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Optimal Reservoir Operation for Irrigation of Multiple Crops

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A model for the optimal operating policy of a reservoir for irrigation under a multiple crops scenario using stochastic dynamic programming (SDP) is developed. Intraseasonal periods smaller than the crop growth stage durations form the decision intervals of the model to facilitate irrigation decisions in real situations. Reservoir storage, inflow to the reservoir, and the soil moisture in the irrigated area are treated as state variables. An optimal allocation process is incorporated in the model to determine the allocations to individual crops when a competition for water exists among them. The model also serves as an irrigation scheduling model in that at any given intraseason period it specifies whether irrigation is needed and, if it is, the amount of irrigation to be applied to each crop. The impact on crop yield due to water deficit and the effect of soil moisture dynamics on crop water requirements are taken into account. A linear root growth of the crop is assumed until the end of the vegetative stage, beyond which the root depth is assumed to be constant. The applicability of the model is demonstrated through a case study of an existing reservoir in India.

INTRODUCTION

Decision making for reservoir releases for irrigation involves many subtle considerations such as the nature and timing of the crop being irrigated, its stage of growth, the competition among different crops for the available water and the effect of a deficit water supply on the crop yield. Water released from the reservoir is utilized by the crops in the form of evapotranspiration. In determining the amount of release from a reservoir, it is therefore necessary to consider the crop water requirement in relation to the crop growth and its yield. Also, in the context of multiple crops, the competition among the crops, when the available water is inadequate to meet the aggregate demand, must be taken into account in deciding upon the release from the reservoir. From the point of view of efficient use of water at the farm level, this implies the need for a single decision-making mechanism for the entire system. The decision should be sufficiently explicit to indicate not only how much water is to be released from the reservoir in a given period but also how much of it should be allocated to a given crop.

The uncertainty in the various hydrologic variables involved, rainfall, evapotranspiration, soil moisture and the reservoir inflow, adds to the complexity of decision making. Mathematical modeling can aid in the development of optimal reservoir operation for irrigation given the complete cropping scenario in the command area. Earlier models developed for optimal reservoir operation for irrigation dealt with different aspects of the problem in different degrees of complexity. Stochastic dynamic programming proved a potential tool in developing reservoir operation models in the recent past [e.g., *Butcher*, 1971; *Torabi and Mobasheri*, 1973; *Dudley and Burt*, 1973; *Mawer and Thorn*, 1974; *Roefs and Guitron*, 1975; *Bogle and O'Sullivan*, 1979; *Oven-Thompson et al.*, 1982; *Stedinger et al.*, 1984; *Esmaeil-Beik and Yu*, 1984; *Goulter and Tai*, 1985; *Karamouz and Houck*, 1982]. *Yeh* [1985] presents a comprehensive state-of-the-art review of the various reservoir operation models.

Whenever reservoir management for irrigation has been discussed, even through variability in reservoir inflow was taken into account [*Hall et al.*, 1968, 1969; *Schweig and Cole*, 1968] most of the approaches treated seasonal crop water demand for irrigation as deterministic. Exceptions to this generalization are the approaches of *Sanford* [1969], *Dudley* [1970], and *Burt and Stauber* [1971] which stressed the need to take account of intraseasonal variations in crop water requirement. *Burt and Stauber* [1971] incorporated stochastic crop water demand but assumed a deterministic water supply. Both stochastic water demand and stochastic addition to storage were considered by *Dudley et al.* [1971*a*, *b*, 1972], *Dudley* [1970, 1972], and [*Dudley and Burt* [1973].

Dudley and Burt [1973] developed an integrated intraseasonal and interseasonal stochastic dynamic programming model to determine an optimum decision rule for intertemporal water application rates and crop acreage decisions for a single crop. Area available for irrigation, soil moisture level, available water (reservoir content) and a measure for the crop production function are treated as state variables. In the application, soil moisture was deleted as a state variable, however. Subsequently, *Dudley et al.* [1976] developed a hierarchy of models to aid decisions in multicrop systems for deterministic crop water requirements. These models use linear programming (LP), simulation and dynamic programming. Linear programming is used to select best crop combinations, simulation to predict changes in reservoir storage and dynamic programming to optimize interseasonal water allocation. *Dudley* [1988] advanced this earlier work by simulating the effects of the optimal decisions under the assumption of a single decision maker. Development of a single model, computationally simple enough for application to reservoir operation for multiple crops, which takes into account stochastic reservoir inflows and stochastic evaporative demands with dynamic soil moisture accounting, still remains a formidable task.

In determining the release policy for irrigation, the decision interval often used is the crop growing season or the individual growth stages of the crop. However, the irrigation release decisions have to be made in a much shorter time

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ıterval, such as a week or two weeks in a real situation. 'herefore, models which aid decision making over large time ıtervals such as a month or a season are inadequate, as they .o not take into account the variability in irrigation demand vithin these time intervals. What is therefore necessary is to letermine the optimal operating policy of the reservoir 'is-à-vis the intraseasonal irrigation requirement of all the :rops, to enable release decisions to be made such that the 'ight amount of water is provided at the right time.

