Reservoir Operation Modelling with Fuzzy Logic

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Abstract. In this article, a fuzzy rule based model is developed for the operation of a single purpose reservoir. The model operates on an 'if - then' principle, where the 'if is a vector of fuzzy premises and the 'then' is a vector of fuzzy consequences. The steps involved in the development of the model include, construction of membership functions for the inflow, storage, demand and the release, formulation of fuzzy rules, implication and defuzzification. The methodology is illustrated through the case study of the Malaprabha irrigation reservoir in Kamataka, India. Reservoir storage, inflow, and demands are used as premises and the release as the consequence. Simulated reservoir operation with a steady state policy provides the knowledge base necessary for the formulation of the Fuzzy rules.

Key words: decision making, fuzzy rules, reservoir operation, uncertainty.

1. Introduction

In his classical review paper on reservoir operation models, Yeh (1985) observed that, despite considerable progress, the research relating to reservoir operation has been very slow in finding its way into practice. Simonovic (1992) has discussed the limitations in the reservoir operation models and the remedial measures to make them more acceptable to the operators. More recently, Russel and Campbell (1996) also emphasised that due to the 'high degree of abstraction' necessary for efficient application of optimization techniques, the applicability of most reservoir operation models is limited. The managers and reservoir operators are often uncomfortable with the sophisticated optimisation techniques used in the models, which are made much more complex by the inclusion of stochasticity of hydrologic variables. The fuzzy logic approach may provide a promising alternative to the methods used for reservoir operation modelling, because, as Russell and Campbell (1996) mention, the approach is more flexible and allows incorporation of expert opinions, which could make it more acceptable to operators. Shrestha *et al.* (1996) also confirm that fuzzy logic is an appropriate tool to consider the impreciseness of variables like inflows, in reservoir operation modelling. Fontane *et al.* (1997) have also dealt with the imprecise nature of objectives in reservoir operation modelling. The fuzzy logic based modeling of a reservoir operation is a simple approach, which operates on an 'if-then' principle, where 'if is a vector of fuzzy explanatory variables or premises such as the present reservoir pool elevation, the inflow, the demand, and time of the year. The 'then' is a fuzzy consequence such as release from the reservoir.

Fuzzy logic has been used in a number of water resources applications but generally as a refinement to conventional optimization techniques in which the usual 'crisp' objective and some or all of the constraints are replaced by the fuzzy constraints. Kindler (1992) used fuzzy logic for optimal allocation of water. The objective function and the constraints were taken as fuzzy and the 'Tsebyschev Polynomial' transformation was applied to transform the fuzzy constraints to suit a linear programming formulation. Bardossy and Disse (1993) applied the fuzzy logic to model the infiltration and water movement in unsaturated zone. Instead of computing the flow with an unrealistically high accuracy their fuzzy rule based model only approximates the flow, which can be measured and interpolated with reasonable accuracy.

In this article, a fuzzy rule based reservoir operation model is developed for a single purpose reservoir. The approach adopted is essentially the same as that of Russel and Campbell (1996) and Shrestha *et al.* (1996), with the difference that the expert knowledge for framing the Fuzzy rules is derived from an explicit stochastic model. A steady state policy derived from a Stochastic Dynamic Programming (SDP) model provides this knowledge base. The stochastic model is used here as an alternative to the expert knowledge that is generally available with experienced reservoir managers. The methodology for building the fuzzy rule based model is independent of the stochastic model, and any other expert knowledge may also be used in its place. The procedure is illustrated through a case study of the Malaprabha irrigation reservoir in Kamataka state, India.

2. Fuzzy Reservoir Operation Model

In modelling of reservoir operation with fuzzy logic, the following distinct steps are followed: (a) Fuzzification of inputs, where the crisp inputs such as the inflow, reservoir storage and release, are transformed into fuzzy variables, (b) Formulation of the fuzzy rule set, based on an expert knowledge base, (c) Application of a fuzzy operator, to obtain one number representing the premise of each rule, (d) Shaping of the consequence of the rule by implication, and (e) Defuzzification. These steps are discussed in the following paragraphs for a general reservoir operation problem. Some basic concepts of fuzzy logic relevant to the present work are given in Appendix. The application is demonstrated through a case study in the subsequent section.

2.1. FUZZIFICATION OF THE INPUTS

The first step in building a fuzzy inference system is to determine the degree to which the inputs belong to each of the appropriate fuzzy sets through the membership functions. The input is always a crisp numerical value limited to the universe of discourse of the input variable and the result of fuzzification is a fuzzy degree of membership (generally between the interval 0 and 1). The problem of constructing a membership function is that of capturing the meaning of the linguistic terms employed in a particular application adequately and of assigning the meanings of associated oprerations to the linguistic terms. The general scenario within which construction of a membership function takes place is as follows. The scenario involves a specific knowledge domain of interest, one or more experts in that domain, and a knowledge engineer. The role of knowledge engineer is to elicit the knowledge of interest from the experts, and to express the knowledge in some operational form of a required type. In the first stage, the knowledge engineer attempts to elicit knowledge in terms of propositions expressed in natural language. In the second stage, the knowledge engineer attempts to determine the meaning of each linguistic term employed in these propositions. The methods employed for constructing a membership function, based on experts' judgement, can be classified as *direct methods* and *indirect methods.* In the direct method, experts are expected to give answers to various questions that explicitly pertain to the construction of a membership function. In the indirect method, experts are required to answer simpler questions, easier to answer and less sensitive to various biases of the subjective judgement, which pertain to the construction of membership function only implicitly. Both the methods may involve one or more experts. A detailed discussion on construction of membership functions and assigning membership values may also be found in Ross (1997). Civanlar and Trussel (1986) and Devi and Sharma (1985) provided methods of constructing membership function from statistical data. Useful discussions related to construction of membership function may be found in Dombi (1990) and Klir and Yuan (1997).

Figure 1 shows the transformation of the storage variable, as an example. For a storage of 300 M m^3 the membership value of the variable storage is 0.7, as seen from the figure.

2.2. FORMULATION OF THE FUZZY RULE SET

The fuzzy rule set is formulated based on expert knowledge. A fuzzy rule may be of the form: *If the storage is low, and the inflow is medium in period t, then the release is low.* The expert knowledge available on the particular reservoir should always be used for formulating the rule base.

