A short-term reservoir operation model for multicrop irrigation

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Abstract An integrated model is developed for short-term yearly reservoir operation for irrigation of multiple crops. The model optimizes a measure of annual crop production, starting from the current period in real time. Reservoir storage at the beginning of a period, inflow during the previous period, crop soil moisture values and crop production already achieved up to the beginning of the period are used as inputs to the model. The solution specifies the reservoir release and optimal irrigation allocations to individual crops during an intra-seasonal period. The model overcomes some of the limitations of an earlier model developed by Mujumdar & Ramesh (1997) by replacing the two dynamic programming (DP) formulations with a single linear programming (LP) formulation. Application of the model is studied through a case study in India.

Un modèle de gestion à court terme d'un réservoir pour l'irrigation de cultures multiples

Résumé Dans ce papier on présente un modèle intégré developpé pour la gestion à court terme d'un réservoir destiné à l'irrigation de cultures multiples. Le modèle optimize une mesure de la production annuelle, partant du moment courant. Les entrées du modèle sont le stock d'eau emmagasiné au début de la période considérée, l'apport au cours de la période précédente, l'humidité des sols et la production réalisée jusqu'au début de la période. La solution indique la lâchure devant être réalisée et l'allocation optimale à chaque culture pour la période courante. Le modèle surmonte un certain nombre de limites d'un modèle précédemment développé par Mujumdar & Ramesh (1997) en remplacant deux formulations antérieures par un programme linéaire unique. Une étude de cas en Inde a permis d'étudier l'application du modèle.

INTRODUCTION

Recent studies on reservoir operation for irrigation have focused on developing long-term steady state operating policies (e.g. Dudley 1988a,b; Dudley & Musgrave, 1988; Vedula & Nagesh Kumar, 1996). Dudley (1988a) developed a methodology for arriving at the steady state operating policy when a single decision maker is responsible for the reservoir releases and cropping pattern. Vedula & Mujumdar (1992) and Vedula & Nagesh Kumar (1996) have developed Stochastic Dynamic Programming (SDP) models integrating the reservoir operation with the field level

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water utilization by crops, for a single reservoir irrigating multiple crops. These models specify the reservoir releases and crop water allocations during intra-seasonal periods for a known state of the system given by the initial reservoir storage, inflow during a period and the soil moisture at the beginning of the intra-seasonal period. These models are essentially planning models which are useful in optimizing the long-term performance of the system. For model application in real time, however, forecasts of inflow/rainfall are needed. Also, the issues addressed by a long-term policy may not be very relevant for short-term real-time operation of a reservoir.

For the short-term operation, a time frame of one year is often considered as the operating horizon. The short-term objective in an irrigation operation problem is typically the maximization of annual crop yield, whereas a long-term steady state model maximizes the expected annual yield. Dariane & Hughes (1991) addressed the problem of real-time reservoir operation for irrigation. The model developed by them considers the crop productivity through crop yield functions. However, the model does not take into account the soil moisture contribution in meeting irrigation requirements. In a recent work, Mujumdar & Ramesh (1997) developed a short-term real-time operation model, specifically addressing these issues. The model consists of two components: one, an operating policy model and the other a crop water allocation model, with both models formulated as deterministic dynamic programming (DP) models. This model is referred to as the DP–DP model in this paper.

The real-time operation model uses forecast inflows for the current period, for which a decision is sought, and for all subsequent periods in the year, to specify the reservoir releases and crop water allocations. The model is formulated to be solved once at the beginning of each period. The soil moisture and reservoir storage state are updated for intermediate periods in the model with forecast inflows and optimal crop water allocations. In addition to reservoir storage and soil moistures of the crops, a crop production measure is also included as a state variable to incorporate the interdependence of crop water allocations. A major limitation of the model is the number of state variables needed in the DP formulation. Apart from the reservoir storage, two state variables are needed for each crop: one for soil moisture and one for the crop production measure. In a real case, typically five to six major crops are grown in the irrigated area. It will be impossible to apply the model to such real cases because of computational intractability caused by the large number of state variables needed. The number of state variables will have to be reduced by using only one soil moisture state variable for all the crops, as used by Vedula & Mujumdar (1992). To overcome this serious limitation, a short-term model is developed in this paper replacing both the DP models of Mujumdar & Ramesh (1997) by a single linear programming (LP) model. All major features of the earlier model are still retained in the new model. The model is formulated to be solved once at the beginning of each period and to derive the reservoir releases and crop water allocations, for known initial reservoir storage and soil moisture values in the area of concern. The model uses inflow forecasts for the current period in real time and all the subsequent periods till the end of the year. Only the current period's decision is actually implemented and the state of the system is updated with the actual values of inflow and soil moisture of crops. The model is applied to a case study of the

Malaprabha irrigation reservoir in Karnataka State, India.

Similar to most real-time operation models, the present model also consists of an inflow forecasting model and an optimization model. The inflow forecasting model is used to obtain updated inflow forecasts at the beginning of each decision interval. These forecasts are used in the optimization model to obtain the release policy for the remaining part of the year starting with the current period in real time. The following sections give details of the model and its application for real-time reservoir operation.