Apart from the need to consider relatively shorter time ntervals, the occurrence of possible competition for water ımong different crops in an intraseason period adds to the :omplexity of the problem. However, policies for optimal allocation of water among competing crops, based on detailed crop information, even with some assumptions to facilitate computations, would be of immense value in planning and real time operation of irrigation reservoirs. The present study deals with reservoir modeling with this purpose in view. To keep the model application computationally tractable, only the reservoir inflows are assumed to be stochastic, however. The present paper, in contrast to the paper by *Dudley et al.* [1976], develops a model for reservoir irrigation of multiple crops with stochastic reservoir inflows but with variable irrigation demands. The demands which vary from period to period are determined from a soil moisture balance equation with soil moisture specified as a state variable in the three state variable stochastic dynamic programming model (the other states being reservoir inflow and storage). Rainfall and evapotranspiration are treated as deterministic in computing the irrigation application to different crops. Irrigation is given to a crop in any period if the available soil moisture falls below a specified level, in which case the amount of irrigation is such as to bring the soil moisture to field capacity. The reservoir storage continuity and soil moisture balance requirements are stated as constraints. The cropped areas and the crop calendar are assumed to be fixed.

The proposed model, which integrates the reservoir release decisions with the irrigation allocation decisions at the field level with respect to each crop in each period, is formulated conceptually to operate in two phases. In the first phase, the model uses deterministic dynamic programming and allocates a given amount of water among all the crops to optimize the impact of the allocation within a period. This allocation is determined for all possible supplies in a given period, for all periods in a year. These "within year" periods are referred to as intraseasonal periods. In the second phase, a stochastic dynamic programming (SDP) model evaluates all the intraseasonal periods to optimize the overall impact of the allocations over a full year. The end result of this two-phase analysis is a set of decisions indicating the reservoir release to be made in each intraseason period and the distribution among the crops of this release available at the crop level (accounting for losses between the reservoir and the application area).

The next section of the paper gives an overview of the second phase of the model, which is solved by stochastic dynamic programming. It includes subsections which discuss the state variables and the recursive equation. Then follows a section which describes the first phase of the model, in which the inputs for use in the second phase are calculated. These inputs consider potential and actual evapotranspiration and soil moisture balances, and represent optimal within-period allocations of water among crops. An application to an actual irrigation operating problem is then presented, followed by some closing comments.

MODEL DEVELOPMENT

The formulation of the stochastic dynamic programming model used to determine the steady state optimal operating policy for a single-purpose irrigation reservoir in the context of multiple crops is discussed in this section. The SDP model is solved using backward recursion beginning from a year chosen sufficiently distant in the future to arrive at a steady state policy. The steady state policy is one which gives a steady state value of the objective function over a year as defined later following equation (5).

State Variables

In a hydrologic system the number of variables influencing the decision is so large that it becomes computationally impossible to consider all of them simultaneously. It is therefore necessary to choose only those variables that influence the decision process the most. The state variables vector ϕ_t in this study is defined as

$$\phi_t = \{S_t, Q_t, \theta_t\}, \tag{1}$$

where S_t is the reservoir storage (volume) at the beginning of period t, Q_t is the reservoir inflow (volume) during period t, and θ_t is the average initial soil moisture (depth per unit root depth) at the beginning of period t.

Stochasticity in the model is introduced through the specification of the reservoir inflow Q_t as the stochastic variable. These inflows are assumed to constitute a simple (or one-step) Markov chain and the process is assumed to be stationary. This implies that the inflow transition probability matrix for any given period within a year does not change from year to year which allows the derivation of a steady state operating policy from the model.

Rainfall and potential evapotranspiration that affect the release policies are, however, taken as deterministic inputs to the model.

Discretization

Each state variable is discretized so that its value at any given time lies in one of several "class intervals" or "classes" through which the possible range of the state variable is divided. Any value within the range of a class interval is represented by a single value for that class interval, referred to as its representative value. The larger the number of such class intervals, the better the approximation of the variable. However, an increase in the number of class intervals would result in an increase in the required computer memory and/or run time. One guiding principle in designing the discretization scheme for the state variables (e.g., reservoir storage and inflow in a specific case) is to see that trapping states are avoided in reservoir simulation.

State Transformation

Let the indices k and l represent the class intervals for the storage, and m and n for the soil moisture, at the beginning of periods t and $t + 1$ respectively. Let i and j be the class intervals for inflow during the periods t and $t + 1$, respectively. Let S_k^t, S_l^{t+1}, θ_m^t, θ_n^{t+1}, Q_i^t and Q_j^{t+1}, be the representative values of the corresponding variables in the respective class intervals.

The reservoir storage transformation is governed by the continuity equation,

$$S_l^{t+1} = S_k^t + Q_i^t - R_{kilt} - e_{klt} \tag{2}$$

where R_{kilt} is the release and e_{klt} the evaporation loss when the initial storage class interval is k, inflow class interval is i and the final storage class interval is l in period t. Equation (2) specifies the release for a given combination of k, i and l for each period t. It may be noted that some of the various theoretical combinations of k, i and l may not be feasible, as they result in a negative value of R_{kilt}.

The soil moisture transition from period to period is governed by the soil moisture continuity equation. The soil moisture balance defines the soil moisture state n at the beginning of the period $t + 1$, given the initial soil moisture state m at the beginning of the period t and the irrigation application in the period t. This aspect is discussed in detail later in the paper.

Decision Intervals in the Context of Multiple Cropping

The length of the decision periods in an irrigation optimization problem depends on two factors: (1) the smallest time interval during which irrigation decisions are to be taken, consistent with the availability of data and (2) the duration of the crop growth stages. Irrigation decisions in the field are usually made weekly or biweekly. The decision interval should accordingly be 2 weeks or less in order to provide a useful guide for operating the reservoir.