2.3. APPLICATION OF FUZZY OPERATOR

Once the inputs are fuzzified, the degree to which each part of the premise has been satisfied for each rule is known. If the premise of a given rule has more than one part, then a fuzzy operator is applied to obtain one number that represents the result

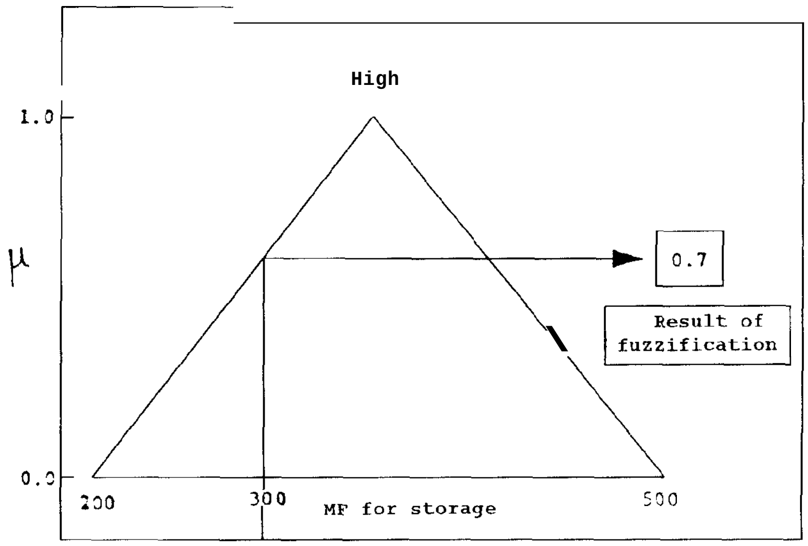

Figure 1. Fuzzy transformation of inputs.

of the premises for that rule. The input to the fuzzy operator may be from **two** or more membership functions: but the output is a single truth value.

The fuzzy logic operators such as the AND or **OR** operators obey the classical two valued logic. The AND operator can be conjunction (min) of the classical logic or it can be the product (prod) of the two parameters involved in it. Similarly the OR method can be the disjunction operation (max) in the classical logic or it can be the probabilistic OR (probor) method. The probabilistic OK method is defined by the equation:

$$\text{probor}(m, n) = m + n - mn. \tag{1}$$

The result of the 'AND' (min) operator tor the example shown in Figure 2 will be 0 3.

For example, consider the input 1 as storage and the input 2 as inflow The premise of the rule consists **of** both the inputs i.e. storage and inflow, Let the rule be ot the nature

if storage is x or inflow is y then

The two different parts of the premise yielded the membership values of 0.3 and 0.8 for storage and inflow. respectively. The fuzzy operator is applied to obtain one number that represents the result of the premise for that rule: in this case the fuzzy OR operator simply selects the maximum of the two values. 0.8. and the fuzzy operation for the rule is complete. If the rule uses the fuzzy probabilistic OR (probor) operator then the result will be calculated by Equation (1), where m and n

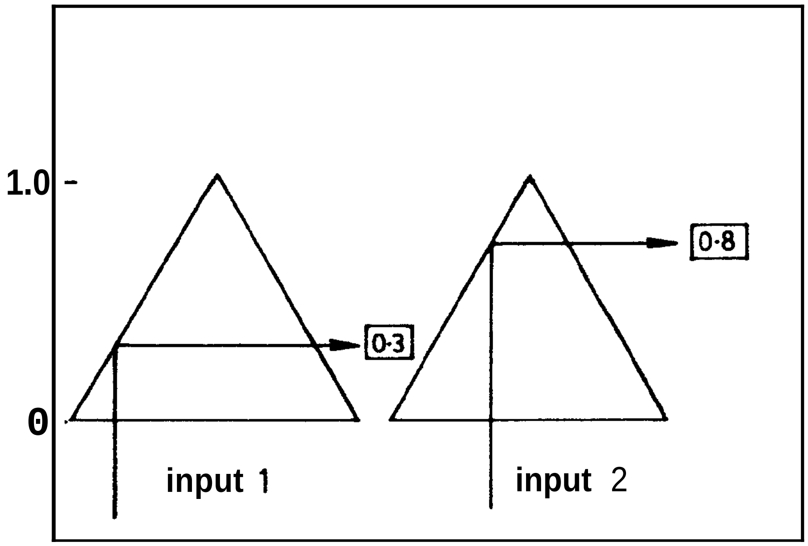

Figure 2. Application of fuzzy operator

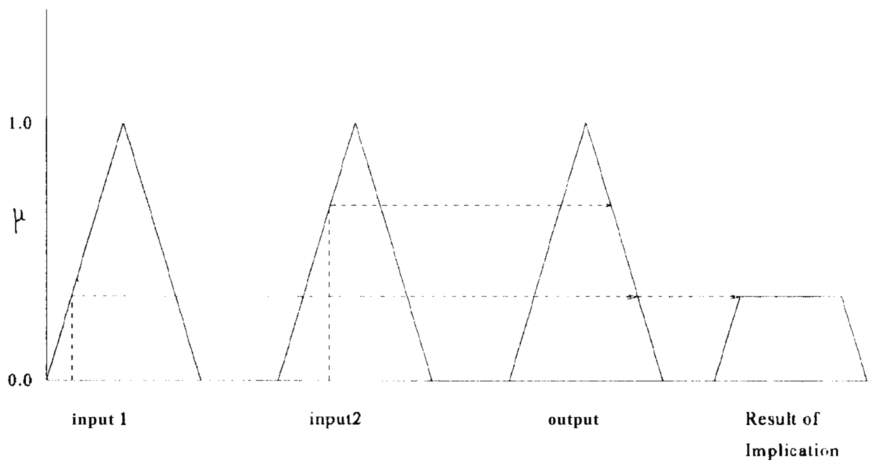

Figure 3. Truncation of output by implication.

correspond to the membership values of both the inputs. In this caw, the result of the probor operator work4 out to be 0.86.