OPERATING POLICY MODEL

A short-term operating policy model must ideally be used in conjunction with a long-term steady state policy. End-of-the-year targets such as over-year storage may be specified from the long-term policies derived with a stochastic optimization model. When such targets are available, the short-term model should use them as end conditions and address the problem of distribution of water within the year. The short-term operating horizon for an irrigation system is typically one year, with the intra-seasonal decision intervals consisting of a week to ten days. The one-year operating horizon is especially relevant in monsoon climates where the annual periodicity of the reservoir inflows is well pronounced. Taking the operating horizon as one year, the operating policy model is formulated as a deterministic LP model to obtain optimal release decisions for the intra-seasonal periods starting from the current period in real time to the last period in the year. Forecast inflows for the current and all subsequent periods in the year, soil moisture of each crop and reservoir storage at the beginning of the current period constitute the inputs to the model. Rainfall in the command area which, along with the soil moisture, determines the crop water requirements, is taken as a deterministic known input. Rainfall during a period is assumed to be uniformly distributed over the entire area of concern. Thus, all crops grown in the area are assumed to receive the same rainfall input during a period.

Choice of appropriate decision intervals for the model is an important step in the model formulation for irrigation reservoirs. If the decision intervals in the model are made equal in length to the growth stages of a crop, not only will all the decision intervals be of different lengths but, in case of multiple crops, the decision intervals will not coincide with growth stages across crops. It is therefore necessary that the decision interval be such that the total time (number of periods) elapsed from the beginning of the crop season to the end of any growth stage be an integral multiple of the decision interval. For modelling purposes, this condition can be achieved by suitably adjusting the lengths of individual growth stages marginally (within acceptable limits). The decision interval in the model is chosen to be such that the growth stages are integral multiples of the decision interval, the interval itself is of a reasonable length consistent with the field needs, and the model resulting from it is computationally tractable for real situations. Typically, for model application the interval is taken as a week or ten days.

Objective function

For a short-term operation of an irrigation reservoir, only the current year is of interest. The returns from the reservoir command during the particular year would therefore form the main basis for formulating the objective function. In the context of irrigation, the ultimate value of the reservoir release depends on how much of it is utilized for crop production and therefore, how the released water is allocated to individual crops. In the operating policy model, the release itself must therefore be determined based on an optimal allocation among crops to maximize the crop yield. A good predictor of the crop yield is the actual evapotranspiration. Several models have been proposed in the literature (e.g. Hiller *et al.*, 1974; Doorenbos & Kassam, 1979) in which the yield ratio (ratio of actual to maximum yield) is related to the ratio of actual to potential evapotranspiration. These models consider the fact that plant growth is a function of the factors that contribute to plant water stress. In general, studies on plant water stress reveal that stress occurs when the actual evapotranspiration (*AET*) is less than the potential evapotranspiration (*PET*). Soil water stress does not occur when *AET* equals *PET*. Under this condition, the crop is assumed to have the optimum growth. With this background, the following objective function is considered for the operation model:

$$\min \sum_{t=t_p}^{T} \sum_{c=1}^{N_t} P_c + Ky_c^t \left(1 - AET_c^t / PET_c^t\right) \tag{1}$$

where t_p is the current period in real time, T is the last period in the year, N_t is the number of crops present in period t, c is the crop index, AET_c^t is the actual evapotranspiration, PET_c^t is the potential evapotranspiration, Ky_c^t is the yield sensitivity factor of crop c in period t, and P_c is a measure of production for crop c actually realized up to the beginning of the period t_p. It may be noted that as the model is applied in real time, the actual value of the crop production function up to the beginning of the current period may be quite different from that predicted when the model was applied in the previous period. The term P_c is the value of the second part of the objective function actually realized from the first period in the year up to the beginning of the current period t_p, i.e.:

$$P_c = \sum_{t=1}^{t_p - 1} Ky_c^t \left(1 - AET_c^t / PET_c^t\right) \tag{2}$$

Addition of this term in the objective function ensures that at every period the allocations are made such that the crop yield in the entire year is maximized, using the latest available information on the state of the system. In the dynamic programming model of Mujumdar & Ramesh (1997), this feature is achieved by introducing an additional crop production state variable for each crop.

The objective function (equation (1)), is based on the additive form of crop production function given by Doorenbos & Kassam (1979). The crop sensitivity factor *Ky* is a measure of reduction in crop yield for a given evapotranspiration deficit, (1 – *AET*/*PET*) occurring in a growth stage. In the objective function it acts as a weighting factor to ensure that, when competition for water exists among crops, the crop with the highest sensitivity to water deficit is given a higher priority for allocation, other factors being equal. The actual allocations from the model to a crop in a period, however, depend also on the available soil moisture, potential evapotranspiration, crop root depth, area of cultivation and severity of competition among crops. The crop sensitivity factors Ky_c^t are specified for individual growth stages of crops. In this study, the Ky_c^t values for all decision intervals within a growth stage are assumed to be the same as the value for the growth stage itself. Such an assumption has also been made in some earlier studies (e.g. Bras & Cordova, 1981), and is not very limiting in estimating the crop production. The *AET/PET* ratio of a crop, that determines the crop yield, depends on the irrigation allocation to the crop vis-à-vis its irrigation requirement and its competition for water with other crops.