The growing season of a crop may be divided into four or five critical growth stages on the basis of the crop response to water stress, such as the establishment, flowering, vegetative and yield formation stages. The crop water requirement and the sensitivity of the crop yield to water deficit are different in the different growth stages. For example, a water deficit of a given magnitude occurring in the vegetative stage of a crop may cause a greater reduction in yield than the same deficit occurring in the yield formation stage. It is therefore important to consider not only how much of a deficit occurs during the crop season but also at which growth stages of the season the deficit occurs.

Furthermore, different growth stages of a crop are often of different lengths. Thus, if the decision intervals in the operation model are made equal in length to the growth stages of a crop, not only will the decision intervals be all of unequal lengths, but in the case of multiple crops, the decision intervals may not coincide with the growth stages of every crop. It is therefore necessary that the decision interval be such that the total time (number of periods) elapsed from the start of the crop to the end of any growth stage of any crop be an integral multiple of the decision interval. For modeling purposes, this condition can be achieved by marginally adjusting the lengths of individual growth stages (normally only by a few days), if necessary. This does not cause any serious error considering the fact that a change of crop growth stage from one stage to a subsequent stage is never abrupt, as the complete change occurs usually over a span of a few days. Within a certain time period, two different crops may be in two different growth stages. However, since the crop characteristics within any growth stage of a crop are known, this does not pose any conceptual difficulty in model formulation.

The Recursive Equation for the SDP Model

Let G denote a measure of the system performance which must be minimized. G is, in general, a function of the release R_{kilt}, the soil moisture θ_m^t and the crop parameters during the period t. The nature and the detailed development of this function is discussed later in this paper. For the present, it is assumed that the value of G for given k, i, l, m and t is known. The objective function of the SDP model is then written as

$$\underset{\{\text{feasible } l\}}{\text{Minimize}}\ E_t[G(k, i, l, m, t)] \qquad \forall k, i, m \tag{3}$$

where $E_t[\ ]$ denotes the expected value, over all the periods in a year, of the function contained in brackets.

The model is initiated at some arbitrary year Y in the future at the last period T. This arbitrary year is chosen sufficiently distant in the future to enable the derivation of a steady state operating policy from the model solution through backward recursion.

Let N define the number of periods remaining till the end of year Y, and $f_t^N(k, i, m)$ represent the sum total of the expected value of the system performance over N time periods (with N periods to go), including the current period t, given that the initial storage is S_k^t, the inflow is Q_i^t and the average initial soil moisture is θ_m^t in the current period t. With only one period remaining ($N = 1$ and $t = T$),

$$f_T^1(k, i, m) = \underset{\{l\}}{\text{Min}}\ G(k, i, l, m, T) \qquad \forall k, i, m \tag{4}$$

where $\{l\}$ denotes feasible l. In general, for period t and stage N,

$$f_t^N(k, i, m) = \underset{\{l\}}{\text{Min}}\ [G(k, i, l, m, t) + \sum_j P_{ij}^t f_{t+1}^{N-1}(l, j, n)] \qquad \forall k, i, m \tag{5}$$

where P_{ij}^t is the transition probability of the inflow, defined as the probability that the inflow in period $t + 1$ will be in state j given that it is in state i in period t. It should be noted that n in the second term in the right-hand side of (5) is the soil moisture state at the beginning of the period $t + 1$, and is a deterministic function of k, i, l, m and t.

Equation (5) is solved recursively, following *Loucks et al.* [1981], until a steady state solution is reached defining the optimal policy $l^*(k, i, m, t)$ for all values of k, i and m and for all periods t. Steady state is reached when $[f_t^{N+T}(k, i, m) - f_t^N(k, i, m)]$ becomes constant for all k, i, m and for all t.

The next section describes how the values of $G(k, i, l, m, t)$ and n are obtained in a deterministic dynamic program which allocates water among crops within a period.

DEVELOPMENT OF THE PERFORMANCE MEASURE

The system performance measure $G(k, i, l, m, t)$ reflects the response of the crops to the level of irrigation applied. When a certain release R is made at the reservoir, its ultimate utility depends both on how much of it is used for crop production and how it is allocated among the various crops. The performance measure $G(k, i, l, m, t)$ is determined based on an optimal allocation of the available water (limited to the release R_{kilt}, adjusted for losses) in a given intraseason period t among the various crops. At this time, the water available for allocation and the initial average soil moisture in the irrigated area are assumed to be given or known. In the case of multiple crops, water is allocated to the various crops taking into account any competition for water that might exist among them in the period t such that a specified objective function (expressing the impact of the allocation on crop growth, described later in (7)) is optimized.

The water available for irrigation may be written as

$$X_{kilt} = \beta R_{kilt} \tag{6}$$

where β is the field irrigation efficiency (ratio of the volume of water available at the farm to the volume of water released at the reservoir).

The actual response of a crop to water application is best described by the crop production function, which can be used to find the relative yield (ratio of actual to potential yield) of a crop for a given amount of water deficit. A good predictor of crop yield is the actual evapotranspiration rate. Studies on plant water stress show that the stress occurs when the actual evapotranspiration, ET_a (depth units), is less than the maximum (potential) rate of evapotranspiration, ET_{max} (depth units). Soil water stress does not occur when ET_a equals ET_{max}, under which condition the plant is assumed to have the optimum growth [*Fogel et al.*, 1976].

