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Mathematical Tools for Irrigation Water Management
An Overview

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Abstract: *Irrigation water management has significant economic implications in developing countries like India. While the structural infrastructure has been created with a huge financial investment in these countries, it is vital that appropriate non-structural measures be adopted for efficient water management. Scientific policies of operation of irrigation reservoir systems need to be developed with the aid of mathematical tools and implemented in practice. In this paper, a brief overview of some mathematical tools for irrigation system operation, crop water allocations and performance evaluation is presented, with a discussion on the work carried out in India by the author's team. Recent tools and techniques of fuzzy optimization and fuzzy inference systems that incorporate imprecision in management goals and constraints and that address the interests of stakeholders are also discussed. Perceptions on issues relating to applicability of the tools to real-life problems, existing gaps between theory and practice and possible hurdles in narrowing such gaps in developing countries are presented.*

Keywords: *Irrigation, reservoir operation, fuzzy sets, fuzzy optimization, performance evaluation, dynamic programming, crop yield.*

Introduction

Irrigated agriculture is by far the largest consumptive user of water in many developing countries like India. Irrigation water management thus has enormous economic implications for these countries. While the structural infrastructure for irrigation – comprising of reservoirs, canal networks, drainage works, and delivery systems – is created at a huge financial investment, a commensurate effort is also essential on developing scientific water management policies. Developments in systems science, operations research, and mathematical modeling for decision making under uncertainty have been usefully exploited for water resource management in many technologically advanced countries. Applications of such mathematical techniques to specify irrigation water management policies in the developing countries – at both macro as well as micro level – and implementing them in practice will lead to significant economic benefits. Considerable work has been carried out in academic institutes in India and other developing countries to develop mathematical models for irrigation water management, addressing issues specific to those countries. In this paper, an overview of some mathematical tools used for irrigation reservoir operations and performance evaluation is presented, with a discussion on work carried out in India by the author's team. The paper does not attempt a general state-of-the-art review, but provides an overview of some models developed by the author over the years for irrigation water management.

Irrigation water management is characterized by uncertainty not only due to randomness of hydrologic variables such as streamflow, rainfall, and evapotranspiration, but also due to imprecision in management goals, constraints, crop response, and stakeholder interests. Mathematical tools used in deriving policies for irrigation water management must address both these uncertainties. The stochastic optimization techniques (such as the stochastic dynamic programming, SDP) explicitly incorporate uncertainties due to randomness of hydrologic variables. The recent tools of fuzzy optimization and fuzzy inference systems address uncertainties due to imprecision in various components of the management problem. In addition, the Monte Carlo simulation technique is used for evaluating the performance of an irrigation system. Table 1 shows the purposes for which these tools are used in water resources management, with a particular emphasis on irrigation water management, along with some recent examples in literature where the tools have been applied. Although the mathematical tools and techniques discussed in this paper are general in nature and are applicable to any irrigation reservoir system, they are particularly useful in extreme water deficit situations, such as those existing in many parts of South India. In most developing economies driven predominantly by agriculture, irrigation is by far the largest consumptive user of water. As an example, Figure 1 shows the estimated surface water requirements for different sectors in India in the year 2010, emphasizing the importance of irrigation water manage-

Table1. Mathematical Tools Commonly Used in Water Resources Management*

Objective	*Mathematical Tools*	*Issues Addressed*	*Recent Literature*
Long-term, steady state reservoir operation for irrigation	Stochastic Dynamic Programming	Uncertainty due to randomness of hydrologic variables; single decision-making mechanisms for reservoir operation and crop water allocations	Houghtalen and Loftis (1988); Dudley (1988); Dudley and Scott (1993); Vedula and Mujumdar (1992); Vedula and Nagesh Kumar (1996); Ravikumar and Venugopal (1998)
Field-level irrigation scheduling	Stochastic/Deterministic Dynamic Programming; Linear Programming	Seasonal and intraseasonal allocation of deficit water; competition among crops; crop yield optimization	Paudyal and Das Gupta (1988); Rao et al. (1990); Azar et al. (1992); Mannocchi and Marcelli (1994); Sunantara and Ramirez (1997); Paul et al. (2000); Anwar and Clake (2001)
Real-time operation of irrigation systems	Dynamic Programming; Linear Programming; Simulation	Adaptive operation; real-time forecasts of hydrologic variables	Dariane and Hughes (1991), Rao et al. (1992); Mujumdar and Ramesh (1997); Wardlaw and Barnes (1999)
Performance evaluation	Monte-Carlo Simulation	Evaluation of reliability, resiliency, vulnerability and productivity index	Hashimoto et al. (1982); Mujumdar and Vedula (1992); Srinivasan and Philipose (1988); Srinivasan et al. (1999); Sahoo et al. (2001)
Conflict Resolution	Fuzzy Optimization	Uncertainly due to imprecision in goals and constraints	Kindler (1992); Fontane et al. (1997); Bender and Simonovic (2000)
Qualification of Imprecision	Fuzzy Inference Systems; Fuzzy Relation Analysis	Formulation of fuzzy rules	Shreshta et al. (1996); Russell and Campbell (1996); Yin et al. (1999); Teegavarapu and Simonovic (2000); Despic and Simonovic (2000); Panigrahi and Mujumdar (2000)

* With emphasis on irrigation water management

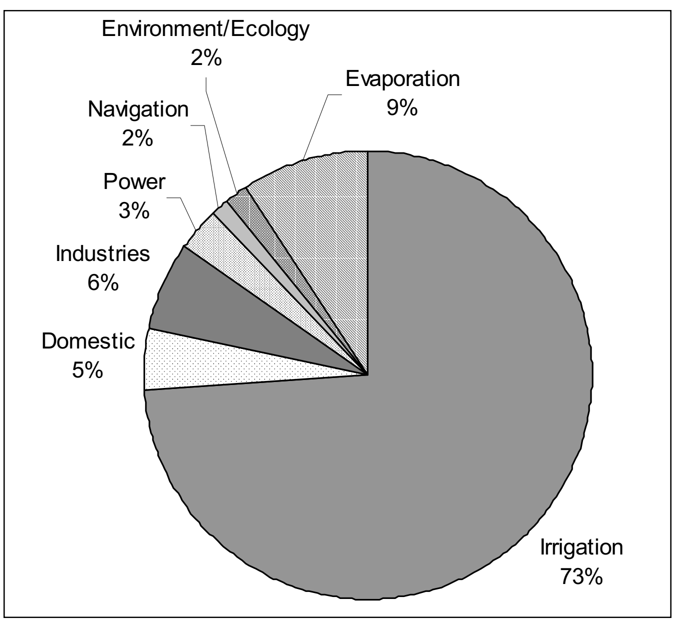

Figure 1. Surface Water Requirements for Different Users – India (2010). Source: Ministry of Water Resources (1999).