Constraints

The model is formulated integrating the reservoir operation with field level water utilization There are thus two major sets of constraints, one corresponding to the reservoir storage and the other dealing with the soil moisture balance of crops. The reservoir release is adjusted for conveyance and application losses, when allocating among crops. The following constraints are considered in the model:

Reservoir storage continuity The reservoir storage continuity is written taking into account the storage-dependent evaporation losses (Loucks *et al.*, 1981) as:

$$(1 + a_t)S_{t+1} = (1 - a_t)S_t + Q_t - R_t - O_t - A_0 e_t \quad t = t_p, t_{p+1}, \ldots, T \tag{3}$$

where $a_t = A_a\, e_t/2$; A_a is the area per unit active storage volume above A_0; A_0 is the water spread area corresponding to the dead storage level; e_t is the evaporation rate in period t; S_t is the storage at the beginning of period t; Q_t is the forecast inflow during the period t; R_t is the release during period t; and O_t is the overflow during period t.

The reservoir storage at the beginning of the current period t_p is known:

$$S_{tp} = S_0 \tag{4}$$

where S_0 is the known live storage of the reservoir at the beginning of the period. In the case of over-year storage, the end condition on the storage is also fixed, and an appropriate constraint is included to force the condition that the storage at the end of the last period T should be at least equal to the specified over-year storage, S_{oy}:

$$S_{T+1} \geq S_{oy} \tag{5}$$

The storage is restricted to the active storage capacity, K, of the reservoir:

$$S_t \leq K \tag{6}$$

Water available for irrigation The reservoir release undergoes conveyance, application and other losses. The water actually applied for irrigation of crops must therefore take into account these losses. When a release R_t is made at the reservoir in a period t, the water available for allocation among crops is given by:

$$X_t = \eta R_t \qquad t = t_p, t_{p+1}, \ldots, T \tag{7}$$

where η is the conveyance efficiency accounting for all losses in the release.

Soil moisture continuity The soil moisture at the beginning of the current period t_p is known for all crops. Starting with this known soil moisture, the soil moisture values at the beginning of all subsequent periods up to the end of the year are computed by the soil moisture continuity, given by:

$$\theta_c^{t+1} D_c^{t+1} = \theta_c^t D_c^t + RAIN_t + x_c^t - AET_c^t + \theta_0 (D_c^{t+1} - D_c^t) - Dp_c^t \quad \forall c, t \tag{8}$$

where θ_c^t is the soil moisture of crop c at the beginning of the period t, D_c^t is the root depth of crop c during period t, $RAIN_t$ is the rainfall in the area of concern in period t, x_c^t is the irrigation application to crop c in period t, AET_c^t is the actual evapotranspiration of crop c in period t, θ_0 is the initial soil moisture in the soil zone into which the crop root extends at the beginning of period $t + 1$, and Dp_c^t is the deep percolation.

In equation (8), the soil moisture values θ_c^t and θ_0 are in units of depth per unit root depth and all other terms are in depth units. The soil moisture, θ_0, is assumed to be known, and in model application it is taken to be equal to the field capacity. Sensitivity of the system performance to variation in θ_0 is discussed in Mujumdar & Vedula (1992). The soil moisture, θ_c^t, irrigation allocation, x_c^t, and actual evapotranspiration, AET_c^t, are all decision variables. The relationship between the *AET/PET* ratio and the available soil moisture is approximated by a linear relationship, with $AET = 0$, when the available soil moisture is zero (corresponding to the actual soil moisture at wilting point) and $AET = PET$ when the available soil moisture is equal to the maximum available soil moisture (corresponding to the actual soil moisture at field capacity). This condition is written as:

$$AET_c^t \leq \frac{\left(\theta_c^t D_c^t + RAIN_t + x_c^t\right) - \theta_w D_c^t}{\left(\theta_f - \theta_w\right) D_c^t} PET_c^t \quad \forall c, t \tag{9}$$

and

$$AET_c^t \leq PET_c^t \quad \forall\, c, t \tag{10}$$

The constraint (equation (10)) is necessary along with equation (9) to restrict the maximum value of the actual evapotranspiration to the potential evapotranspiration. The denominator in equation (9) is the maximum available soil moisture of crop c in period t and the term $(\theta_c^t D_c^t + RAIN_t + x_c^t) - \theta_w D_c^t$, is the actual available soil moisture after the addition of rainfall and irrigation application. The crop root depth in period t is assumed to be known, and in model application an appropriate root depth model may be used.