Objective Function

On the basis of the production function given by *Doorenbos and Kassam* [1979], the following objective function is considered for the optimal allocation of the available water among different crops in a period t:

$$\text{Min} \sum_{c=1}^{C} ky_c^t[1 - ET_{a,t}^c/ET_{\max,t}^c] = G(k, i, l, m, t) \tag{7}$$

where c is the crop index, ky_t^c is the yield factor of the crop c corresponding to the growth stage to which the period t belongs, and C is the numbr of crops in period t.

The yield factor ky reflects the sensitivity of the crop yield to a water deficit. A higher ky would, in general, mean a higher reduction in the crop yield for the same amount of water deficit. The yield factors are available in the literature only for individual growth stages of a crop. It is assumed, in the present study, that the yield factors for the different time periods constituting a growth stage are the same as those for the growth stage. This assumption has been made in earlier investigations [*Bras and Cordova*, 1981; *Rao*, 1985].

The value of $G(k, i, l, m, t)$ is zero in (7) if the volume of water available, X_{kilt}, is greater than or equal to the total water requirement of all the crops, as there is no competition for water, and therefore no moisture deficit. This permits water allocation to individual crops such that $ET_a = ET_{max}$. The competition for water exists only if $X_{kilt} < \sum_{c=1}^{C} \text{IRR}_c^t$ where IRR_c^t is the irrigation requirement of crop c in period t in volume units. The available water in such a case is allocated by solving the allocation problem (equation (7)) by a procedure discussed subsequently. The details of the different components required for solving the allocation problem are discussed below before the procedure used for its solution is discussed.

Irrigation Requirement of a Crop

The irrigation requirement IRR_c^t of a crop c in a given period t depends on the initial soil moisture level and the rainfall contribution to the soil moisture. It is determined as follows.

The irrigation policy (used in the present study) is to apply irrigation to a crop c in period t only when the available soil moisture (soil moisture above the permanent wilting point) in the root zone is below $(1 - d)(Z_f - Z_w)\, D_c^t$, where d is the soil moisture depletion factor (expressed as a fraction) and D_c^t is the root depth of the crop c in period t in depth units, and Z_f and Z_w are the moisture levels at field capacity and at permanent wilting point, respectively, expressed in depth per unit root depth. The amount of irrigation in such a case is such that if sufficient water is available the soil moisture in the root zone is raised to the field capacity.

Thus, the irrigation requirement of a crop, c, during a period t, IRR_c^t (in volume units) is given by,

$$\begin{aligned} &\text{IRR}_c^t = 0 \qquad (\theta_m^t - Z_w)D_c^t + \text{RAIN}_t \geq (1 - d)(Z_f - Z_w)D_c^t \\ &\text{IRR}_c^t = [Z_f D_c^t - (\theta_m^t D_c^t + \text{RAIN}_t)]\text{AREA}_c \qquad \text{otherwise} \end{aligned} \tag{8}$$

where RAIN_t is the rainfall in period t and AREA_c is the area of crop c. It is assumed that all the rainfall infiltrates into the soil and contributes to soil moisture storage.

In (8), IRR_c^t is expressed in volume units, AREA_c is in area units, RAIN_t and D_c^t are in depth units and Z_f, Z_w and θ_m^t are in depth per unit depth (of root zone) units.

Actual Evapotranspiration ET_a

The actual evapotranspiration ET_a is given by [*Doorenbos and Kassam*, 1979],

$$\begin{aligned} &ET_a^t = ET_{\max} \qquad Z_t \geq (1 - d)(Z_f - Z_w) \\ &ET_a^t = \frac{Z_t}{(1 - d)(Z_f - Z_w)} ET_{\max} \qquad \text{otherwise} \end{aligned} \tag{9}$$

where Z_t is the available soil moisture in the root zone in period t (expressed as depth per unit depth of root zone), determined by (10) below.

Determination of the actual evapotranspiration requires the knowledge of the available soil moisture at a given time in the root zone, while the root depth itself increases progressively with time. A simple linear root growth is assumed in the model. The root is assumed to attain its maximum depth at the end of the third growth stage (flowering) and remain constant thereafter till the end of the crop season. The root depth in any period is approximated by the depth of the root corresponding to the midpoint of the period.

Soil Moisture Balance

The following discussion of soil moisture balance is relevant to each of the crops considered. The crop subscript, c, in the terms of (10), (11) and (12) below is, therefore, omitted for simplicity. It should be noted that the term RAIN_t is the

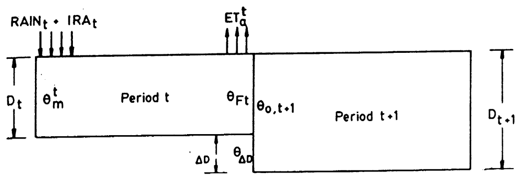

Fig. 1. Definition sketch for soil moisture transition for a crop.

same for all the crops in a given period. The values Z_f and Z_w, being functions of the soil type, are the same for all the crops in all the periods, as all the crops are assumed to be grown on the same type of soil.

The available moisture in period t, Z_t, for a given crop, is determined for a known irrigation application by

$$Z_t D_t = (\theta_m^t - Z_w)D_t + \mathrm{IRA}_t + \mathrm{RAIN}_t \tag{10}$$

where D_t is the root depth of crop in period t and IRA_t is the irrigation applied during the period t (depth units).