ment in that country. Agricultural productivity, however, remains poor in most developing countries because of poor water management policies. In an exhaustive and lucid review of irrigation water management in India, Sarma (2002) has discussed several critical issues related with poor agricultural productivity. Similar situations exist in many other developing countries also, where irrigation activity has expanded enormously over the years but is very inefficient technologically, economically, and environmentally (Chaturvedi, 1992). Increasing the efficiency of irrigation through scientific operating policies therefore forms a major nonstructural measure for irrigation water management in such developing countries.

The irrigation reservoir system to which most models discussed in this paper may be applied is schematically shown in Figure 2. The techniques are discussed with reference to this general system. The applications considered are for obtaining reservoir operating policies – both steady state, long term policies as well as real-time operating policies – for field level crop water allocations, and for performance evaluation of the entire irrigation system as shown in Figure 2. It is emphasized here that the tools discussed in the paper do not constitute an exhaustive set of mathematical tools available for applications to irrigation water management. The discussion of the tools is organized according to the purpose for which the tools are used, as follows: Stochastic Dynamic Programming for long term operation, Deterministic Dynamic Programming for real-time operation, Monte Carlo Simulation for performance evaluation, Fuzzy Optimization for conflict resoultion, and Fuzzy Inference Systems for quantifying imprecison.

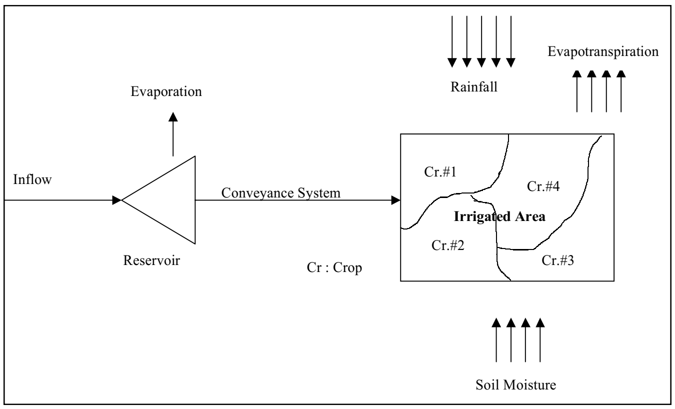

Figure 2. Components of a Typical Surface Water Irrigation System.

Stochastic Dynamic Programming

Decision making for reservoir releases for irrigation involves many subtle considerations such as the nature and timing of the crop being irrigated, its stage of growth, competition among crops for a limited amount of available water, and the effect of a deficit water supply on the crop yield. From the point of view of efficient use of water at the field level, a single decision-making mechanism for the entire system – consisting of the reservoir and the delivery subsystem – is implied in the integration of the decisions at the reservoir level with those at the field level. The decisions should be sufficiently explicit to indicate not only how much water is to be released from the reservoir in a given time period, but also how much of it should be allocated to a given crop. The uncertainty in various hydrologic variables involved – reservoir inflow, rainfall, evapotranspiration, and the soil moisture – adds to the complexity of decision making. Stochastic dynamic programming (SDP) can aid in the development of long-term reservoir operating policies for irrigation, given the complete cropping scenario in the command area.

The irrigation release decisions are to be made in short time intervals such as a week, ten days, or, at the most, two weeks. Mathematical models which aid decisions over larger time intervals such as a month or a season are inadequate, as they do not take into account the variability in irrigation demand within these time intervals. Mathematical models have, therefore, been developed to determine a long-term operating policy considering the intraseasonal irrigation demands of the crops and competition among them in the face of a deficit supply (e.g., Dudley, 1988; Vedula and Mujumdar, 1992). The crop irrigation demands, which vary from period to period, are determined from a soil moisture balance (Figure 3), starting with a known soil moisture (subscripted by *m*, in Figure 3) at the beginning of a period to obtain soil moisture at the end of the period (subscripted by n). Decision models integrating the reservoir release decisions with the irrigation allocation decisions at the field level, with reference to each crop in each intraseasonal period, are formulated to conceptually operate in two phases. In the first phase, deterministic dynamic programming is used to allocate a known amount of water among all crops to optimize the "impact" of allocation in a period (with impact measured with respect to the evapotranspiration deficit and sensitivity of the crop yield to a deficit supply in that period). Such allocations are determined for all possible supplies in a given period, for all periods in a year. In the second phase, a stochastic dynamic programming model evaluates all the intraseasonal periods to optimize the overall impact of the allocation over a full year. The end result of this two-phase analysis is a set of decisions indicating the reservoir releases to be made in each intraseason period and the distribution among the crops of this release available at the crop level after accounting for losses between the reservoir and the application area.

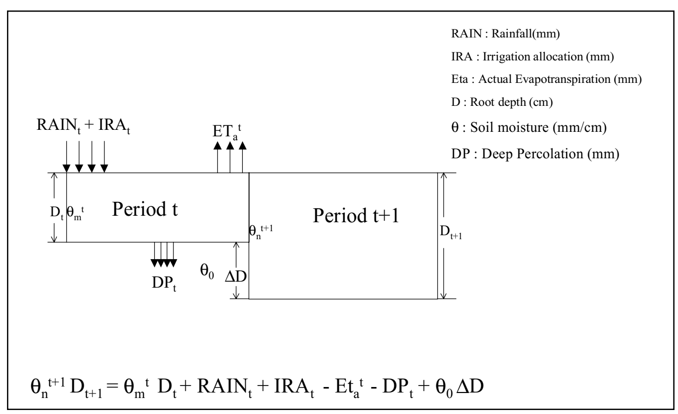

Figure 3. Soil moisture balance.