2.4. IMPLICATION

The fuzzy operator operates on the input fuzzy sets to provide a single value corresponding to the inputs in the premise The next step is to apply this result on the output membership function to obtain a fuzzy set lor the rule This is donc by

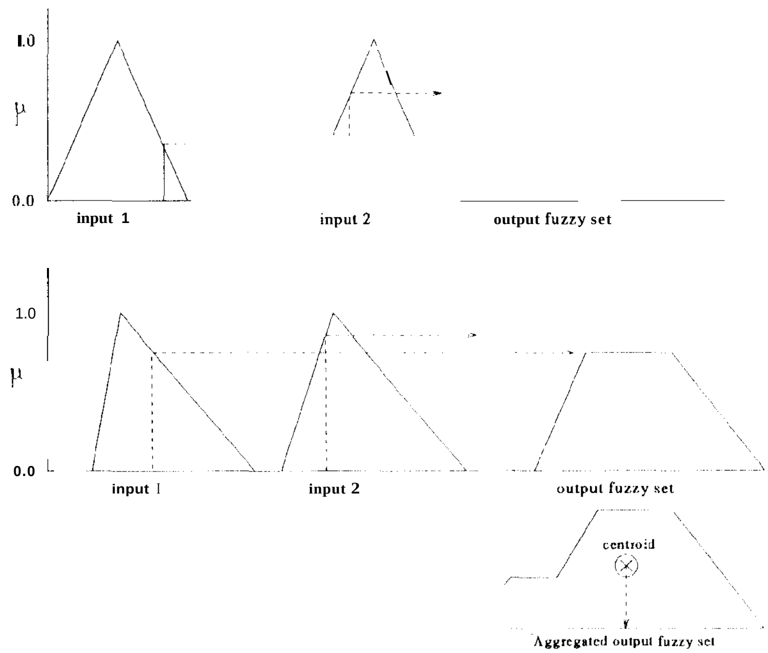

Figure 4. Aggregation of Fuzzy output

the implication method. The input for the implication method is a single number resulting from the premise, and the result of implication is a fuzzy set. Implication occurs for each rule by the AND method which truncates the output fuzzy set, or the prod method which scales the output fuzzy set.

The implication method is shown in Figure 3. In the figure, the truncation of the output fuzzy set is done at the higher **of** the two membership function values, resulting from the input 2, because the 'OK' fuzzy operator is used on the inputs. In applying the implication ***to*** a set of rules, weightages **may** be attached **to** different rules to distinguish them from each other based on priorities.

2.5. AGGREGATION

Aggregation is the unification ot' the output **of** each rule by merely joining them. When an input value belongs to the intersection of the two membership functions, fuzzy rules corresponding to both the membership functions are invoked. Each of these rules, after implication, specifies one output fuzzy set. The two output fuzzy sets are then unified to get one single output fuzzy set. Aggregation is shown in Figure 4. If more than one input lies in the intersection regions, all corresponding rules are invoked and aggregation is carried out on the output fuzzy sets. Aggregation occurs once for each variable. The input of the aggregation process is the list

of truncated output function returned by the implication process of each rule. The output of the aggregation process is one fuzzy set lor each output variable. The aggregation methods are given by: max (maximum),probor (probabilistic or), and sum (sum of each rules output).

2.6. DEFUZZIFICATION

The result obtained from implication is in the form of a fuzzy set. For application. this is defuzzified. The input for the defuzzification process is a fuzzy set and the result is a single crisp number. The most common defuzzification method is the 'centroid' evaluation, which returns the center of area under the curve. Other methods for defuzzification include 'bisection'. which returns the bisection of the base of the output fuzzy set; 'middle of maximum', which returns the value **of** middle of maximum **of** the aggregation of the truncated output fuzzy subsets; 'largest of maximum', which returns the value **of** largest of maximum of the aggregation of the truncated output fuzzy subsets; and 'smallest of maximum', which returns the value of minimum of maximum of the aggregation of the truncated output fuzzy subsets. For a general discussion **on** defuzzification, see Hellendoorn and Thomas (1993).

The defuzzification methods of maximum membership like largest. of maximum(LOM), middle of maximum(MOM) or smallest of maximum(SOM) are derived from the probabilistic method of maximum likelihood. The maximum membership defuzzification scheme suffers from two fundamental problems. First, the mode distribution of the output fuzzy sets is not unique. For continuous membership functions this leads to infinitely many modes. Second, the maximum memberschip method ignores the information **in much** of the output fuzzy set. In practice the output fumy set is highly asymmetric. Therefore infinitely many output distributions can share the same mode. Yager and Filev (1993) have shown that. the selection problem can be implemented by converting the output. fuzzy subset into a probability distribution and use this probability distribution to select the element either via performance or by calculation of an expected value.

In 'centroid' method of defuzzification we directly compute the real valued output as a normalized combination of membership values. It is given by the expression

$$G = \frac{\sum_{i=1}^{n} y_i m_B(y_i)}{\sum_{i=1}^{n} m_B(y_i)}, \tag{2}$$

where G is the centroid of the truncated fuzzy output set B. $m_B(y_i)$ is the membership value of the element y_i in the fuzzy output set B, and n is the number of elements. The centroid method of dcfuzzitication yields a unique crisp number while using all the information in the output distribution B. In the present work the centroid method is used for defuzzification.

Table I. Periodwise demands and average inflows

Period in 10-daily unit	Demand in M m^3	Average inflow in M m^3
June 1–10	10.500	14.907
June 11–20	10.575	16.867
June 21–30	41.895	46.189
July 1–10	35.565	167.092
July 11-10	37.620	192.497
July 21–31	52.575	152.935
August 1–10	59.370	156.676
August 11–20	53.475	139.616
August 21–31	55.230	xx.299
September 1–10	56.160	57.927
September 11–21	56.160	49.879
September 21–30	25.620	56.091
October 1–10	16.635	53.638
October 11–20	25.830	44.880
October 21–31	26.620	34.850
November 1–10	36.570	13.442
November 11–20	45.318	1x.974
November 21–30	40.815	16.520
December 1–10	45.360	9.849
December 11–20	54.015	6.243
December 21–31	58.935	5.519
January 1–10	51.375	5.756
January 11–20	49.275	5.431
January 21–31	50.880	5.071
February 1–10	50.655	3.887
February 11–20	39.300	3.684
February 21–28	27.150	3.520
March 1–10	27.150	3.006
March 11–20	0.000	3.056
March 21–31	0.000	2.740
April 1–10	***0.000***	3.195
April 11–20	***0.000***	3.629
April 21–30	***0.000***	4.162
May 1–10	0.000	11.562
May 11–20	0.000	5.574
May 21–31	***0.000***	11.025

3. Case Study Application

The methodology discussed in the previous section is used for modeling of operation of Malaprabha reservoir situated in the Krishna basin of Karnataka state, India. It is a single purpose irrigation reservoir which has been in operation since 1973. Located in the northern region of Karnataka state, the reservoir has a major portion of the irrigation area in black cotton soil. The major crops grown in the command area are cotton, wheat, maize, safflower, groundnut and pulses. The reservoir has a gross storage capacity of 1070 M m^3 and a live storage capacity of 870 M m^3. A water year (from June 1 to May 31) is divided into 36, 10-day periods. The duration of the last few periods was increased by one day each to compensate for the additional number of days over 360 in a given year. Table I gives the average inflows and average demands for the 36 periods in a year. The demands shown in the table are the water requirements of the Right Bank crops at the filed level. For release decisions at the reservoir, these demands are multiplied by a factor of 1.75 to account for requirements of the left bank crops and all losses. The reservoir serves a command area in a drought prone region in South India, where frequent water shortage occurs. There is a good scope for implementing scientific operating policies in the case study. A number of studies in the past have already addressed the problem of optimal operation for the case study (e.g., Mujumdar and Vedula, 1992; Vedula and Mujumdar, 1992; Mujumdar and Ramesh, 1997). However the optimization models used in those studies are far too complex and data-intensive to be of immediate use for actual operation. The fuzzy logic model presented in this article, on the other hand, is mathematically simple and provides implementable, near optimal operating policies. Since the model is not mathematically complex. the technology transfer is expected to be more effective.