The initial soil moisture, θ_m^t, is assumed known. This is the representative value of the soil moisture class interval m for which the value of the performance measure $G(k, i, l, m, t)$ is being determined. The depth, D_t, of the root zone is computed from the root depth model. Rainfall is treated as deterministic and the average rainfall in period t is used as RAIN_t.

Thus, for any given level of irrigation application IRA_t, the available soil moisture Z_t can be determined. This soil moisture level is then used to determine the actual evapotranspiration (ET_a) from (9). The resulting soil moisture at the end of the period t, θ_{Ft}, is computed by the equation,

$$\theta_{Ft} D_t = \theta_m^t D_t + \mathrm{IRA}_t + \mathrm{RAIN}_t - \mathrm{ET}_a^t \tag{11}$$

where ET_a^t is the actual evapotranspiration in period t.

The value θ_{Ft} should be adjusted to represent the initial soil moisture to be used for the subsequent period $t + 1$ as the root depth for $t + 1$ is different from that of t. Thus,

$$\theta_{0,t+1} = \mathrm{Min}\left(\frac{\theta_{Ft}D_t + \theta_{\Delta D}(D_{t+1} - D_t)}{D_{t+1}},\ Z_f\right) \tag{12}$$

where $\theta_{0,t+1}$ is the initial soil moisture at the beginning of the period $t + 1$ and $\theta_{\Delta D}$ is the soil moisture in the layer of soil added to the root zone in the previous period, assumed to be known (in the present case, a constant value equal to the field capacity).

Starting with an average soil moisture value of θ_m^t over all the crops, the soil moisture balance determines the value of $\theta_{0,t+1}$ for each crop. The class interval to which the average value of $\theta_{0,t+1}$ over all the crops belongs is denoted as n. The representative soil moisture of this class is θ_n^{t+1}. Figure 1 schematically represents the soil moisture transition between two adjacent time periods t and $t + 1$.

Allocation Problem

The problem of water allocation among different crops, for known X_{kilt} and θ_m^t in period t is solved as a one-dimensional allocation problem using the backward moving algorithm of dynamic programming. This needs to be solved only when competition for water exists among the crops.

The objective function for the allocation problem is specified by (7). As the allocation problem is always solved for a given period t, the symbol t appearing as either a subscript or a superscript is omitted henceforth in this section for convenience in presentation.

Each crop constitutes a stage in the dynamic program, i.e., there are as many stages as there are crops. The state variable is the amount of water q_r available at a given stage r for allocation among all the stages up to and including that stage. The crops are indexed $c = 1, 2, \cdots, u$ and the stages are indexed $r = 1, 2, \cdots, R$. In backward recursion then, $c = 1$ corresponds to $r = R$, $c = 2$ to $r = R - 1$, and so on. The last crop $c = u$ corresponds to $r = 1$. Numerically, $R = u = C$, the total number of crops.

Let $g_{rm}(q_r)$ be the minimum value of the objective function for given m when q_r is allocated to r stages and $q(r)$ be the allocation made at the rth stage to the crop corresponding to that stage. The allocation $q(r)$, which is in volume units, is divided by the area of the crop for which it is allocated to get IRA_t, the irrigation depth applied to the crop, which is used in the determination of the available soil moisture Z_t (equation (10)) which in turn is used to compute ET_a (equation (9)).

At stage 1 ($r = 1$), i.e., with only the last crop remaining for allocation,

$$g_{1m}(q_1) = \underset{q(1)}{\mathrm{Min}}\left[ky_u\left(1 - \frac{\mathrm{ET}_a}{\mathrm{ET}_{\max}}\right)_u\right] \qquad 0 \le q(1) \le q_1 \le X_{kilt} \tag{13}$$

The value $q(1)$ divided by the area of the crop u gives the irrigation application IRA_t defined earlier. The ET_a value in (13) is then obtained using (10) and (9).

The recursive relation for the rth stage may be expressed as

$$g_{rm}(q_r) = \underset{q(r)}{\mathrm{Min}}\left[ky_{u-r+1}\left(1 - \frac{\mathrm{ET}_a}{\mathrm{ET}_{\max}}\right)_{u-r+1} + g_{(r-1)m}(q_r - q(r))\right] \qquad 0 \le q(r) \le q_r \le X_{kilt} \tag{14}$$

At the last stage ($r = R$), all the crops remain to be allocated and $q_R = X_{kilt}$.

$$g_{Rm}(q_R = X_{kilt}) = \underset{q(C)}{\mathrm{Min}}\left[ky_1\left(1 - \frac{\mathrm{ET}_a}{\mathrm{ET}_{\max}}\right)_1 + g_{(R-1)m}(q_R - q(R))\right] \tag{15}$$

Here, $g_{Rm}(X_{kilt})$ represents the value of the objective function for optimal allocation of the available water X_{kilt} for a given m among all the crops in the time period t under consideration. Thus

$$G(k, i, l, m, t) = g_{Rm}(X_{kilt}) \tag{16}$$

$$\forall\ k, i, l \text{ (feasible)}, m \text{ and } t.$$

When the allocation model is solved, X_{kilt} would have been allocated fully and optimally among the various crops. The optimal allocations are used in the soil moisture balance