Figure 4 shows the state transformation for the SDP problem for reservoir operation for irrigation. The four state variables being considered are: reservoir storage, inflow, soil moisture, and rainfall in the command area. The recursive relationship for this four-state variable SDP model is written as (Mujumdar and Vedula, 2000)

$$f_s^t\ (k,i,m,p) = \underset{\{1\}}{Max}\ [G(k,i,l,m,p,t) + \sum_q \sum_j P^{t+1}(q)\ P_{ij}^{\ t}\ f_{s-1}^{\ t+1}\ (l,j,n,q)] \qquad \forall\ k,i,m,p \qquad (1)$$

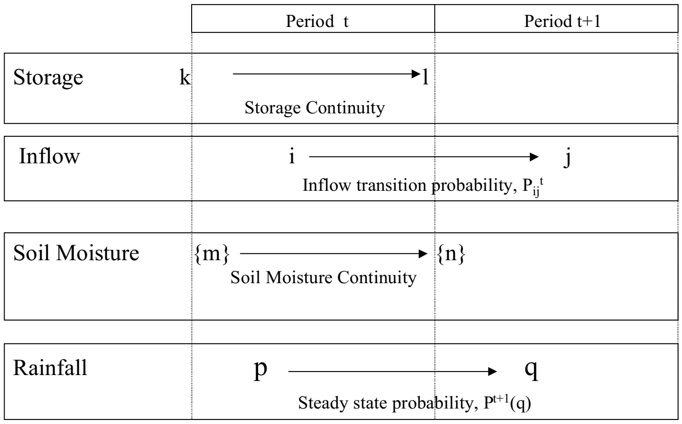

Figure 4. State variables and their transformation in the Stochastic Dynamic Programming Model.

where $f_s^t\ (k,i,m,p)$ is the optimal value of the objective function in period t with s stages to go, with reservoir storage class k, inflow class i, soil moisture class m, and rainfall class p; $G(k,i,l,m,p,t)$ is a measure of the system performance, corresponding to the discrete classes k, i, l, m and p in period t, and is obtained by solving the allocation problem; $P^{t+1}(q)$ is the probability of rainfall in class q in period t+1, and, $P_{ij}^{\ t}$ is the transition probability of inflow being in class j in period $t+1$ given that it is in class i in period t.

When this equation is solved recursively, a steady state solution is reached fairly quickly (within about four to five cycles). The decision variable in the optimization is the end-of-the-period storage class l, in a period t for given values of k, i, m and p. The steady state operating policy is specified as the optimal end-of-the-period storage class, l*, to be maintained for a given initial storage, inflow, rainfall, and soil moisture of each crop. The optimal release from the reservoir is obtained from the storage continuity corresponding to the storage class interval l*. The crop water allocations out of this release are specified through the allocation model, after accounting for conveyance and other losses. It must be noted that the SDP model quickly falls prey to the "curse of dimensionality," as the number of state variables increases. For a case study of the Malaprabha Reservoir in South India (Mujumdar and Vedula, 2000) the SDP model expressed in Equation 1, bordered on computational intractability with 15 class intervals for storage, four for inflow, five for soil moisture of each crop (five crops) and four class intervals for rainfall, with 36 within one-year time periods. The model, therefore, remains a single purpose, single reservoir model, and an extension to include multiple reservoirs or multiple purposes is possible only by sacrificing a great deal of crop level details (e.g., by excluding soil moisture and rainfall state variables) in the model.

As the policy specifies a steady state operation, the cropping pattern used in the SDP model is assumed to remain fixed over the entire operating horizon. In most developing countries, where the land holdings are small, it is often the individual farmers who decide which crops to be grown in a season rather than the government agencies responsible for irrigation water management. Therefore, it is unrealistic to expect that the cropping pattern will remain unaltered over the operating horizon. The long term, steady state operating policy resulting from a SDP model should be viewed essentially as a planning aid, in which a planned cropping pattern is implied. Separate real-time operating policies need to be derived for annual operations with actual cropping patterns adopted. A good interaction between a steady state operation and a real time operation may be ensured by specifying constraints on end-of-the-year reservoir storages to be maintained – consistent with long term policies – during real-time operation.

Deterministic Dynamic Programming

Deterministic dynamic programming is used for arriving at adaptive real-time operating policies for an irrigation reservoir system. The real-time operation model is formulated as a dynamic programming problem (Mujumdar and Ramesh, 1997) for solution once at the beginning of each intraseasonal period. It uses forecasted inflows for the current period in real time (for which a decision is sought) and for all subsequent periods in the year. These forecasts themselves are obtained using the latest available information on previous period's inflow. Solution of the real-time operation model gives the release decisions and the crop water allocations for all periods in the year starting from the current period. Only the decisions on release and allocations for the current period are implemented and the state of the system (comprising of the reservoir storage, soil moisture of each crop and a crop production measure indicating the current state of the crop) is updated at the end of the period. Because the model is applied in real-time, the productive value of previous allocations to a crop, up to the beginning of the current period in real time would be known. This information is taken as an input to the model in deriving optimal releases for the subsequent periods in the year. The model thus updates the release decisions from period to period, making use of the latest available information. Actual rainfall in the command area, reservoir inflow, current production status, and actual soil moistures of the crops all contribute to the updating of the release decisions for the subsequent periods. The irrigation allocation to a crop in a period is based on: (1) its current production status, which is the net effect of water supplied to the crop (through irrigation allocations and precipitation) from the beginning of the season up to the beginning of that period; (2) available soil moisture in the root zone of the crop; and (3) competition for water with other crops. The first two conditions are introduced in the mathematical model through the use of two state variables, a crop production state variable and a soil moisture state variable for each crop. The third condition of competition with other crops is introduced through use of crop yield factors, K_y, in the objective function which indicate the sensitivity of a crop to a deficit supply, and which vary with the crop growth stages. The state variable for crop production indicates the production potential of a crop from the current period to the end of the crop season. Features of the real-time operation model are shown in Figure 5. In this figure, t_p is the current period in real-time, T is the last period in the year, and I_t is the inflow during period t. The operating policy model, which scans across time periods from the current period to the last period in the year is formulated as a deterministic dynamic programming model. The allocation model, which provides decisions on crop water allocations in an intraseasonal period, is solved for a given amount of available water, known soil moisture of each crop, and a given crop production measure of each crop.