The fuzzy logic tool box available with the MATLAB package, version 4.2c, is used for developing the model (MATLAB, 1995). The inputs to the fuzzy system are inflows, storage, and time-of-year. The demand is assumed to be uniquely defined for a period, and hence the variable time-of-year (the period number) is taken as the equivalent input. The output is the release during the period. For the inputs and output operations the logical and implication operators arc taken as (with conventional Fuzzy notation).

And Method	=	'Min';
Or Method	=	'Max';
Imp Method	=	'Min':
Defuzz Method	=	'Centroid'.

Where the 'And' and 'Or' method corresponds to the conjuction (min) arid disjunction (max) operation of classical logic. The implication method 'Min' produces a clipped output fuzzy set and the defuzzification method 'Centroid' is as discussed earlier in section 'Defuzzification of the Rules'.

3.1. FUZZIFICAT10N OF INPUTS

The degree to which a particular measurement of inflow or storage is high, low or medium depends on how we define the fuzzy sets of high inflow/low storage etc. This definition may arise from statistical data or neural clustering of historical data or from pooling the response of experts. In this case study of reservoir operation the SDP program was used in lieu of expert's opinion. Thus the ranges for membership function were derived from the SDP discretisation. As the number of membership functions for input/output increases, the error decreases. But as the number of membership functions increases the number of rules in the rule base also increases. There is, thus, a trade off between the number of membership functions and the number of rules, so that an acceptable near optimal (minimum error) solution is achieved. This is analogous to the dimensionality problems encountered in the Dynamic Programming (DP) models for reservoir operation. The ranges for membership functions for reservoir storage, inflow and release are fixed, keeping in view the computational considerations. The 36 ten-day periods in a year arc divided into three distinct groups for the purpose of discretisation: Periods 1 to 12 (June to September), periods 13 to 24 (October to January), and periods 25-36

(February to May). All periods in a group will thus have me sam6 Membership functions for inflow, storage and release. The storage, inflow and the release were assigned the triangular membership functions. The time-of-year (i.e., the 10-daily period number) was assigned the membership function of a straight line, hut for computational convenience, a triangle with a very small base was used. As an example, typical membership functions of the variables for period 22, January 1 10, (when crop demand is high and the average inflow during the period is low) is shown in Figure 5. It may be noted that the range of inflows for various membership functions for period 22 is the same as that for any other period between periods 13 to 24. As seen from Table **I,** the average inflow in period 22 (January I-10) is very low (5.75 M m^3), and consequently all classes of membership functions will not be represented in this period. Thus, although five membership functions have been defined for inflow during the period, in actual practice, only the first two may he represented for that period.

3.2. FORMULATION OF FUZZY RULE BASE:

In a fuzzy system, the rules are generally formed by using 'expert knowledge'. 1;' the reservoir operation model of Russell and Campbell (1996), for example, the fuzzy rules are formed based on the actual historical operation of the reservoir. In the present study, the expert knowledge is derived from a long term steady state operation of the reservoir. For this purpose, a steady slate policy is derived with stochastic dynamic programming (SDP), using the ten-day inflow transition probabilities. The objective function used in the SDP is to minimise the expected value of the squared deficit of release from the irrigation demand. Reservoir storage and the inflow during a period form the two stale variables in the SDP model. Four

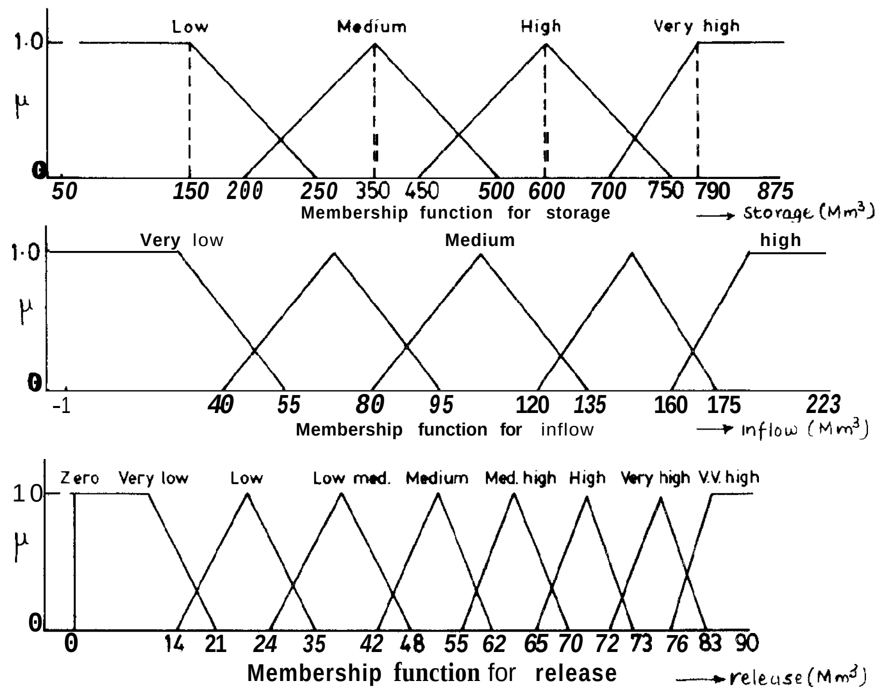

Figure 5. Membership functions for period 22.

inflow class intervals and 15 storage class intervals are used. The storage discretisation scheme is adopted from Vedula and Mujumdar (1992). The mathematical formulation of the SDP model is adopted directly from Loucks *et al.* (10X1). With 36 time periods in a year and four inflow class intervals, 36 transition probability matrices, each of size 4 x 4 arc defined. The general recursive equation of the SDP model is written, for period t and stage n as, (with backward recursion)

$$f_t^n(k, i) = \text{Min}\left[B_{kilt} + \sum_j P_{ij}^t f_{t+1}^{n-1}(l, j)\right] \forall k, i; l \text{feasible} . \quad (3)$$

where $f_t^n(k, i)$ is the expected value **of** the squared deficit upto and including period t, corresponding to the stage n in the algorithm, B_{kilt} is the sqaured deficit in the period t, corresponding to k, i and l, k is the initial storage state (class),i is the inflow state, P_{ij}^t is the inflow transition probability (probability **of** transition from state i in period t to state j in period $t+1$) and l is the storage state at the end of the period t. This recursive equation is solved until steady state policy is achieved. The steady state policy specifies, for a given period, the end-of-the period storage state (class) l^*, for a given initial storage state k and the inflow state i during a period. As an example. the steady state policy for period no. 22, is shown in Figure 6.

