'$

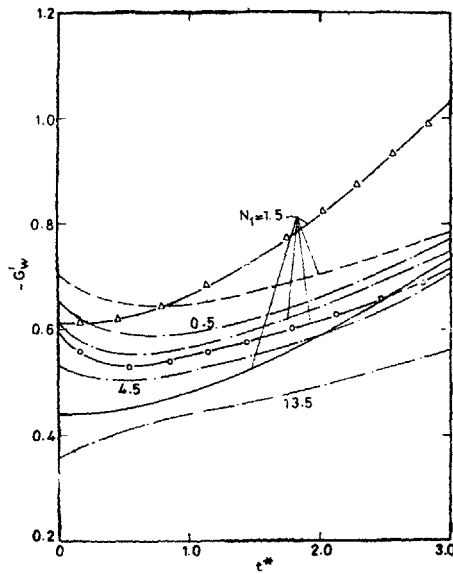


FIG. 4. (c) Heattransfer parameter $-G_w'$ for $\varphi(t^*) = 1 + \epsilon t^{*2}$, $Pr = 0.7$, $A = 0.0$, $\lambda = 5.0$, $Br = 0.0$.

—, $\xi = 0.0$, $N_3 = 1.5$, $\varphi_1(t^*) 1.0$; —·—·—, $\xi = 0.5$, $N_3 = 1.5$, $\varphi_1(t^*) = 1.0$; ———, $\xi = 1.0$, $N_3 = 1.5$, $\varphi_1(t^*) = 1.0$; —○—, $\xi = 0.5$, $N_3 = 4.5$, $\varphi_1(t^*) = 1.0$; —△—, $\xi = 0.5$, $N_3 = 1.5$, $\varphi_1(t^*) = 1 + \epsilon_2 t^*$.

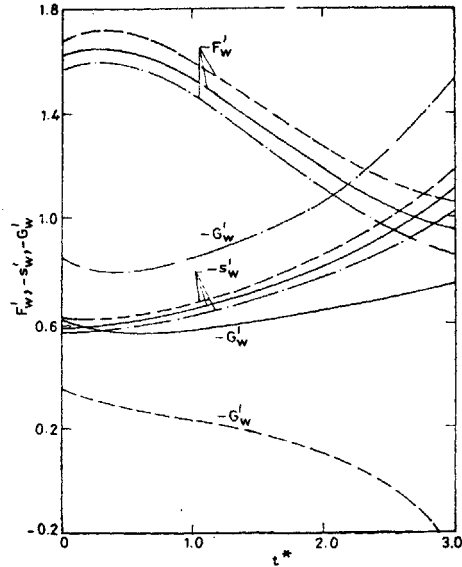


FIG. 5. Skin-friction parameter F'_w , microrotation-gradient parameter $-s'_w$ and heat-transfer parameter $-G'_w$ for $\varphi(t^*) = 1 + \epsilon t^{*2}$, $\varphi_1(t^*) = 1.0$, $N_1 = 1.5$, $N_3 = 1.5$, $A = 0.0$, $\lambda = 5.0$, $Pr = 0.7$, $\xi = 0.5$. ———, $Br = 0.0$; - · - · - ·, $Br = -0.2$; ———, $Br = 0.2$.

flow ($\lambda > 0$), and the velocity overshoot increases as λ increases. However it decreases as t^* increases. There is no velocity overshoot either for purely forced flow ($\lambda = 0$) or buoyancy opposed flow ($\lambda < 0$). The velocity overshoot is because buoyancy force ($\lambda > 0$) gives rise to a favourable pressure gradient resulting in velocity which adds to the forced convection velocity. The buoyancy opposed flow ($\lambda < 0$) gives to adverse

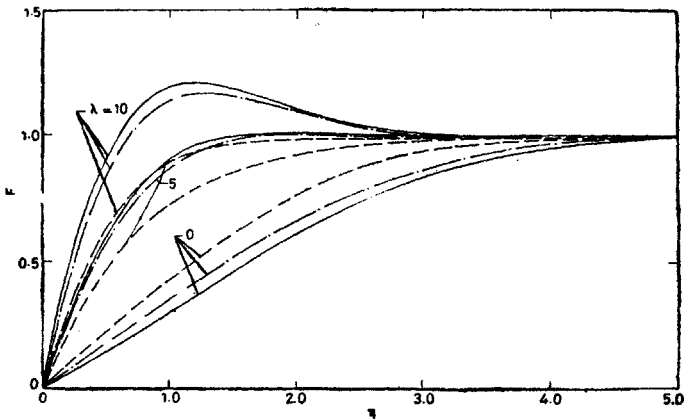


FIG. 6. (a) Velocity profile in the x direction F .

pressure gradient which reduces the forced convection velocity. As the combined effect of buoyancy force $\lambda > 0$ and forced convection force decrease with time which results in reduction in velocity overshoot with time.

The microrotation profile $-s$ and the temperature profile G are shown in Figs. 6(b)-(c). It is observed that the profiles $-s$ and G are significantly affected by the parameter λ and the effect becomes more pronounced as time t^* increases. The profiles $-s$ and G become more steep as t^* and λ increase.

