DYNAMICAL SYSTEMS THAT MIMIC FLOW TURBULENCE

R. NARASIMHA and G. S. BHAT
Centre for Atmospheric Sciences, Indian Institute of Science, Bangalore 560 012, India.

ABSTRACT

The relevance of recent developments in nonlinear dynamical systems towards understanding the onset of turbulence in open flows has often been questioned as the behaviour in all such systems studied to-date is fundamentally different in several respects from that of strong turbulence. In particular, such systems exhibit only slow chaos, which furthermore is not persistent as the control parameter is continuously varied beyond the value critical for onset of chaos. Based on physical considerations of the dynamics of interacting large and small eddies, we devise here a nonlinear system which mimics the strong, fast turbulence characteristic of open shear flows, revealing the possibility of establishing closer connections between dynamical chaos and turbulence.

1. INTRODUCTION

RECENT developments in the theory of dynamical systems exhibiting chaotic behaviour (summarized for example in ref. 1) have raised the question whether turbulence in fluid flows could be understood as dynamical chaos. Several proposals on possible routes to chaos have been made[2–4], and these have found some support from observations in bounded flows such as convection in a box[5] or Taylor-Couette flow[6]. However, the existence of any connection between the "dynamical chaos" exhibited by such systems and turbulence in *open* flows like boundary layers, jets, etc. has often been questioned. It is generally felt[4,7] that the chaotic phenomena observed in dissipative systems governed by nonlinear ordinary differential equations may be relevant only to the onset stage of turbulence, i.e. to "weak" turbulence.

More specifically, there are at least three issues that need to be faced[7].

(i) Chaotic dynamical systems do not exhibit a strong cascade process, of the kind generally considered an essential feature of turbulence[8], where energy put in at low wave numbers or frequencies produces strong fluctuations at very high wave numbers or frequencies. The spectrum in the dynamical systems studied to-date (e.g. the Lorenz equations[9]) tends to fill up at low (rather than high) frequencies by period-doubling or other similar mechanisms: i.e. the chaos is 'slow'.

(ii) In fluid flows, especially those that are open (e.g. boundary layers), the critical value of a control parameter like the Reynolds number at onset of turbulence is *not* unique (even for a given flow), but depends strongly on environmental disturbance levels[10].

(iii) For values of the control parameter beyond the critical value, turbulence invariably persists in the flow, whereas in dynamical systems chaos and order often alternate in narrow windows, and indeed chaos eventually tends to disappear as the control parameter is increased (once again, the Lorenz system is a good example[9]).

The question that arises is whether the above criticisms apply only to systems considered to-date or whether other systems exist that more closely mimic the essential characteristics of 'strong' flow turbulence listed above. The present work is an attempt to devise the simplest possible systems that exhibit the generic behaviour of strong turbulence.

2. GENERAL CONSIDERATIONS

The idea underlying the present models is to treat turbulent flow as interaction between motions at different scales: the emphasis is not on any particular flow such as that over a flat plate or behind a cylinder, but rather on general physical arguments valid for a wide class of turbulent flows. For this purpose we divide spectral (or wave number) space into two broad regions, one where nonlinearity and external disturbances play the major role, representing the so-called large scale or large eddy motion, and the other where viscous dissipation is dominant, representing small or Kolmogorov scale motion. These two scales are coupled by a nonlinear energy transfer mechanism often called the cascade process, schematically illustrated in figure 1 (see e.g. discussion in reference 8).