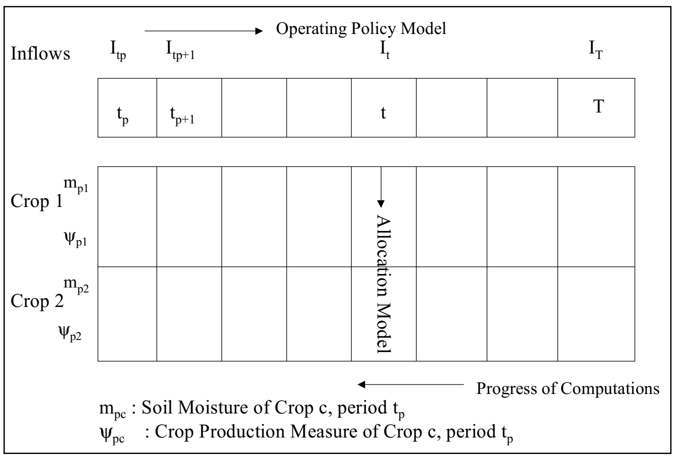

Figure 5. Real-time operational model.

The recursive relationship for the operation model is written, for any intermediate period t, as,

$$f_t^j (k,\mathbf{M},\psi_b)=Max[\varphi (k,l,\mathbf{M}, \psi_b,t)+f_{t+1}^{j-1}(l,\mathbf{N}, \psi_e)] \quad (2)$$

where $\mathbf{M}$ is the soil moisture vector, at the beginning of period, t; ψ_b is the crop production measure vector, at the beginning of period, t; $\mathbf{N}$ is the soil moisture vector at the end of period, t; ψ_e is the crop production measure vector, at the end of period, t; $j (k,l,\mathbf{M}, \psi_b,t)$ is the system performance measure at period t; and, $f_t^j (k,\mathbf{M},\psi_b)$ is the optimal accumulated system performance measure up to period t.

The system performance measure, $\varphi (k,l,\mathbf{M}, \psi_b,t)$, is obtained from the solution of the allocation model. The real-time operation model needs to be solved once at the beginning of each period in real-time specifying the current state of the system, in terms of the crop production measure, crop soil moisture, and reservoir storage. The crop production measure may be computed from production functions, based on the actual allocations made to the crop up to the beginning of the current period in real-time. Solution of the real time operation model specifies the reservoir release for all subsequent periods, including the current period along with crop water allocations.

Monte Carlo Simulation

When an operating policy is derived for an irrigation reservoir based on an objective function, the policy itself does not, in general, indicate how the system will actually perform, unless a criterion to this effect is embedded in the objective function. While a systems analyst is interested in arriving at the operating policy, the decision maker in charge of actual operations would look for implications of using the policy through answers to questions such as how often the system will fail and how quickly it will recover from a failure under a given policy of operation. It is therefore important to study implications of reservoir operation with a given policy while considering the interests of the decision maker. The Monte Carlo simulation technique is useful in generating large sequences of hydrologic variables that may then be used in developing a database on simulated operation of the system. Such a database is useful in evaluating the expected performance of the system. Three performance measures that are commonly used (e.g., Hashimoto et al., 1982) for performance evaluation of reservoir systems are reliability, resiliency, and vulnerability. In the context of irrigation, the vulnerability, which indicates the effect of failure, is replaced by a productivity index, which reflects the overall crop productivity under a given policy of operation.

The reliability of a system is defined as the probability that the system output is satisfactory. In case of irrigation systems, various interpretations of the concept of satisfactory output are possible. The system output in a period may be measured in terms of simply the water availability at the field level or it may be related to the soil moisture through the actual evapotranspiration of crops. Taking the output to be satisfactory in period t, whenever the water available for irrigation at the field level (after accounting for all losses from reservoir to the application area) is at least equal to the total irrigation requirement of all crops present in that period, the reliability may be computed as $P\ [X_t \geq \Delta_t]$, where X_t is the water available at the field level in period t, and Δ_t is the total irrigation requirement in period t. This definition of reliability simply reflects the likelihood of a non-failure without specifying the extent of the failure, when one occurs. It still provides a good measure of the ability of the system to provide the required irrigation.

While the system reliability gives the likelihood of a satisfactory performance, the resiliency gives the likelihood of the system recovery from a failure, once a failure occurs. If the recovery from failure is slow, it may have serious implications on the crop yield. A higher resiliency would mean a quicker recovery and hence one would prefer policies having high resiliency as well as high reliability. Mathematically, the resiliency γ, is defined (Hashimoto et al., 1982) as the inverse of the expected value of T_u, the length of time (number of periods, subsequent to a failure period) that the system output remains unsatisfactory. This is determined as

$$\gamma = P\ (\xi_{t+1} \in V_{t+1} / \xi_t \in U_t) \qquad (3)$$

where ξ_t is the system output in period *t,* U_t is a set of unsatisfactory (failure) outputs in period *t* and V_t is a set of satisfactory (success) outputs in period t. Expressed in words, the resiliency is given by the probability that the system output in period *t+1* is satisfactory, given that it is unsatisfactory in period *t*. In case of irrigation systems, the system output, ξ_t, indicates, for a given operating policy, the water available at the field level for allocation among crops in period *t*. The sets V_t and U_t are defined in terms of the amount of water required by the crops for meeting the evapotranspiration demands, accounting for the crop soil moistures in period *t*.

It is interesting to note that even with a high reliability and resiliency, the effect of failures can be quite significant on the crop yield, if such failures occur in critical preriods of the growth season of the crop and/or if the extent of failure (deficit) is so large as to cause permanent damage to the crop. The performance of the system must also, therefore, be measured with respect to the crop yield resulting from a policy. An index called the productivity index, η, is defined as a measure of the relative yields of the crops. It is defined as the probability that the average of the relative yields among all crops in a year is greater than a specified value $y*$. That is,

$$\eta = P[(y/y_m)_{av} \geq y*] \qquad (4)$$

where $(y/y_m)_{av}$ is the average relative yield resulting from the policy, determined from appropriate crop production functions, which relate the crop yield to evapotranspiration deficits occuring during various growth stages of the crop. The three performance indicators — reliability, resiliency, and productivity index — are computed from the simulated operation of the reservoir system. The Monte Carlo simulation technique is used to generate reservoir inflow sequences for a sufficiently long period of time (typically for one to two hundred years), and for a specified failure event, the simulation results are examined for occurrence of failure and for the transition of failure to a success state. Reliability and resiliency are computed simply by a relative frequency approach, whereas the productivity index is computed from a crop production function (e.g., Doorenbos and Kassam, 1979), knowing the irrigation allocation and soil moistures for each crop during each time period, from the simulated operation.