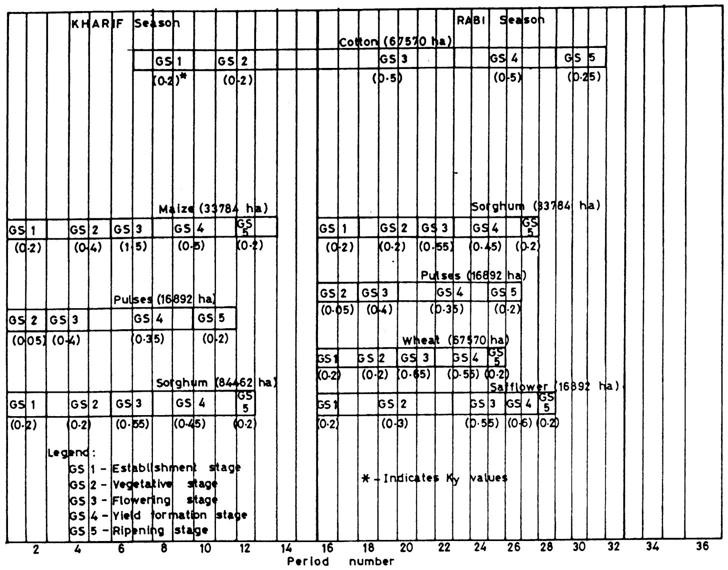

Fig. 2. Crops and crop calendar.

(equations (10), (11) and (12)) to obtain n. The value of n thus obtained and of $G(k, i, l, m, t)$ obtained from (16) are carried into the recursive equation of the SDP, equation (5).

The limitation in using an average soil moisture over all crops in the formulation is discussed later in the general remarks section.

APPLICATION TO AN EXISTING RESERVOIR

The application of the model is demonstrated through a case study of the Malaprabha Reservoir in the Krishna Basin, Karnataka state, India. The reservoir is a single-purpose irrigation reservoir on Malaprabha River, a tributary to Krishna, and has been in operation since 1973. Located in the northern region of Karnataka state, the reservoir has a major portion of the irrigated area (71%) in black cotton soil (montmorrillonitic, expansive clay with very little organic matter, categorized as CH as per the unified soil classification system). The model is applied to the crops grown on this single soil type. The major crops grown are cotton, wheat, sorghum, maize, safflower and pulses. Figure 2 shows the principal crops and the crop calendar for the black soils area.

There are two principal cropping seasons, kharif (monsoon season) and rabi (nonmonsoon season) and eight crops in all, three in kharif, four in rabi, and one two-seasonal crop, cotton. It is to be noted that if the same crop is planted at a different time in the year, as far as the model is concerned, it has to be treated as a separate crop.

In terms of assessing the water available from the Malaprabha Reservoir, the crop water requirement of the crops in the black soil area is esimated to be two thirds of the requirement for the total area served by the reservoir. Hence, in the model application two thirds of the release from the reservoir was assumed to be available for irrigating the crops in the area. The optimal policy for the reservoir operation was derived based on allocating this available water among the crops grown in the black soil area. A decision interval of 10 days was chosen for the study. Daily inflows for a period of 35 years (June 1951 to May 1986) were used to develop the transition probability matrices for the 10-day periods. The inflows prior to the construction of the reservoir were taken from a nearby downstream gauging station. The growth stages of the crops were adjusted to be multiples of the decision intervals (of 10 days) with the pertinent information on crop growth stages being taken from *Doorenbos and Kassam* [1979]. A value of 0.45 for d was used in (8).

Discretization of the State Variables

The discretization of the inflows and the reservoir storages was performed concurrently to avoid trapping states. The possible range of inflows was identified from the historical data and divided into four class intervals in each period. The discrete values representing the class intervals were chosen at their midpoints (except in the fourth class interval containing the extreme flows in which the representative value was chosen based on the relative frequency of occurrence of such inflows).

A number of simulation runs were carried out with historical inflow data using the standard operating policy (a policy in which the release in any period equals the demand or the available water, whichever is less) with each period's total crop demands (computed ignoring the soil moisture contribution but taking rainfall into account), each time with a different number of class intervals for storage. It was found that 12 intervals quite often resulted in trapping states in simulation. When the number was increased to 15, trapping states were avoided. The grid of storage discretizations used in this study is different for different periods. The details of the discretization of the state variables with their respective class intervals, model formulation, and application are given by *Mujumdar* [1988].