Table II. Typical results for one year of simulation of SDP policy

Time period (t)	Storage (S_{kt})	Inflow (I_t)	Storage class (k)	Inflow class (i)	Opt. stor. class (l^*)	End of period storage $S_l^* t+1$	Evap. loss (E_{kl^*t})	Release (R_{kil^*t})
10 days	M.Cu.m					M.Cu.m		
1	200.00	90.52	7	4	3	216.50	5.28	21.00
2	264.24	13.87	4	2	4	303.50	6.03	0.00
3	272.08	42.05	4	3	4	303.50	6.07	5.46
4	303.50	30.36	4	1	4	303.50	3.96	26.40
5	303.50	19.03	4	1	4	303.50	3.96	15.07
6	303.50	91.34	4	2	3	216.50	3.69	105.15
7	286.00	87.86	4	2	4	303.50	3.91	66.45
8	303.50	76.26	4	2	4	303.50	3.96	72.30
9	303.50	52.53	4	2	3	216.50	3.69	110.46
10	241.88	30.83	3	2	5	390.50	4.14	0.00
11	268.57	16.93	4	2	5	390.50	4.25	0.00
12	281.25	20.50	4	2	2	129.50	3.45	51.24
13	237.05	07.23	3	4	9	738.50	4.22	0.00
14	340.06	2x.23	4	2	6	477 50	1.71	0.00
15	364 53	16.26	5	2	3	303.50	3 42	53.25
16	324.12	14.31	4	2	4	303.50	3.32	31 60
17	303.50	13.36	4	2	4	303.50	3.25	10.11
18	303.50	13.41	4	2	4	303.50	3.27	10.14
19	303.50	5.00	3	2	4	303.50	2.81	3.00
20	303.50	0.15	4	1	4	303.50	2.61	0.00
21	301.04	0 27	4	1	4	303.50	2.61	0.00
22	29X.70	0.29	4	1	3	216.50	2.41	80.06
21	21650	0.32	3	1	2	129.50	2.06	85.26
24	120.50	0.32	2	1	3	216.50	2.06	0.00
25	127.76	0.37	2	1	-	129.50	1.96	0.00
26	126.17	0.37	2	1	2	129.50	2.05	0.00
27	124.48	0.37	2	1	2	129.50	2.05	0.00
2X	122.80	0.29	2	1	1	43.00	2.32	54.30
29	66.47	0.34	1	1	1	43 00	2.32	0.00
30	64.49	0.27	1	1	1	43.00	1.57	0.00
31	61.19	0.17	1	1	1	43.00	3.05	0.00
12	58.31	0.15	1	1	1	43.00	3.53	0 00
33	54.93	0.51	1	1	1	33.00	3.51	0.00
74	51.93	0.20	1	1	1	43.00	3.73	0.00
35	4X.41	0.22	1	1	1	43.00	3.69	0.00
36	44.93	0.16	1	1	1	43.00	3.67	0.00

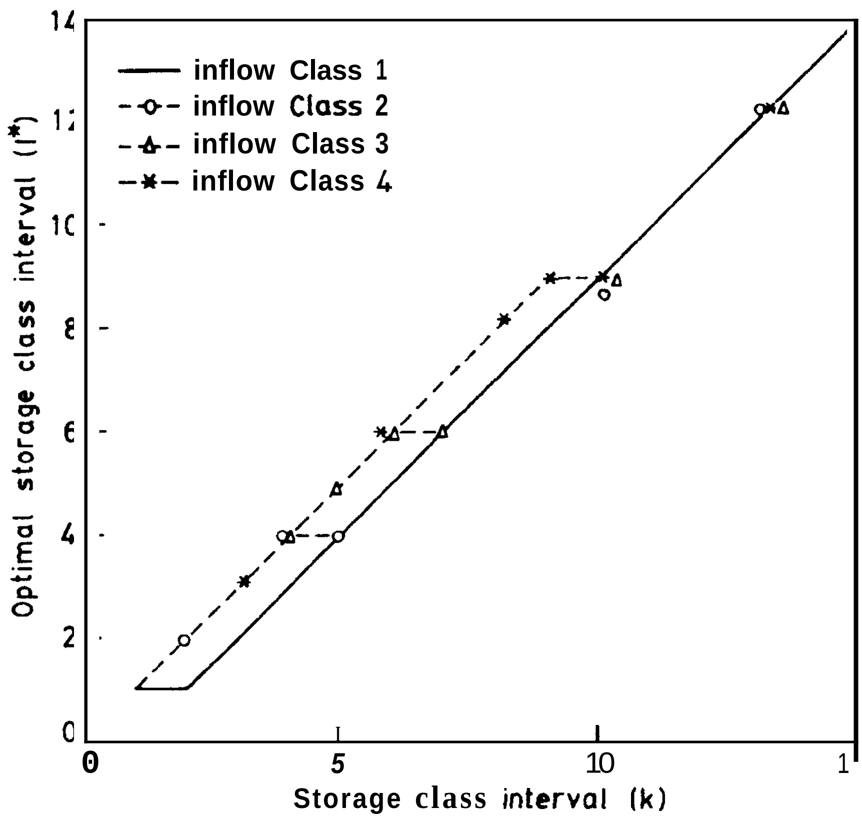

Figure 6. Steady state policy for period 22.