Allocation limit The total water available for irrigation, X_t, corresponding to the release, R_t, must equal the water actually allocated to the crops, i.e.:

$$X_t = \sum_c x_c^t A_c \quad \forall t \tag{11}$$

where A_c is the area of crop c under irrigation. The upper limit on the soil moisture θ_c^t is the field capacity. Any moisture in excess will go out of the root zone as deep percolation, which is ensured through soil moisture balance equation (7).

$$\theta_c^t \leq \theta_f \quad \forall\, c, t \tag{12}$$

The objective function (equation (1)), together with the soil moisture balance equations (8), (9) and (10) and allocation limits (equations (11) and (12)), achieves the optimal allocation of a known amount of water among the competing crops. In the earlier DP–DP model, the problem of optimal crop water allocation was addressed by a separate DP model, with state variables for crop production and soil moisture for individual crops. The optimized performance measure of the crop water allocation DP model was then used as an input to the operating policy DP model. In the present LP model, on the other hand, the operating policy and the crop water allocations are simultaneously obtained. The reservoir-related constraints, equations (3)–(6), essentially achieve the objective of the operating policy model of the earlier work. The link between the reservoir operation and crop water allocation is achieved through the water availability constraint, equation (7).

The LP model given by equations (1) and (3)–(12) is solved from the current period, t_p, in real time up to the last period, T, in the year. The reservoir storage, S_{tp}, at the beginning of the period t_p, the actual inflow, I_{tp-1}, during the previous period $t_p - 1$, and the values of soil moisture, θ_c^{tp}, at the beginning of the period t_p for all crops, c, are known. Inflow forecasts, Q_t, for the periods starting from the current period, t_p, to the last period, T, are obtained by a suitable inflow forecasting model that uses the latest available actual inflow, I_{tp-1}. The inflow forecasts are used as single-valued estimates of inflows in the deterministic LP model, in preference to using any other values such as the average inflows. The forecasting model may work well only for one-step-ahead forecasts, and may perform poorly for further $(T - t_p)$ steps ahead, as used in the model. As the time progresses in the year, however, fewer forecasts would be needed and a better forecasting accuracy may be obtained. The rainfall inputs may be provided in a similar way, although in monsoon climates with low coefficient of variation for rainfall, mean rainfall values may be used as a deterministic input.

RESULTS AND DISCUSSION

The real-time operation model developed in the study was applied to the irrigated area served by the right bank canal of Malaprabha irrigation reservoir in Karnataka, India. The reservoir has been in operation since 1973, and inflow records are available at the reservoir site for 38 years. The reservoir command area consists predominantly of black cotton soil with major crops grown in the irrigated area being cotton, maize, wheat, sorghum, pulses and safflower. The field capacity, θ_f for the soil is 3.5 mm cm^{-1} and the wilting point θ_w is 1.0 mm cm^{-1}. For model application, a decision interval of 10 days was considered, with a year consisting of 36 intra-seasonal periods. The last few periods in the year were marginally adjusted to account for the number of days in excess of 360 days in a year. For model application, each crop was assumed to have five growth stages: establishment, vegetative, flowering, yield formation and ripening. The crop root growth was assumed to be linear across time. The root was assumed to grow from zero depth at the beginning of the crop season to its full depth at the end of the flowering stage and to remain constant thereafter till the end of the crop season. The crop root depth in a period was taken as the average of the root depths at the beginning and end of the period. The crop growth stages were made integral multiples of ten days. Such a marginal adjustment of the growth stage lengths would not be very limiting since the crop growth stages are not sharply defined and only a range of values is available for each growth stage.

Figure 1 shows the crop calendar for the right bank command of the reservoir. A year consists of two seasons, the Kharif season, spanning from periods 1 to 15 and the Rabi season, extending from periods 16 to 31. There are no crops sustained by irrigation after period 31. The potential evapotranspiration values of the crops are given in Table 1. A one-step-ahead inflow forecasting model identified for the site in an earlier study by Mujumdar & Nagesh Kumar (1990) was used to provide the inflow forecasts for the entire horizon consisting of $(T - t_p)$ periods. The inflow forecast for the current period, t_p, was based on the actual realized inflow during the previous period, $t_p - 1$. The forecasts for all subsequent periods were based on the forecast inflow during the preceding period. This procedure was used in preference to using simply the average (or any other fixed) values for the inflows, because the

Table 1 Potential evapotranspiration values of the crops.

Kharif season					Rabi season					
Period	Potential evapotranspiration (mm)				Period	Potential evapotranspiration (mm)				
	Maize	Pulses	Sorghum	Cotton		Sorghum	Pulses	Wheat	Safflower	Cotton
1	13.05	9.14	10.44	–	16	13.64	11.93	13.64	13.64	42.62
2	13.05	9.14	10.44	–	17	13.64	11.93	27.28	27.28	42.62
3	44.30	31.32	39.15	–	18	25.57	21.06	27.28	27.28	42.62
4	37.65	26.58	33.28	–	19	23.33	18.66	24.88	24.88	38.88
5	38.85	34.27	34.23	–	20	24.09	24.09	38.56	25.70	40.16
6	56.56	35.35	54.20	–	21	38.15	24.88	39.80	26.53	41.46
7	52.08	32.44	49.91	21.70	22	40.37	26.32	26.32	28.08	43.87
8	42.56	33.60	35.84	22.40	23	29.01	27.20	27.20	43.52	32.64
9	43.95	34.69	37.00	23.13	24	29.95	28.08	28.08	44.93	33.69
10	39.43	45.65	33.20	33.20	25	35.12	48.29	10.98	52.68	39.51
11	39.43	45.65	33.20	33.20	26	32.78	45.07	–	28.68	36.87
12	23.66	–	22.83	33.20	27	20.93	–	–	26.63	34.24
13	–	–	–	25.40	28	–	–	–	26.63	34.24
14	–	–	–	51.12	29	–	–	–	–	26.64
15	–	–	–	52.69	30	–	–	–	–	26.64
					31	–	–	–	–	26.64