An imortant question to be addressed in the performance evaluation of irrigation projects is whether the system is more vulnerable to high frequency, low impact failures or to low frequency, high impact failures. This is addressed by the productivity index given by Equation 4. The effect of failures (deficit supplies) on the crop output is, in a broad sense, provided by the productivity index. The relative yield of a crop y/y_m used in Equation 4 is a function not only of the amount of water deficit, but also of the timing of the deficit. Indeed, the crop yield is greatly affected by the timing of the deficit. If a deficit occurs during a critical growth stage of a crop, it will have a greater adverse impact on crop yield than the same deficit occuring in another growth stage that is not so critical. Crop production functions, used to estimate the relative yields of the crops take into account the sensitivity of crop yield to deficit supply during various growth stages, and thus the productivity index provides important information on the effect of both the frequencies of failures and the extent of failures.

Fuzzy Optimization

In most decision-making problems related to water management, the objectives and/or constraints are often imprecisely stated and conflict with each other. For example, in an irrigation water management problem, we may not have a clear (crisp) demarcation of crop yield returns. Instead what we would be looking for is to achieve "as high a crop yield as possible," when severe water deficits would constrain the irrigation in certain drought periods to "as low an allocation as possible." As the number of stakeholders increases in a decision-making process, imprecision as well as conflict is bound to increase. Such imprecision and conflict in objectives (as well as constraints) may be incorporated in the decision models by using the concept of the fuzzy decision making introduced by Bellman and Zadeh (1970).

Bellman and Zadeh (1970) considered the following approach for decision making when there are fuzzy goals and fuzzy constraints for the set of alternatives (the decision vector), as decision making in a fuzzy environment. A fuzzy goal *G* and a fuzzy constraint *C* are fuzzy sets on the set of alternatives, which are characterized by the membership functions μ_G : [0,1] and μ_C : [0,1], respectively.

The fuzzy decision *Z* is defined as the intersection of a fuzzy goal *G* and a fuzzy constraint *C*. In other words, the fuzzy decision is defined by

$$Z = G \cap C \qquad (5)$$

and its membership function is defined by

$$\mu_Z(x) = min\ (\mu_G(x),\ \mu_C(x)) \qquad (6)$$

where x is the decision variable.

In the general case in which r number of fuzzy goals G_1, G_2, G_r, and h number of fuzzy constraints C_1, C_2, C_h exist, the fuzzy decision Z is defined as the intersection of these.

$$Z = G_1 \cap G_2 \cap \ldots \cap G_r \cap C_1 \cap C_2 \cap \ldots . C_h \qquad (7)$$

The fuzzy decision Z is characterized by the membership function,

$$\mu_Z(x) = \text{Min}\ (\mu_{G1}(x),\ \mu_{G2}(x),\ \ldots..\mu_{Gr}(x),\ \mu_{C1}(x),\ \ldots\mu_{Ch}(x)) \qquad (8)$$

The optimal fuzzy decision corresponds to that x with the maximum degree of membership in Z. That is, the objective is to find x^*, such that,

$$\mu_Z(x^*) = max\ \mu_Z(x) = max\{min(\mu_{G1}(x),\ \mu_{G2}(x), \ldots\ldots \mu_{Gr}(x), \mu_{C1}(x),\ \mu_{C2}(x), \ldots\ \mu_{Ch}(x))\} \qquad (9)$$

This concept is shown in Figure 6 for one fuzzy goal G, and one fuzzy constraint C. In practical situations, a large number of fuzzy goals and fuzzy constraints will be present.

The space of alternatives (that is, the decision space) is generally restricted also by precisely defined constraints and bounds known as the crisp constraints (e.g., reservoir mass balance, channel carrying capacity constraints, soil moisture limits of field capacity and wilting point, etc.). Incorporating such crisp constraints, the fuzzy optimization problem is stated as follows (e.g., Zimmerman, 1985; Kindler, 1992; Rao, 1993; Sakawa, 1995; Sasikumar and Mujumdar, 1998).

$$Max\ \ \lambda \qquad (10)$$

Subject to

$$\mu_i\ (x) \geq \lambda \qquad \forall\ i \qquad (11)$$

$$g_j\ (x) \leq 0 \qquad \forall\ j \qquad (12)$$

$$0 \leq \lambda \leq 1 \qquad (13)$$

The objective function and the constrains together define λ as a "Maximum of minimum membership function," and is interpreted as maximization of the minimum satisfaction among all stakeholders. In an irrigation management problem, the stakeholders may include decision makers at the field level, reservoir level, and administrative level along with the farmers and the village groups. The decision variable x corresponds to irrigation water allocation. The response of the stakeholders to water allocations are incorporated in the decision models through the fuzzy membership functions $\mu_i(x)$, where i refers to a particular stakeholder. The upper and lower bounds of λ reflect two extreme scenarios in the system. The upper bound, λ =1, indicates that all the goals have been completely satisfied and therefore represents a no-conflict scenario. The lower bound, λ =0, indicates that at least one goal has a zero satisfaction level and therefore represents a total-conflict scenario. Any intermediate value of λ represents the degree of conflict that exists in the system. Kindler (1992) has demonstrated the use of fuzzy optimization for allocating water resources to multiple users in a river system. The level of water requirement together with the tolerance to a deficit in required quantity is interpreted in terms of the user's satisfaction. The objective of a user regarding the water quantity requirement is then transformed into a fuzzy goal. In the case in which the available water resources are insufficient to completely satisfy all the users, it is aimed to maximize the minimum satisfaction in the system through fuzzy optimization. The criterion of such an allocation has been termed a "fair-compromise" principle.