The reservoir release during a period may be determined for a given **initial** storage and inflow during the period, using the steady state policy. To formulate the fuzzy rules, the reservoir operation is simulated with the steady state policy, for 30 years, using historical inflow data. Table II shows one year of simulation results. This simulation forms the basis for the fuzzy rule base. The simulation results contain 30 values of reservoir storage. associated inflow and the release for each **of** the 36 within year periods. These values are matched to the respective fuzzy sets (e.g., low, medium etc.) based on the interval to which they belong. The database so generated will thus have information such as, 'Storage: Low; Inflow: Medium; Release: Low' and so on for each of the 36 periods for the 30 years **of** simulation. This database is used to formulate the fuzzy rules, for individual periods. From the database, information is picked up on the consequence (e.g., Release: Low) for each premise (e.g., Storage: Low and Inflow: very low and Time-of-Year: 22). Where one premise leads to more than one consequence (e.g., Release: Medium and Release: High), in the database for the same period in different years, the Fuzzy rule for the condition is formulated based on the average value of the simulated release. Suppose, for example, in a particular year in simulation, for period 22 (January 1–10), the storage, inflow, and release values from the SDP simulation are 180, 10 and 40, respectively. From the membership function definitions (Figure 5) these values are traced to 'low', 'very low' and 'low-med' of storage, inflow and release membership functions, respectively. If there are no other years in simulation for which the premise (Storage: Low, and Inflow: very low) is obtained for period 22, then the rule formulated in this case will be:

If the storage is *low* and the inflow is *very low* and *time-of-year* is *period* 22 then *release* is *low-med*.

If however there are more than one year in simulation for which the premise (Storage: Low and Inflow: very low) is obtained for the same period, and the consequences are different, the fuzzy rule is formulated for the premise based on the average value of the consequences. To illustrate this, for the above example, suppose in any two years (out of 30 yr of simulation), the storage, inflow and release are 180, 10, 40 and 100, 25, 55 for the two years respectively. As may be verified from Figure 5, the premise arising out of this data would be 'Storage: Low: and Inflow: very low' for both the cases, but the consequences are 'Release: Low-Medium', for one year and 'Release: Very-High' for the other. In such a case the consequence is formulated based on the average value of the release (i.e., (40 + 55)/2 = 47.5). This value is traced to the membership of both medium and medium-high from Figure 5. Since, the value 47.5 has a higher membership value for the membership function 'Medium', the fuzzy rule is written as

If the storage is *low* and the inflow is *very low* and *time-of-year* is *period* 22 then *release* is *medium*.

The number of rules in the fuzzy rule base is $\prod_{i=1}^{n} c_i$, where c_i is the number of classes in the ith variable and n is the number of variables. With increasing number of classes for the variables, a greater accuracy may be achieved. However, a very large rule base leads to dimensionality problems.

3.3. APPLICATION OF FUZZY OPERATOR

The premise part of the rule is assigned one membership value through this operation. For example, in period 22, a storage of 180 corresponds to a membership value of 0.7 in the membership function for 'low' storage, and the inflow of 10 corresponds to 1.0 in the membership function for 'very low' inflow. The result of application of the fuzzy operator 'AND' will be to pick the minimum of these two (viz.. 0.7) and assign it to the premise of the rule.

3.4. IMPLICATION, AGGREGATION AND DEFUZZIFICATION

The result obtained from application of the fuzzy operator is applied to the membership function of the consequence of the rule, and one output fuzzy set is obtained. For the example mentioned above, the consequence is 'Release: Medium'.

The membership function of this fuzzy set is truncated as shown in Figure 3, at the level of 0.7 – which corresponds to the membership value **of** the premise assigned in the previous step. and the resulting truncated output fuzzy set is obtained. if any of the input values belong to the intersecrion region of two membership functions, the rules corresponding to both the membership functions need to be invoked. In such a case the same set **of** input values will yield different output sets. **To** obtain one output set corresponding to these different output sets, aggregation of output fuzzy sets (Figure 4) is carried out. The end result of implication and aggregation, if necessary, is a truncated fuzzy set for release which is defuzzified using the centroid method to obtain the crisp value of relese.

4. Results

The rules are expressed in a symbolic for-in, where the meaning of various symbols are as follows

'==' : *Logical Equality*

'=' : *Assignment operator*

'⇒' : *Implication*

'&' : *Fuzzy Operator (AND)*

Table III. Fuzzy rule base for period # 22

Premise of the rule	Output
(storage==very_high) and (inflow==low) and (time_of_year==Jan1)	(release=very_high)
(storage==high) and (inflow==low) and (time_of_year==Jan1)	(release=very_high)
(storage==high) and (inflow==very_low) and (time_of_year==Jan1)	(release=medium)
(storage==medium) and (inflow==low) and (time_of_year==Jan1)	(release=very_high)
(storage==medium) and (inflow==very_low) and (time_of_year==Jan1)	(release=low_med)
(storage==low) and (inflow==low) and (time_of_year==Jan1)	(release=medium)
(storage==low) and (inflow==very_low) and (time_of_year==Jan1)	(release=low)

As a typical example. the rules tor period 22 (January 1–10) are presented here For this period, membership functions were constructed lor four levels of storage,

Table IV. Sensitivity to defuzzification method

Method of defuzzification	Reliability	Resiliency
Centroid	76%'	42%
Bisection	78%	45%
LOM	79%	32%
MOM	77%	43%
SOM	64%	40%

five levels of inflow and nine levels of release. Since this period falls in the group October to January (periods 13–24), the range:, of storage and inflows are fixed based on maximum and minimum values among all these periods (periods 13–24). The storage was classified as: 'low', 'medium', 'high', and 'very-high' in the range of [50 875]; the inflow as, 'very-low'. 'medium', 'high', 'very-high' in the range of [–1 220]; the release was divided into nine class interval:, as, 'zero', 'very-low', 'low-med', 'medium', 'med-high', 'high', 'very-high'. and 'v.v. high'. in the range of [0 90]. The rules for the period 22 are given in Table III. It may be noted that. combinations for the premise, other than those given in Table III, do not occur for period 22.

In simulation, the fuzzy rules are used as follows: Knowing the reservoir storage and inflow levels (i.e., high, medium etc.), appropriate fuzzy rule for the period is invoked. The fuzzy operator, implication and aggregation together yield a fuzzy set for the release. A crisp release is then obtained by using the centroid of the fuzzy set, Figure 7 shows for a year, the release obtained by applying the fuzzy rules.

For evaluating the performance, we used the two performance criteria, reliability and resiliency (Hashimoto *et al.*, 1982). When simulated over a period of 30 yr, the fuzzy rule based operation yielded a reliability of 76% and a resiliency of 42%. The SDP operation. on the other hand, yielded a lower reliability of 54% and a higher resiliency of 56% compared to the fuzzy operation. The factors that contribute to the performance of the two operations are the objectives of operation, the discretisation scheme6 and even the inflow distribution. which are all case specific. In this particular case, although the fuzzy rule base was derived based on the SDP policy, the two operations – the fuzzy operation and the SDP operation – resulted in different values of the performance indices, because of the way the two policies are used in operation, the difference in the discretisation schemes and the nature of the objective function in the SDP operation. To examine the sensitivity of results to different methods of defuzzification, simulation is carried out with other methods of defuzzificaton. Table III gives the reliability and resiliency for the fuzzy rule based operation with different defuzzitication method. It is seen that the results are not very sensitive except for smallest of maximum (SOM) method.