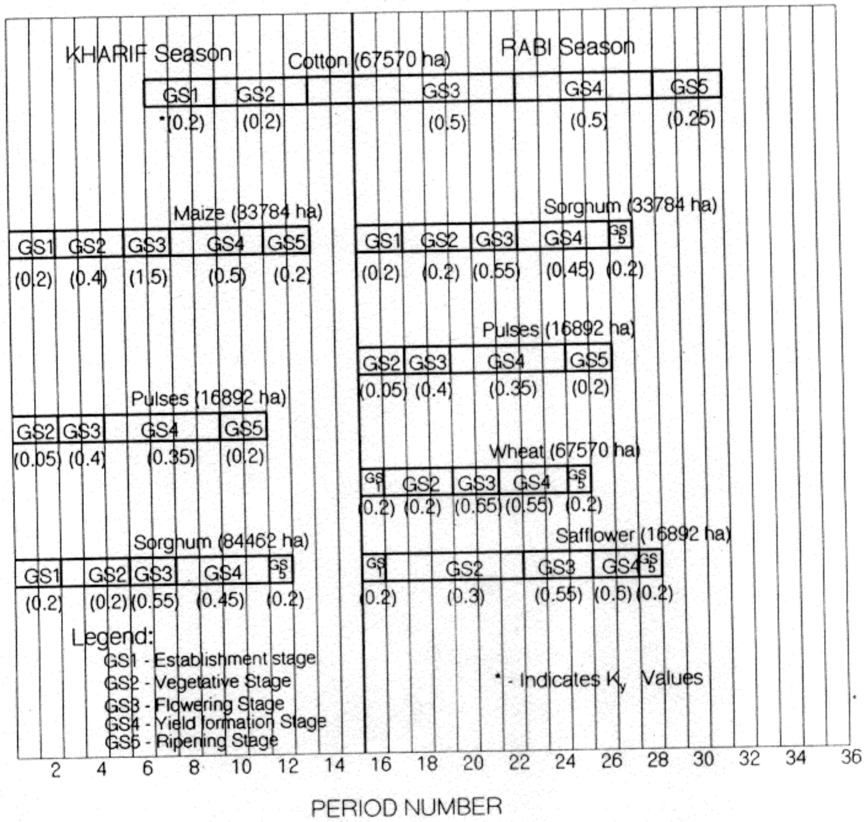

Fig. 1 Crop calendar.

forecasts could be progressively updated based on actual inflows realized, in real time. The uncertainty due to inflow forecasts is still present in the model.

For the case study, water deficit exists mainly in the Rabi season, since the Kharif crops are supported to a large extent by the monsoon rains. Table 2 gives a typical output from the model run for period 16 (first period in the Rabi season). It must be noted that the results presented in the Table were obtained by one run of the model for period 16. The inputs required for this run are: the storage at the beginning of the period, the inflow during the previous (i.e. 15th) period and the soil moisture values of the crops at the beginning of period 16. As seen from Fig. 1, the two-seasonal crop cotton was already in advanced growth stage at the beginning of period 16, whereas the other crops were just at the beginning of the first growth stage. The soil moisture of cotton and the initial reservoir storage shown in Table 2 are the values obtained by real-time simulation till the end of the 15th period, for the particular year. The soil moisture values of all other crops were assumed to be at field capacity, since the root depths are very small and consequently the soil moisture required to keep the root zone at field capacity is very low, which would be available at the time of sowing. The inflow values shown in Table 2 are all forecast inflows

Table 2 Typical results of model solution (current period in real time = 16).