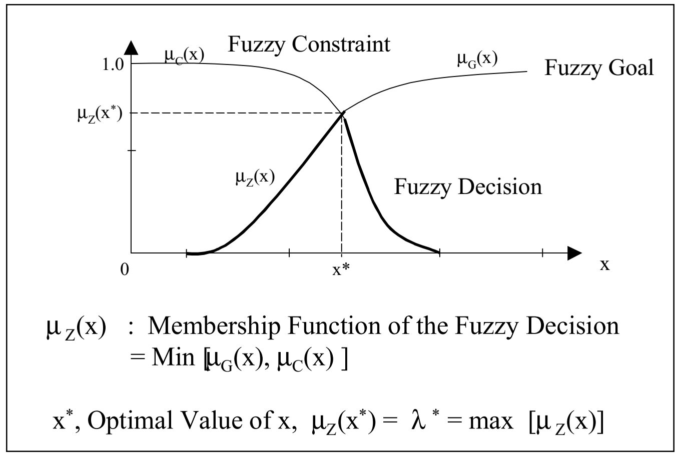

Figure 6. Fuzzy Decision

Fuzzy Inference Systems (FIS)

Uncertainties due to imprecision exist in most decision-making processes. In case of irrigation water management problems, these may stem from our lack of clear (crisp) knowledge on how a crop yield responds to variations in farm practices (e.g., type of seedlings, fertilizers, land preparation methods, etc.), climatic variables (such as temperature, humidity, etc.) and from a need to include social and cultural "equity" criteria in the decision-making process. Imprecision also stems from lack of adequate data, in which case assigning probability distributions to a random variable (such as reservoir inflow) may become impossible, and we may need to treat the variable as a fuzzy variable.

The fuzzy inference systems operate on "if – then" principle, where the "if" part is a vector of fuzzy explanatory variables or premises, and the "then" part is a fuzzy consequence. Operations such as fuzzification of inputs, formulation of fuzzy rule sets, application of the fuzzy operator, implication, aggregation, and finally defuzzification are carried out in obtaining decisions. Applications of the fuzzy rule based reservoir operations may be seen in Russell and Campbell (1996), Shreshta et al., (1996) and Panigrahi and Mujumdar (2000). Despic and Simonovic (2000) have provided an indepth discussion on the aggregation operators used in fuzzy inference systems.

Consider the example of addressing imprecision in the effect of temperature and farm practices on the relative yield deficit of a crop. The relative yield deficit, the temperature and the farm practices are all expressed as fuzzy sets and appropriate membership functions are defined as shown in the Figure 7. The farm practices are mapped to (0,1) with the aid of agriculture experts' input. These membership functions reflect the degree to which a particular variable belongs to a fuzzy set. For example, a relative yield deficit of 0.45 belongs to the fuzzy set, "Medium" deficit with a degree of 1, whereas a deficit of about 0.5 belongs to the fuzzy set of "Medium High" with a degree close to about 0.1 and also to the fuzzy set, "Medium" with a degree of close to about 0.5. Once the membership functions are defined for all fuzzy sets, the fuzzy rules are formulated based on experts' input on agronomic considerations. Typical fuzzy rules for this problem are expressed as, for example, "if temperature is *low* and farm practice is *poor* then the relative yield deficit is *medium,*" "if temperature is *low* and farm practice is *average* then relative yield deficit is *medium low*," and "if temperature is *optimum* and farm practice is *good* then relative yield deficit is *low.*" Such fuzzy rules are then used in conjunction with the crop water allocation model to optimally allocate a given amount of water among crops.

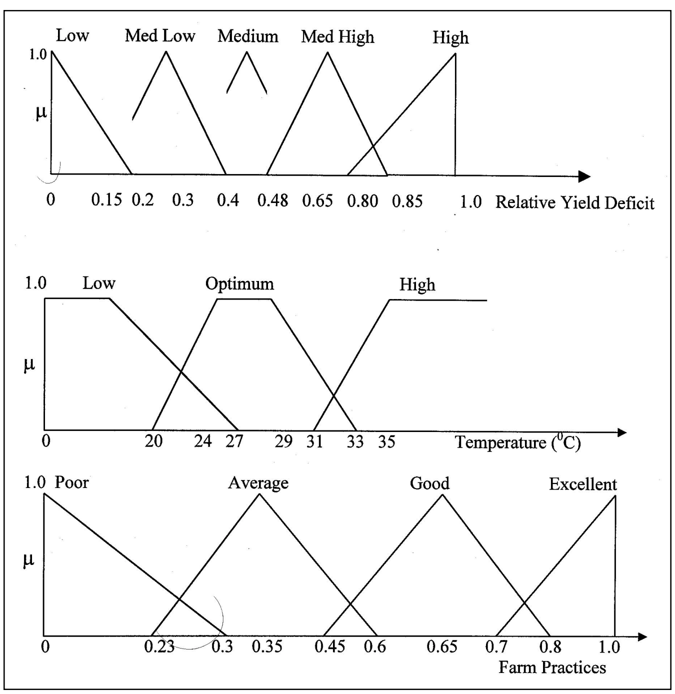

Figure 7. Fuzzy membership functions for relative yield deficit, temperature and farm practices.

Concluding Remarks

Four decades of research on development of mathematical models for water resources management has resulted in a number of useful mathematical tools for decision making. However, the actual implementation of results of the research has been rather slow. Simonovic (1992) has discussed causes and possible remedial measures for closing the gap between theory and practice in reservoir operation decision problems. An important conclusion drawn in that study states: "in order to achieve the future direction of closing the gap, there must be further integration of new technologies (like expert systems) with existing simulation and optimization tools for reservoir analysis." This conclusion is especially relevant in the case of irrigation water management in developing countries, where the institutional mechanisms of technology transfer are rather poor. Decision support systems (e.g., Simonovic and Savic, 1989; Simonovic, 1991; Simonovic, 2000; Ahmad and Simonovic, 2002; Simonovic, 2002) that integrate mathematical models in a user friendly, knowledge-based, and object-oriented computer software serve this very important purpose of technology transfer to provide implementable decisions

In most developing countries, the gap between theory and practice in irrigation water management is very wide, and most of the systems studies remain restricted to the academic arena. In a study of systems policy analysis for irrigation in India, Chaturvedi (1992) revealed several reasons for existence of such a gap:

- Despite a gradual shift of orientation of decision- making bodies from big dams and structural infrastructure options to wider aspects of technology and systems analysis, institutional constraints have greatly hindered the progress of real-life applications;
- Considerable capability exists in academic institutions, but they suffer from two handicaps – first the interaction of academics with real life problems is much more limited than in their counterpart institutions in advanced countries, and second, their facilities, and more importantly, their environment is essentially that of the Third World, despite very generous support from the Government; and
- The best approach for closing the gap is through academic linkages and developing networks at the academic level, focusing on applied research in the context of real-life problems.