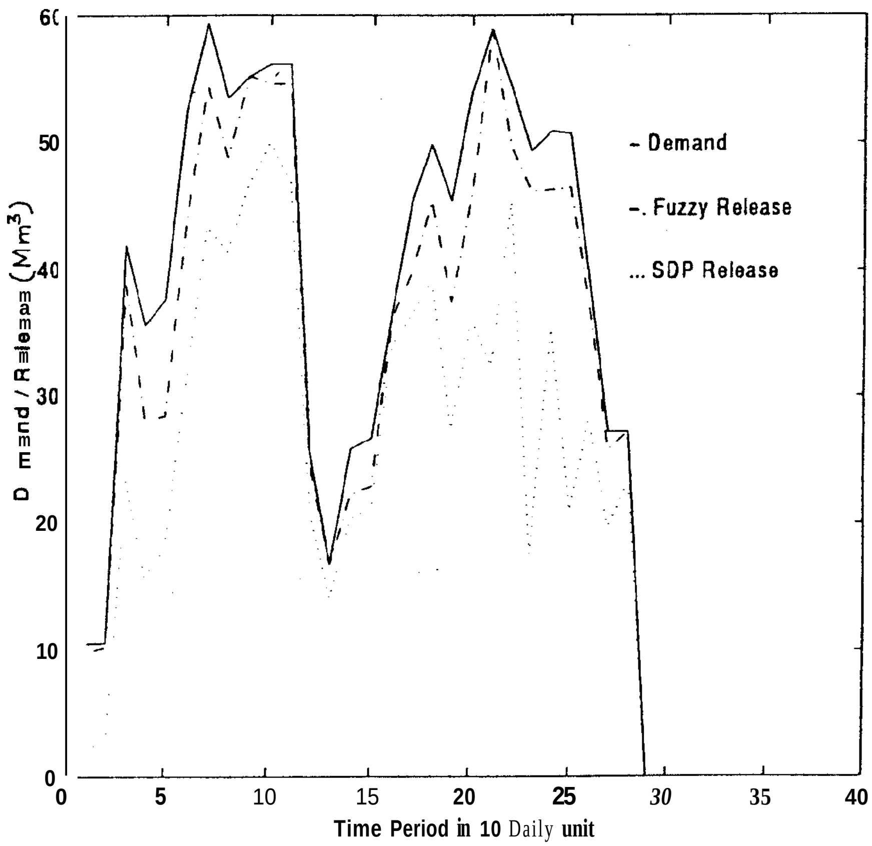

Figure 7. Comparison of SDP and fuzzy releases.

4.1. GENERAL REMARKS

The fuzzy logic model for reservoir operation, presented in this work, attempts to provide an implementable reservoir operation policy. As Russell and Campbell (1996) point out, the fuzzy logic approach by itself'. is not an alternative to the more conventional optimisation techniques. Rather, it provides an opportunity for the reservoir operators to participate in formulating the rule base, and hence may be more acceptable to them than the policies derived using complex optimization models. The approach, however, suffers from dimensionality problems. As the number of fuzzy sets increases, the dimensionality of the problem grows multiplicatively. The fuzzy rules, required for the specification of the policy may be defined by an expert or a group of experts. In the present work, the fuzzy rules have been derived from a simulated operation of the reservoir with a SDP operating policy. It may be noted that the SDP model is used in this work only as an alternative to an expert who is presumed to have a good insight to provide the fuzzy rules for operation, and that the **SDP** model itself is not essential for the approach presented here.

A limitation of the present work is that simplistic membership functions have been used for inflow and reservoir storage. The results presented here could be quite sensitive to the nature of the membership functions. This limitation is conceded to, because the primary aim of the article is to demonstrate the utility of fuzzy logic in reservoir operation. A good scope exists for further studies on defining appropriate membership functions and defuzzification methods.

5. Conclusion

A methodology to construct a fuzzy rule based system for reservoir operation is presented in this article. An advantage of the fuzzy rule based reservoir operation is that complex optimisation procedures are avoided, and linguistic statements such as 'low inflow' 'poor rainfall' etc., may be readily incorporated. As a result, the operators may feel more comfortable in using such models. While a fuzzy rule based model is easy to develop and adopt for operation, it suffers from the curse of dimensionality, and therefore the applications of fuzzy logic to reservoir operation problems may remain limited to single reservoir systems.

6. Appendix – Some Basic Concepts of Fuzzy Logic

Zadeh (1965) pioneered the development of fuzzy logic. Some basic concepts of the fuzzy logic and the operations used in the article are discussed briefly in this appendix. Most of this discussion is adopted from the text books, Klir and Folger (1995), Zimmermann (1996), Kosko (1996) and Ross (1997).

A *membership function* (MF) is a function – normally represented by a geometric shape – that defines how each point in the input space is mapped to a membership value between 0 and 1. The input space is refered as *universe of discourse*. If X is the universe of discourse and its elements are denoted by x, then a fuzzy set A in X is defined as a set of ordered pairs

$$A = \{x, \mu_A(x) | x \in X\}, \tag{4}$$

where $\mu_A(x)$ is called the membership function of x in A. Thus the membership function maps each element of X to a membership value between 0 and 1. A membership function can be of any valid geometric shape with appropriate equations describing them. Some commonly used membership functions are of bell, triangular and trapezoidal shape.

An important step in applying fuzzy methods is the assessment of the membership function of a variable, which parallels the estimation of probability in stochastic models. For reservoir operation modeling purposes, the membership functions required are those of inflow, storage, demand and release. When the standard deviation is not large it is appropriate to use a simple membership function consisting of only straight lines, such as a triangular or a trapezoidal membership function. In this work, for simplicity and to demonstrate the applicability of the method. the triangular membership function has been considered, for all the variables.

The equation describing the triangular membership function is given by

$$f(x;a,b,c) \begin{aligned}[t] &= 0\,, && x \leqslant a \\ &= \tfrac{x-a}{b-a}\,, && a \leqslant x \leqslant b \\ &= \tfrac{c-x}{c-b}\,, && b \leqslant x \leqslant c \\ &= 0\,, && c \leqslant x\,. \end{aligned} \tag{5}$$

The parameters a and c locate the 'feet' of the triangle and the parameter b locates the peak. The triangle in Figure 1 shows an example of such a membership function. For this example $a = 200$, $c = 500$ and $b = 1$. The resulting membership function may or may not be a symmetrical membership function. This allows a departure from the usual assumption **of** normally or symmetrically distributed error around the most likely value. This flexibility is one of the advantages **of** fuzzy sets when errors are not symmetrically distributed. Tho membership functions may bc overlapped or they may be disjointed from one another. Kosko (1996) observed that the fuzzy controller attained its best performance when there is a overlapping in the adjacent membership functions. A good rule **of** thumb is that the adjacent fuzzy set of values should overlap approximately 25% (Kosko, 1996).