Period	Storage (10^6 m^3)	Inflow (10^6 m^3)	Release (10^6 m^3)	Allocations (10^6 m^3) Cr1	Cr2	Cr3	Cr4	Cr5	Soil moisture (mm cm^{-1}) Cr1	Cr2	Cr3	Cr4	Cr5
16	450.00	10.47	78.76	29.68	1.64	1.73	1.71	4.62	1.63	3.50	3.50	3.50	3.50
17	378.24	9.81	66.12	7.55	2.69	5.61	3.99	13.23	3.50	3.50	3.50	3.50	3.50
18	318.77	10.09	88.82	8.24	3.79	14.00	4.01	14.38	3.50	3.50	3.50	3.50	3.50
19	237.29	6.40	59.46	7.79	0.00	7.96	0.00	13.99	3.50	3.46	3.50	3.46	3.50
20	182.10	5.37	42.51	7.95	8.42	0.00	4.20	0.69	3.50	3.06	3.46	3.05	3.50
21	143.14	4.69	37.50	8.66	5.58	0.00	4.51	0.00	3.50	3.50	3.26	3.13	3.02
22	108.64	4.41	7.13	0.00	0.00	0.00	3.56	0.00	3.50	3.50	3.04	3.24	2.68
23	104.32	4.27	32.02	8.67	4.79	0.00	2.55	0.00	3.10	3.19	2.81	3.31	2.52
24	75.03	4.10	37.30	12.06	2.64	0.00	3.96	0.00	3.23	3.33	2.59	3.21	2.40
25	40.41	3.48	21.21	7.18	2.72	0.00	0.74	0.00	3.50	3.32	2.39	3.20	2.31
26	21.29	3.14	13.41	6.70	0.00	0.00	0.00	0.00	3.50	3.29	2.06	2.93	2.28
27	9.63	3.00	11.28	5.64	0.00	0.00	0.00	0.00	3.50	3.11	1.79	2.77	1.00
28	0.00	2.87	1.17	0.58	0.00	0.00	0.00	0.00	3.47	3.00	1.00	2.77	1.00
29	0.00	2.57	0.20	0.10	0.00	0.00	0.00	0.00	3.21	1.00	1.00	1.00	1.00
30	0.52	2.35	0.00	0.00	0.00	0.00	0.00	0.00	3.02	–	–	–	–
31	0.00	2.73	0.27	0.14	0.00	0.00	0.00	0.00	2.84	–	–	–	–

Note: Release includes left bank release and conveyance losses. Allocations shown are only to crops of the right bank command. Storage and soil moisture shown are values at the beginning of the period. Inflow values shown in the table are all forecast inflows.

Cr1: cotton; Cr2: sorghum; Cr3: pulses; Cr4: wheat; Cr5: safflower.

Table 3 Results of real-time simulation for a year (Rabi season).

Period	Storage (10^6 m^3)	Forecast inflow (10^6 m^3)	Actual inflow (10^6 m^3)	Soil moisture (mm cm^{-1}) Cr1	Cr2	Cr3	Cr4	Cr5	Release (10^6 m^3)	Allocations (10^6 m^3) Cr1	Cr2	Cr3	Cr4	Cr5
16	450.00	10.47	6.84	1.63	3.50	3.50	3.50	3.50	78.76	29.68	1.64	1.73	1.71	4.62
17	375.52	8.12	3.91	3.50	3.50	3.50	3.50	3.50	66.12	7.55	2.69	5.61	3.99	13.23
18	310.81	8.18	14.41	3.50	3.50	3.50	3.50	3.50	90.16	8.24	4.12	1.40	4.34	14.38
19	232.69	8.31	12.84	3.50	3.50	3.50	3.50	3.50	75.98	7.79	4.03	7.96	4.23	13.99
20	167.64	7.03	11.97	3.50	3.50	3.46	3.50	3.50	40.36	7.95	3.93	0.00	4.27	4.03
21	137.69	7.17	6.89	3.50	3.49	3.26	3.50	3.11	45.01	8.66	5.70	0.00	3.43	4.71
22	98.07	5.75	4.47	3.50	3.50	3.04	3.50	2.85	17.34	8.67	0.00	0.00	0.00	0.00
23	83.75	4.31	10.53	3.50	3.19	2.81	3.31	2.66	26.63	5.93	4.79	0.00	2.59	0.00
24	66.29	7.11	11.34	3.50	3.33	2.59	3.21	2.52	13.16	0.00	2.63	0.00	3.95	0.00
25	63.22	6.08	5.42	3.22	3.32	2.38	3.20	2.41	33.55	13.29	2.78	0.00	0.70	0.00
26	33.88	4.31	6.16	3.50	3.29	2.05	2.93	2.38	6.49	0.00	3.24	0.00	0.00	0.00
27	32.51	4.68	18.82	3.19	3.31	1.79	2.77	–	13.53	6.76	0.00	0.00	0.00	0.00
28	37.00	10.85	17.80	3.21	3.19	–	2.77	–	12.65	6.33	0.00	0.00	0.00	0.00
29	41.23	8.13	4.81	3.21	–	–	–	–	12.65	6.33	0.00	0.00	0.00	0.00
30	32.40	3.37	4.32	3.28	–	–	–	–	2.07	1.04	0.00	0.00	0.00	0.00
31	33.13	4.15	6.50	3.13	–	–	–	–	0.21	0.10	0.00	0.00	0.00	0.00

Note: Release includes left bank release and conveyance losses. Allocations shown are only to crops of the right bank command. Storage and soil moisture values shown are values at the beginning of the period.

Cr1: cotton; Cr2: sorghum; Cr3: pulses; Cr4: wheat; Cr5: safflower.