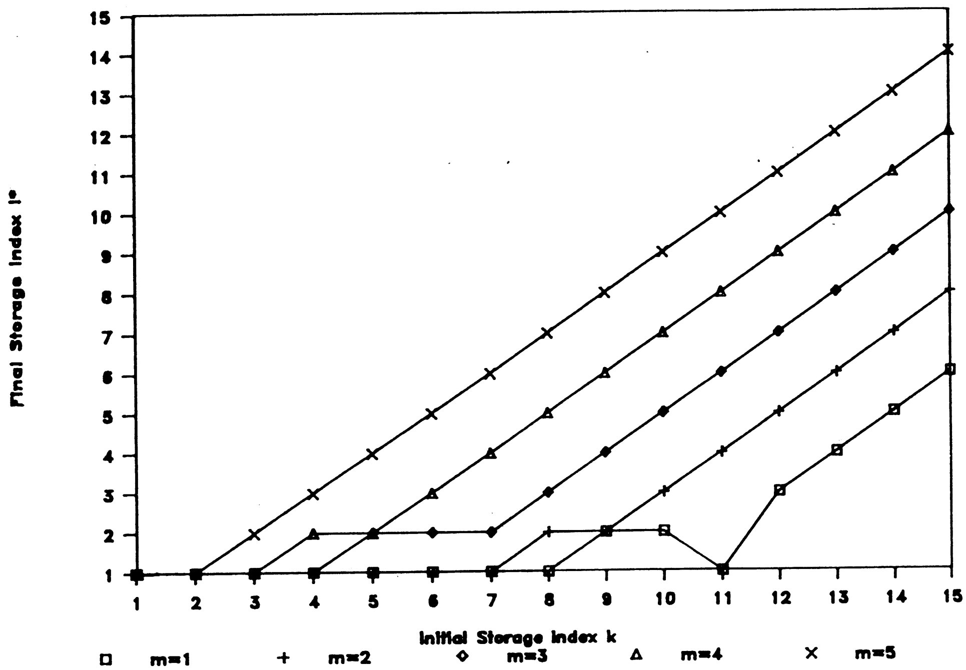

Fig. 3. Steady state policy for period t = 22; inflow state i = 1.

Optimal Operating Policy

The solution was carried out in two phases. The first phase consists of the determination of the values of $G(k, i, l, m, t)$ and the associated values of n for each feasible combination of k, i and l for every m and for each period t. The second phase uses these values to solve the recursive equation, defined by (5).

In solving the allocation problem when competition for water exists, the available water X_{kilt} for allocation was divided into 12 discrete values. The individual crop allocations were selected for each of these 12 values. With the number of storage classes set at 15, the inflow classes at four, the soil moisture classes at five and with 36 within-year periods, there are 162,000 possible values of the function $G(k, i, l, m, t)$. In the model, however, the allocation problem needs to be solved only for feasible combinations of k, i and l, and only when irrigation is required, and even in that case, only when there is competition for water among the crops.

On a 32-bit minicomputer (Horizon III, made by Hindustan Computers Limited, India) the computation of $G(k, i, l, m, t)$ values for all possible combinations for all the 36 periods (phase 1) took approximately 90 min of CPU time. The solution yielded a steady state policy after four annual cycles.

A typical presentation of the steady state policy obtained from the model for different inflow states is shown in Figures 3 and 4 for a given time period. These figures give the policy for period 22 for inflow class intervals 1 and 3 respectively. Period 22 lies in the rabi season (nonmonsoon season) and all crops of the rabi season are represented in this period (Figure 2). Figure 3 gives l^* (optimal end-of-the-period storage state interval) values for each given initial storage state k (at the beginning of the period) and initial average soil moisture state m (at the beginning of the period) when the inflow class interval is 1 (inflow range 0 to 3 Mm^3; represented by Q_1^{22} = 1.5) in that period. Similarly Figure 4 gives the steady state policy for the inflow class interval 3 for any combination of k and m in period 22. These figures are used as follows.

For example, to determine the optimal reservoir release in period 22, in which the inflow is in state 1, the known initial storage and the average initial soil moisture level in the cropped area are entered in Figure 3. When k = 9 (storage range, 372–409 Mm^3, represented by S_9^{22} = 390.5) and m = 4 (initial soil moisture range, 1.92–2.38 mm/cm), Figure 3 gives the optimal final storage index l^* = 6 (storage range, 261–298 Mm^3, represented by S_6^{23} = 279.5). The optimal reservoir release, R_{kilt} for k = 9, i = 1, l = 6 and t = 22, is then given by (2) with the evaporation, e_{klt}, computed from the known evaporation rate for t = 22 and the average reservoir surface area (corresponding to S_9^{22} and S_6^{23}, the beginning and end of the period storages, respectively). The associated value of X_{kilt} (with β = 0.7 in (6) used in the study) and its optimal allocation to the various crops in that period are then traced through the model solution.

The steady state policy thus obtained for all the 36 periods can be used in conjunction with a suitable 10-day inflow forecasting model when operating the reservoir in real time. An alternative to using the inflow forecasts is to derive policies which do not depend on the knowledge of the current period's inflow for implementation at the beginning of the period, by identifying either a final storage volume target, subject to limitations on the releases, or reservoir