To improve water management policies for irrigation in developing countries, thus, there is an urgent need to bring together the expertise of the academia and the extensive field experience of the decision makers. In this context, an encouraging aspect of the financial support being provided by international institutions for irrigation projects in developing countries is that the funding agencies often insist on systems studies to be carried out as a part of the planning exercise, and such compulsions indeed help encourage an interaction between the decision makers and the academic institutions. In many recent irrigation projects in India (e.g., the Upper Krishna Project in South India), where financial assistance has been received from international institutions (such as the World Bank), systems studies employing mathematical models have been carried out at the planning stage, and such studies have helped in arriving at best compromise solutions in the presence of a conflict between individual states.

A distinct feature of the irrigation systems operation in developing countries is that the water allocation decisions are often forced by strong socio-political considerations rather than technological ones. As a result, utility of the mathematical models developed purely based on scientific and technological considerations (such as most models discussed in this paper) for real-life applications will be extremely limited, and the models may at best be useful as planning tools. Mathematical models developed for aiding irrigation decisions, therefore, must be oriented towards addressing such issues in addition to the scientific ones for better acceptability. Inclusion of various stakeholders' interests must be ensured in the models. This may be ideally accomplished by involving the stakeholders in all phases of model development and testing. Crisp idealization of various aspects related to crop yield response, operational objectives and long term goals of irrigation development need to be relaxed, and instead of providing single valued "optimal" decisions, the models should be capable of providing a band of acceptable, near optimal (pareto-optimal or non-inferior) solutions.

These conditions imply that a great deal of imprecision and conflict should be addressed in the mathematical models. Multi-objective optimization tools (e.g., Loucks et al., 1981) of goal programming, compromise programming, and surrogate worth trade-off are useful in addressing conflicting objectives of stakeholders. Recent tools of fuzzy optimization and fuzzy inference systems are also useful in including the stakeholder interests in the decision models. Application of the fuzzy sets theory for modeling uncertainty due to imprecision in water resources management is relatively recent and provides an opportunity for useful research contributions in future. Studies for defining appropriate fuzzy membership functions for various stakeholders to reflect their perceptions accurately in the mathematical models would contribute a great deal in addressing the stakeholders interests in irrigation decisions. Such studies would also ensure an effective multidisciplinary interaction.

Simultaneous considerations of both types of uncertainty – uncertainty due to randomness of hydrologic and climatic variables and that due to imprecision and subjectivity in management goals and constraints – in a single integrated model would render the model more realistic. In addition to these two sources of uncertainties, there also exists the model uncertainty – uncertainty inherent in the mathematical models themselves: uncertainty resulting either from the choice of an incorrect model with correct (deterministic) parameters or from the choice of a correct model with incorrect (uncertain) parameters. In real situations, such partition is seldom feasible. However, in the analysis of model uncertainty, such a distinction is very useful. Uncertainties in the models may supplement the uncertainties in the inputs themselves, and thus the output uncertainty may get compounded. Since irrigation water management models employ a number of constituent sub-models (such as models for soil moisture balance, crop yield response and evapotranspiration relationships) it is essential that the model uncertainties are also analyzed and the results interpreted correctly before implementation. Results from such integrated mathematical models, capturing accurately the multidisciplinary content of the problem, would then serve as effective non-structural measures for irrigation water management.

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References

Ahmad, S. and S.P. Simonovic. 2002. "A Decision Support Tool for Evaluation of Impacts of Flood Management Policies." *Hydrologic Sciences and Technology Journal* 17, No. 1–4: 11–23.

Anwar A.A. and D. Clarke. 2001. "Irrigation Scheduling Using Mixed-Integer Linear Programming." *Journal of Irrigation and Drainage Engineering* 127, No. 2: 63–69.

Azar A.H., V.V.N. Murty, and H.N. Phien. 1992. "Modeling Irrigation Scheduling for Lowland Rice with Stochastic Rainfall." *Journal of Irrigation and Drainage Engineering* 118, No. 1: 36–55.

Bellman, R.E. and L.A. Zadeh. 1970. "Decision-making in a Fuzzy Environment." *Management Science* 17, No. 4: B141–B164.

Bender, M. J. and S.P. Simonovic. 2000. "A Fuzzy Compromise Approach to Water Resources Planning under Uncertainty." *Fuzzy Sets and Systems* 115, No. 1: 35–44.

Chaturvedi, M.C. 1992. "Irrigation and Drainage – Systems Policy Analysis and India Case Study." *Journal of Water Resources Planning and Management* 118, No. 4: 445–464.

Dariane, A.B. and T.C. Hughes. 1991. "Application of Crop Yield Functions in Reservoir Operation." *Water Resources Bulletin* 27, No. 4: 649–656.

Despic, O. and S.P. Simonovic. 2000. "Aggregation Operators for Soft Decision Making in Water Resources." *Fuzzy Sets and Systems* 115, No. 1: 11–33.

Doorenbos, J. and A.H. Kassam. 1979. "Yield Response to Water." FAO Irrigation and Drainage Paper 33. Italy: FAO.

Dudley, N.J. 1988. "A Single Decision-maker Approach to Irrigation Reservoir and Farm Management Decision Making." *Water Resources Research* 24, No. 5: 633–640.

Dudley, N.J. and B.W. Scott. 1993. "Integrating Irrigation Water Demand, Supply and Delivery Management in a Stochastic Environment." *Water Resources Research* 29, No. 9: 3093–3101.

Fontane, D.G., T. Gates, and E. Moncada. 1997. "Planning Reservoir Operations with Imprecise Objectives." *Journal of Water Resources Planning and Management* 123, No. 3: 154–162.