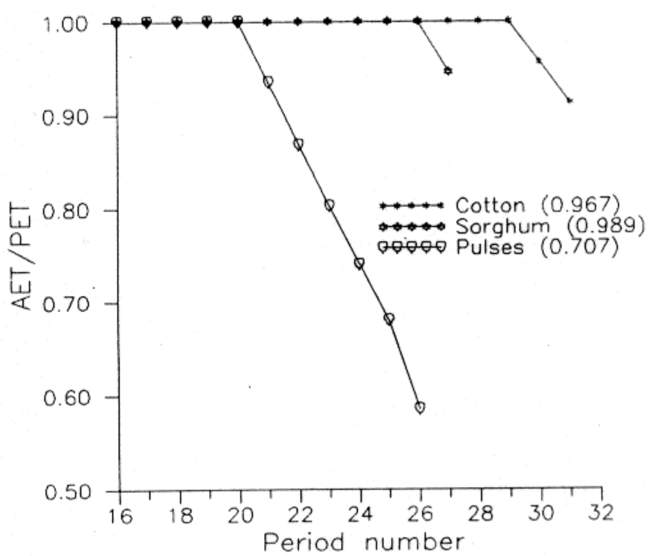

Fig. 2 Response of cotton, sorghum and pulses (values in brackets indicate relative yields).

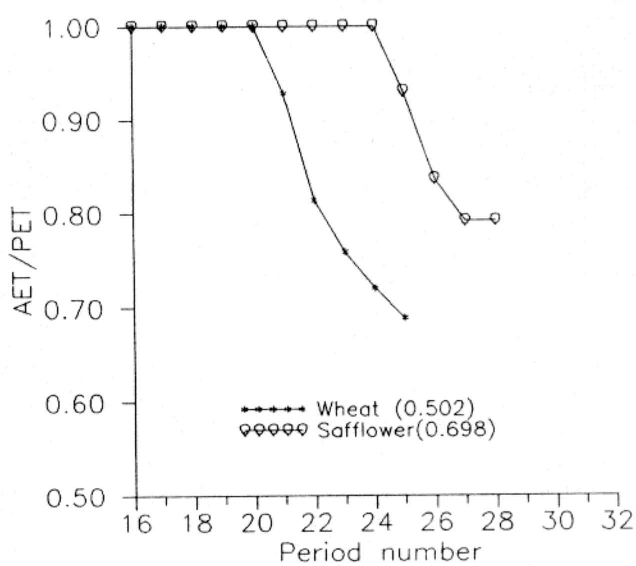

Fig. 3 Response of wheat and safflower (values in brackets indicate relative yields).

from an AR(1) model which uses the actual inflow, 11.32 10^6 m^3 for period 15 as an input. In real time, only the release and allocation decisions for the current period, i.e. period 16, are meant to be implemented. The reservoir storage and the soil moisture values were updated at the end of the period with the actual values of inflow and rainfall, to allow optimal release and allocation decisions to be made. Table 3 gives the results of a real-time simulation carried out for the Rabi season of a typical year. It may be noted that as the simulation progressed from period to period, the number of periods included in the optimization decreased and therefore the size of the problem decreased, taking less computation time for one run. Figures 2 and 3 show the responses of the different crops to irrigation allocations resulting from the policy. The relative yields of the crops resulting from the policy are indicated in brackets on the Figures. The particular year chosen for simulation was a low flow year, and the reservoir storage at the beginning of the Rabi season was quite low. The short-term operation methodology, by updating the release decisions from period to period, ensured that the adverse effect of water deficit on the crop yield was minimized.

Comparison with the DP–DP model

The major features of the present LP model and the earlier DP–DP model are similar. It would be interesting to compare the results from the two models when applied to the same data set. In this work, however, only an approximate comparison could be carried out because of the severe computational limitations of the DP–DP model. It must be noted that, although the two models serve the same objective, because of the solution algorithms, discretization in the DP models, and linearity assumptions in the LP model, results from the two models can be quite different. The following details of the DP–DP model application are relevant while examining the results from the two models:

(a) the reservoir storage was discretized into 15 storage class intervals, with a nonuniform discretization scheme discussed in Vedula & Mujumdar (1992);
(b) in the allocation model, water available for irrigation was divided into 30 discrete values;
(c) a single soil moisture state variable, being the average soil moisture, was used to represent the soil moisture state of all the crops;
(d) the soil moisture state variable was discretized into five class intervals;
(e) a soil moisture depletion factor of 0.45 was used in the *AET–PET vs* soil moisture relationship; and
(f) a state variable for the crop production was not included because of computational considerations.

In simulations with the DP–DP model, it was observed that the discretization of state variables had a significant effect on crop water allocations. Also, since the crop production state variable was not considered in the DP–DP model application, the interdependence of allocations to a crop among different periods was not accounted for. The allocation problem, in the absence of the crop production state variable, was solved independently of the actual allocations made to a crop during the previous periods. Table 4 gives a comparison of the release and crop water allocations resulting from the two models, when the current period, $t_p = 16$. These results correspond to the case presented in Table 2. A small difference in the total release from the two models is due to the difference in the storage-dependent evaporation

Table 4 Comparison of results of LP (I) and DP–DP (II) models.