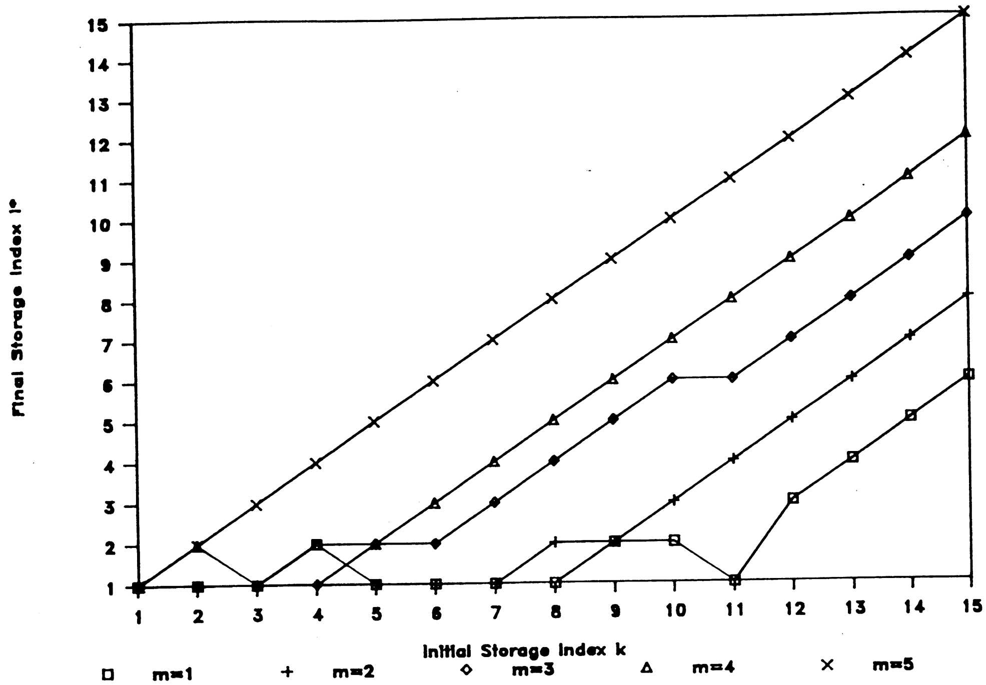

Fig. 4. Steady state policy for period $t = 22$; inflow state $i = 3$.

release targets, subject to limitations on the final storage volumes, respectively, in each period t, as indicated by *Loucks et al.* [1981].

GENERAL REMARKS

The main contribution of the paper lies in the integration of the decision-making mechanism at the reservoir level and the farm level taking into account soil moisture dynamics and root growth of multiple crops when reservoir inflows are stochastic. Assumptions are made in the model formulation to keep the application computationally tractable. The crops and the crop calendar are assumed to be fixed in the model.

A major weakness of the model lies in the averaging of the soil moisture among all the crops at the beginning of the period. This can, however, be avoided by defining a soil moisture state variable for each crop in the formulation. While there can be no conceptual difficulty in doing this, computational complexities could render the present model unworkable in a practical situation. In the context of multiple crops under a single soil type, however, the approximation is believed to be better than for multiple soil types with widely varying moisture-holding capacities (such as noncohesive and cohesive soils). The error introduced in using an average value for the initial soil moisture in the model can be assessed only by reformulating the problem with additional soil moisture state variables, one for each crop. Without such a rigorous assessment, any judgment on the validity of the procedure used can only be speculative.

Another limitation of the study is the assumption that the rainfall and the evapotranspiration in the irrigated area are considered deterministic, while the reservoir inflow is considered stochastic. This, however, may not be a serious limitation where the reservoir is meant to serve for irrigation of a drought-prone area that is subjected to relatively very low rainfall. The impact of the assumption in the present study was examined. In the present case study, Malaprabha Reservoir serves to supply irrigation water to drought-prone areas in the north Karnataka state. Although the average annual rainfall in the irrigated area is 623 mm, most of it occurs in three months (July, August and September) while the rainfall is nil or practically negligible in most other months. The coefficient of variation of weekly rainfall is significantly higher than the coefficient of variation of the corresponding weekly evaporation. Therefore, the sensitivity of the steady state policy in the present study was examined by varying only the rainfall. The steady state policy was derived in each case by setting the rainfall at four different levels: zero, 0.67, 1.0, and 1.67 times the average rainfall in each time period. It was found that l^* values for given k, i, m, t were identical for the first two levels of rainfall, and also for the last two levels. The two sets of l^* values differed for 20% of the total possible combinations of k, i, m, and t. The differences ocurred mostly in periods in which the rainfall is relatively high. Even in these periods of relatively high rainfall, the shift in the optimal final storage index l^* has been only marginal (less than three class intervals in most cases). It is recognized that consideration of a random rainfall contribution to crop demands does, however, pose conceptual difficulties. One way is to specify the rainfall as a stochastic variable. This involves the definition of a rainfall transition probability matrix and a joint probability density function for reservoir inflow and rainfall.

Future research should be directed toward treating both reservoir inflow and the crop water demand stochastically.

The representation of the value of the state variable belonging to a given class interval by its midpoint, as used in this study, can be improved by using appropriate interpolation techniques. Such techniques would enable f_{t+1} and $G(k, i, l, m, t)$ to be estimated for all values of the state variables, not just the class midpoints. This would allow the release variable to be continuous, yielding better solutions.

Lastly, a limitation in the formulation caused by the use of dynamic programming in the present study is recognized. The interdependency of the crop production in a given period on water allocations in other periods is not directly reflected in the model. Instead, allocation policies for each time period are derived first, based on water availability, and optimized over the year for the objective of minimizing the impact of water deficit on an integrated measure of relative yields in crop production.

CONCLUSIONS

A three state variable stochastic dynamic programming model has been developed to obtain an optimal steady state reservoir operating policy integrating the reservoir release and field allocation decisions in a single model which takes into account soil moisture dynamics and crop growth at the field level. Reservoir inflow is considered stochastic, while the crop irrigation requirements are computed based on deterministic rainfall and evapotranspiration in the irrigation area. The optimal operating policy specifies the reservoir release and crop water allocations for the various crops in any given intraseason period for known initial storage, inflow, and initial soil moisture in the cropped area. The impact of water deficit on crop yield, the effect of soil moisture dynamics on crop water requirements and possible competition for water among the crops in an intraseasonal period are taken into account.

The model has been applied to the black soils area of the Malaprabha Reservoir in Karnataka state, India.

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