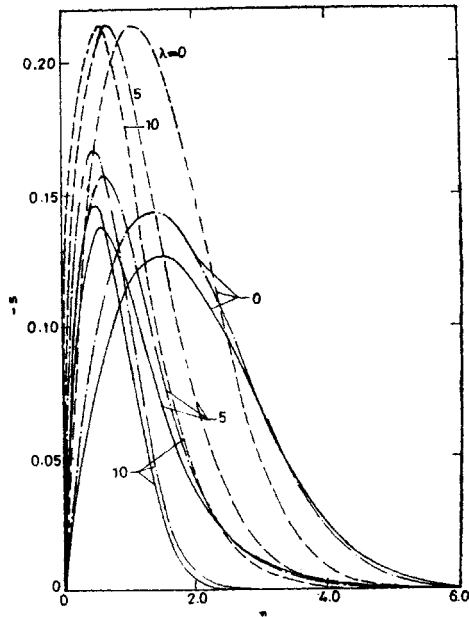


FIG. 6. (b) microrotation profile $-s$.

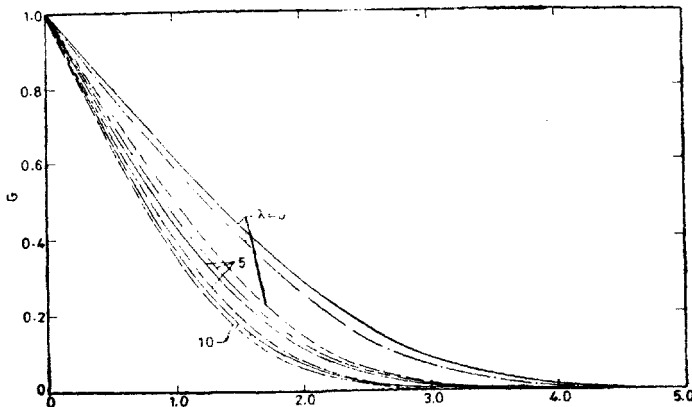


FIG. 6. (c) Temperature profile G for $\varphi(t^*) = 1 + \epsilon t^{*2}$, $\varphi_1(t^*) = 1.0$; $N_1 = 1.5$, $N_3 = 1.5$, $Pr = 0.7$, $A = 0.0$, $Br = 0.0$, $\xi = 0.5$, -----, $t^* = 0.0$; -.-.-.-, $t^* = 1.0$; ———, $t^* = 2.0$.

The skin-friction, microrotation-gradient and heat-transfer parameters (F_w' , $-s_w'$, $-G_w'$) for oscillatory free-stream velocity $\varphi(t^*) = 1 + \epsilon_1 \sin^2(\omega^* t^*)$ are shown in

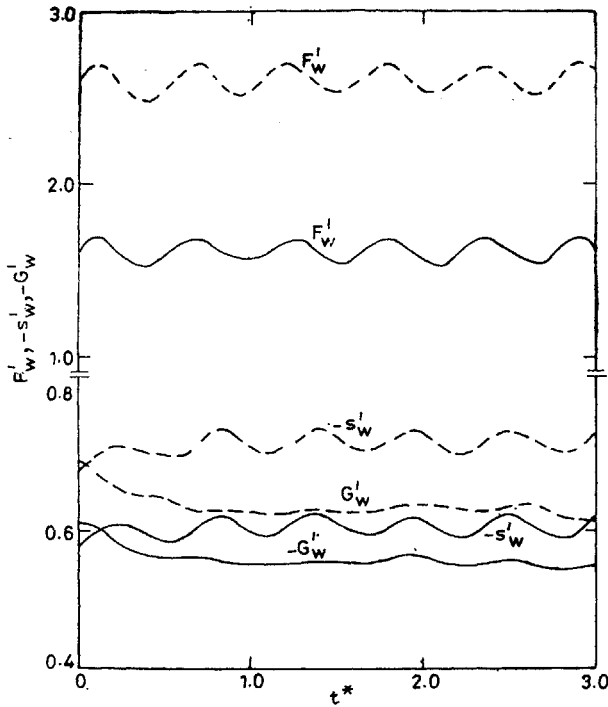


FIG. 7. Skin-friction parameter F_w' , microrotation-gradient parameter $-s_w'$ and heat-transfer parameter $-G_w'$ for $\varphi(t^*) = 1 + \epsilon_1 \sin^2(\omega^* t^*)$, $\varphi_1(t^*) = 1.0$, $N_1 = 1.5$, $N_2 = 1.5$, $Pr = 0.7$, $A = 0.0$, $\lambda = 5.0$, $Br = 0.0$. $\cdots\cdots$, $\xi = 0.5$; --- $\xi = 1.0$.

Fig. 7. It is clear from this figure that the parameters F_w' , $-s_w'$ and $-G_w'$ oscillate as time t^* increases but the oscillations are more for large ξ .

4. CONCLUSIONS

The skin friction, microrotation gradient and heat transfer are strongly dependent on the buoyancy parameter, coupling parameter, mass transfer and unsteadiness in free-stream velocity. The effect of microrotation parameter on microrotation gradient is appreciable whereas its effect on skin friction and heat transfer is comparatively small. The Prandtl number, dissipation parameter and the variation of the wall temperature with time affect the heat transfer significantly whereas skin friction and microrotation gradient are weakly affected by it. Buoyancy parameter induces over-

shoot in the velocity profiles. The magnitude of the velocity overshoot increases as the buoyancy parameter increases and it decreases as time increases.

ACKNOWLEDGEMENT

The author wishes to thank Professor G. Nath for many helpful suggestions.

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