Period	Release (10^6 m^3)		Allocations (10^6 m^3)									
			Cr1		Cr2		Cr3		Cr4		Cr5	
	I	II	I	II	I	II	I	II	I	II	I	II
16	78.76	0.00	29.68	0.00	1.64	0.00	1.73	0.00	1.71	0.00	4.62	0.00
17	66.12	25.00	7.55	8.75	2.69	1.25	5.61	0.00	3.99	2.50	13.23	0.00
18	88.82	15.00	8.24	2.25	3.79	3.00	14.00	1.50	4.01	0.00	14.38	0.75
19	59.46	15.00	7.79	2.25	0.00	0.00	7.96	3.00	0.00	0.00	13.99	2.25
20	42.51	65.00	7.95	10.83	8.42	5.42	0.00	0.00	4.20	16.25	0.69	0.00
21	37.50	10.00	8.66	0.00	5.58	0.00	0.00	2.50	4.51	0.00	0.00	2.50
22	7.13	85.00	0.00	17.00	0.00	8.50	0.00	0.00	3.56	12.75	0.00	4.25
23	32.02	105.50	8.67	15.83	4.79	10.56	0.00	0.00	2.55	21.10	0.00	5.28
24	37.30	55.00	12.06	8.25	2.64	11.00	0.00	2.75	3.96	0.00	0.00	5.50
25	21.20	55.00	7.18	8.25	2.72	11.00	0.00	2.75	0.74	0.00	0.00	5.50
26	13.41	57.50	6.70	14.38	0.00	9.59	0.00	0.00	0.00	0.00	0.00	4.80
27	11.28	1.68	5.64	0.42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.42
28	1.17	1.17	0.58	0.42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.17
29	0.20	2.50	0.10	1.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
31	0.27	0.27	0.14	0.14	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Total	**497.16**	**493.62**	**110.94**	**90.02**	**32.27**	**60.32**	**29.30**	**12.50**	**29.23**	**52.60**	**46.91**	**31.42**
$A_c \sum_{t=16}^{31} PET_c^t$			394.20		110.33		48.57		178.40		66.97	

I: LP model; II: DP–DP model.

Cr1: cotton; Cr2: sorghum; Cr3: pulses; Cr4: wheat; Cr5: safflower.

A_c: area of crop c; PET_c^t: potential evapotranspiration of crop c in period t.

losses. As a rough indicator of relative crop water requirements, the product of crop area and total potential evapotranspiration for each crop, is also given in Table 4. The actual crop water requirements computed in the models will be quite different from these values because of accounting for rainfall, soil moisture, root depth and soil moisture depletion in the models. It must be noted that it will not be possible to conclude, based on the results alone, whether one model is superior to the other. On the one hand, most features of the problem may be accurately represented in the DP–DP formulation, but the resulting model will be dimensionally too large for application to a real system., In the LP model, on the other hand, representation of crop production functions, reservoir storage losses, and *AET–PET* relationships, which are in fact, all nonlinear, requires the linearity assumption. This is a limitation of the LP formulation, but the resulting model is very easily applicable to a real system.

Parameters such as the root depth and the yield sensitivity factors which play an important role in the crop water allocations in both the models cannot, in general, be estimated accurately. A sensitivity study of the crop water allocations in the DP–DP model indicated that the allocations are more sensitive to the root depth than to the yield factors. An increase of 10% in the maximum root depth of a crop, while keeping all other parameters the same, indicated an increased allocation of about

22% to that crop, and the resulting deficit distributed to other crops. A similar increase in the yield factors *Ky* indicated an increase of 10% in the crop water allocation. However, it may be noted that more rigorous studies are necessary before a general conclusion on the sensitivity of allocations to these parameters can be drawn.

CONCLUDING REMARKS

A deterministic LP model is developed in this paper for the short-term annual operation of irrigation reservoirs. The model presented in this paper is similar in many ways to the DP–DP model developed by Mujumdar & Ramesh (1997) for real-time reservoir operation for irrigation. A major difference between the present LP model and the earlier DP–DP model is that, while in the earlier model two dynamic programming formulations were used—one for obtaining crop water allocations and the other for deriving the operating policy—in the present work, a single integrated LP formulation was used to serve the same purpose of specifying reservoir release and crop water allocations. The major limitations of the DP–DP model are: (a) a large number of state variables are needed for its application in real situations, making it computationally intractable in many cases, and (b) the crop water allocation model, being solved externally during each intra-seasonal period for a given set of state variables in the operation policy model, restricts the allocations to a set of discrete values only. Both these limitations are overcome in the present model by replacing the two DP formulations of the earlier model by a single LP formulation. However, because the problem is addressed in a LP framework, the nonlinearities in the crop production functions, reservoir area capacity relationships and variation of *AET* with soil moisture are all approximated with linear functions. These may be more accurately modelled in the DP–DP framework. The uncertainty due to inflow forecasts, present in the DP–DP model, are also present in the LP model, since it is a deterministic model. Future work is directed towards modelling in a stochastic framework, with uncertainty of inflows and crop water demands both addressed in a single integrated model.

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