The problem in using fuzzy membership function with wider base, is that they yield large errors in the response than do narrower ones. The trade off should bc such that the fuzzy rule base describe the physical processes as well as possible without losing the completeness of the rule system. Civanlar and Trussel (1986) provide some guidelines to construct the membership function for fuzzy sets whose elements have a defining feature with a known probability density function (pdf) in the universe of discourse.

6.1. FUZZY RULES

A fuzzy rule system is defined as the set **of** rules which consists of sets of input variables or premises $A_{i,k}$, in the form of fuzzy sets with membership functions μ_A, and a set of consequences ***B***, also in the form of a fuzzy set. Typically a fuzzy if-then rule assumes the form

if x is **A** then y is B

where **A** and B are linguistic values defined by fuzzy sets on the ranges (universe of discourse) X and Y, respectively. The 'if' part of the rule 'x is A' is called as *antecedent* or *premise*, while the 'then' part of the rule 'y is B' is called *consequence*.

The premise is an interpretation that returns a single number between 0 and 1, whereas the consequence is an assignment that assigns the entire fuzzy set B to the output variable y. Interpreting the fuzzy rule of the kind 'if-then' involves distinct steps such as: first evaluating the premise (which involves *fuzzifying* the input and applying *fuzzy operators*) and second, applying that result to the consequence (*implication*). In the case of binary or two-valued logic, if the premise is true then the consequence is also true. But in a fuzzy statement involving a fuzzy rule, if the antecedent is true to some degree of membership, then the consequent is also true to that same degree. Also, if the premise of the rule has multiple parts such as

if x *is* **A** and y *is* **B** and z *is* C, then ...

then all the parts of the premise are calculated simultaneously and resolved to a single number using the logical operators.

The consequence of a rule also can have several parts like

if x *is* **A** and y *is* B and z *is* C, then m *is* N and o *is* P, etc.,

in which case all the consequences are affected equally by the result of the premise The consequence specifies a fuzzy set to be assigned to the output. The *implication* then modifies that fuzzy set to the degree specified by the premise. The most common methods to modify the output fuzzy set are truncation using the min function or scaling using the prod function. To obtain the output of the entire set of rules as a single number, it is required to defuzzify the output fuzzy set.

References

Bardossy, A. and Disse, M.: 1993, Fuzzy rule based models for infiltration, *Water Resour. Res.* **29**(2) 373–382.

Civanlar, M. R. and Trussel, H. J.: 1986, Constructing membership functions using statistical data *Fuzzy Sets and Systems* **18**, 1–13.

Devi, B. B. and Sharma, V. V. S.: 1985, Estimation of fuzzy membership from histograms *Information Sci.* **35**(1), 43–59.

Dombi, J.: 1990, Membership function as an evaluation, *Fuzzy Sets and Systems* **35**, 1–21.

Fontane, D. G., Gates, T. G. and Moncada, E.: 1997. Planning reservoir operations with imprecis objectives, *J. Water Resour. Plann. Manag.* **123**(3), 154–162.

Hashimoto, T., Louks, D. P. and Stedinger, J. R.: 1982, Reliability, resiliency, and vulnerabilit criteria for water resources system performances and evaluation, *Water Resour. Res.* **18**(1 14–20.

Hellendoorn, H. and Thomas, C.: (1993), Defuzzification in fuzzy controllers, *J. Intelligent Fuzz Syst.* **1**(2), 109–123.

Kindler, J.: (1992), Rationalizing water requirements with aid of fuzzy allocation model, *J. Wate Resour. Plann. Manag.* **118**(3), 308–323.

Klir, G. J. and Folger, T. A.: 1995, *Fuzzy sets, Uncertainty, and Information*, 4th edn, Prentice Ha of India Limited (original edition: Prentice Hall Inc., Englewood Cliffs, 1988).

Klir, G. J. and Yuan, Bo: 1997. *Fuzzy Sets and Fuzzy Logic*, Prentice Hall of India (original editio Prentice Hall Inc., Englewood Cliffs, 1995).

Kosko, B.: 1996, *Neural Networks and Fuzzy Systems*, Prentice Hall of India (original editio Prentice Hall Inc., Englewood Cliffs, 1992).

MATLAB: 1995, *Fuzzy Logic Toolbox*, The Math Works Inc.

Mujumdar, P. P. and Vedula, S.: (1992), Performance evaluation of an irrigation system under some optimal operating policies, *Hydrol. Sci. J.* **37**(1), 13–26.

Mujumdar, P. P. and Ramesh, T. S. V.: 1997, Real-time reservoir operation for irrigation, *Water Resour. Res.* **33**(5), 1157–1164.

Ross, T. J.: 1997, *Fuzzy Logic with Engineering Applications*, McGraw Hill International Editions, Electrical Engineering Series.

Russel, S. O. and Campbell, P. E.: 1996, Reservoir operating rules with fuzzy logic programming, *J. Water Resour. Plann. Manag.* **122**(3), 165–170.

Shrestha, B. P., Duckstein, L. and Stakhiv, E. Z.: 1996, Fuzzy rule based modeling of reservoir operation, *J. Water Resour. Plann. Manag.* **122**(4), 262–269.

Simonovic, S. P.: 1992, Reservoir system analysis: Closing the gap between theory and practice, *J. Water Resour. Plann. Manag.* **118**(3), 262–280.

Vedula, S. and Mujumdar, P. P.: (1992), Optimal reservoir operation for irrigation of multiple crops, *Water Resour. Res.* **28**(1), 1–9.

Yager, R. R. and Filev, D.: (1993), On the issue of defuzzification and selection based on a fuzzy set, *Fuzzy Sets and Systems* **55**, 255–271.

Yeh, W., W.-G.: 1985, Reservoir management and operation models: A state of the art review, *Water Resour. Res.* **21**(12), 1797–1818.

Zadeh, L. A.: 1965, Fuzzy sets, *Information and Control* **8**, 338–353.

Zimmermann, H. J.: (1996), *Fuzzy Set Theory and Its Application*, Allied Publishers Limited, New Delhi (original edition: Kluwer Academic Publishers, 1991).