Hashimoto, T., D.P. Loucks, and J.R. Stedinger, J.R. 1982. "Reliability, Resiliency and Vulnerability Criteria for Water Resources System Performance Evaluation." *Water Resources Research* 18, No. 1, pp. 14-20.

Houghtalen, R.J. and J.C. Loftis. 1988. "Irrigation Water Delivery System Operation via Aggregate State Dynamic Programming." *Water Resources Bulletin* 24, No. 2: 427–434.

Kindler, J. 1992. "Rationalizing Water Requirements With Aid of Fuzzy Allocation Model." *Journal of Water Resources Planning and Management* 118, No. 3: 308–323.

Loucks, D.P., J.R. Stedinger, and D.A. Haith. 1981. "Water Resources Systems Planning and Analysis." Englewood Cliffs, New Jersey, USA: Prentice Hall, Inc.

Mannocchi, F. and P. Mecarelli. 1994. "Optimization Analysis of Deficit Irrigation Systems." *Journal of Irrigation and Drainage* Vol. 120, No. 3: 484–503.

Mujumdar, P.P. and S. Vedula. 1992. "Performance Evaluation of an Irrigation System Under Some Optimal Operating Policies." *Hydrological Sciences Journal* 37, No. 1: 13–26.

Mujumdar, P.P. and T.S.V. Ramesh. 1997. "Real-time Reservoir Operation for Irrigation." *Water Resources Research* 33, No. 5: 1157–1164.

Mujumdar, P.P. and S. Vedula. 2000. "Modeling for Optimal Reservoir Operation for Multicrop Irrigation." Technical Report of the Research Project, No. INCOH/3/97. Roorkee, India: Indian National Committee on Hydrology.

Panigrahi, D.P. and P.P. Mujumdar. 2000 "Reservoir Operation Modelling with Fuzzy Logic." *Water Resources Management* 14: 89–109.

Paudyal, G.N. and A.D. Das Gupta. 1990. "Irrigation Planning by Multilevel Optimization." *Journal of Irrigation and Drainage Engineering* 116, No. 2: 273–291.

Paul, S., S.N. Panda, and D. Nagesh Kumar. 2000. "Optimal Irrigation Allocation : A Multilevel Approach." *Journal of Irrigation and Drainage Engineering* 126, No. 3: 149–156.

Rao, N.H., P.B.S. Sarma, and S. Chander. 1990. "Optimal Multicrop Allocation of Seasonal and Intraseasonal Irrigation Water." *Water Resources Research* 26, No. 4: 551–559.

Rao, N.H., P.B.S. Sarma, and S. Chander. 1992. "Real-time Adaptive Irrigation Scheduling under a Limited Water Supply." *Agricultural Water Management* 20: 267–279.

Rao, S.S. 1993. "Optimization Using Fuzzy Set Theory." In *Structural Optimization : Status and Promise*. M.P. Kamat, ed. American Institute of Aeronautics and Astronautics: 637–661.

Ravikumar, V. and K. Venugopal. 1998. "Optimal Operation of South Indian Irrigation Systems." *Journal of Water Resources Planning and Management* 124, No. 5: 264–271.

Russell, S.O. and P.F. Campbell. 1996. "Reservoir Operating Rules With Fuzzy Programming", *Journal of Water Resources Planning and Management* Vol. 122, No. 3: 165–170.

Sahoo, G.B., R.L. Loof, C.L. Abernethy, and S. Kazama. 2001. "Reservoir Release Policy for Large Irrigation System." *Journal of Irrigation and Drainage Engineering* 127, No. 5: 302–310.

Sakawa, M. 1995. "Fuzzy Programming." In *Fuzzy Sets for Management Decisions*. K.Asai, ed. Bruke: IOS Press Inc.:57–78.

Sarma, P.B.S. 2002. "Water Resources and Their Management for Sustainable Agricultural Production in India." In *Research Perspectives in Hydraulics and Water Resources*. Rama Prasad and S. Vedula, eds. Singapore: World Scientific (in print).

Sasikumar, K. and P.P. Mujumdar. 1998 "Fuzzy Optimization Model for Water Quality Management of a River System." *Journal of Water Resources Planning and Management* 124, No. 2: 79–88.

Shreshta, B.P., L. Duckstein, and E.Z. Stakhiv. 1996. "Fuzzy Rule Based Modeling of Reservoir Operation." *Journal of Water Resources Planning and Management* 122, No. 4: 262–269.

Simonovic, S.P. 1991. "Knowledge-based Systems and Operational Hydrology." *Canadian Journal of Civil Engineering* 18: 1–11.

Simonovic, S.P. 1992. "Reservoir Systems Analysis : Closing Gap Between Theory and Practice." *Journal of Water Resources Planning and Management* 118, No. 3: 262–280.

Simonovic, S.P. 2000. "Tools for Water Management: One View of the Future." *Water International* 25, No.1: 76–88

Simonovic, S.P. 2002. "Non-structural Measures for Sustainable Management of Floods." *Water International* 27, No. 1.

Simonovic, S.P. and D.A. Savic. 1989. "Intelligent Decision Support and Reservoir Management and Operations." *Journal of Computing in Civil Engineering* 3, No. 4: 367–385.

Srinivasan, K. and M.C. Philipose. 1998. "Effect of Hedging on Over-Year Reservoir Performance." *Water Resources Management* 12: 95–120.

Srinivasan K., T.R. Neelakantan, S.P. Narayan, and C. Nagarajukumar. 1999. "Mixed-Integer Programming Model for Reservoir Performance Optimization." *Journal of Water Resources Planning and Management* 125, No. 5: 298–301.

Sunantara J.D. and J.A. Ramirez. 1997. "Optimal Stochastic Multicrop Seasonal and Intraseasonal Irrigation Control." *Journal of Water Resources Planning and Management* 123, No. 1: 39–48.

Teegavarapu, R.S.V. and S.P. Simonovic. 1999. "Modeling Uncertainty in Reservoir Loss Functions Using Fuzzy Sets." *Water Resources Research* 35, No. 9: 2815–2823.

Vedula, S. and P.P. Mujumdar. 1992. "Optimal Reservoir Operation for Irrigation of Multiple Crops." *Water Resources Research* 28 , No. 1: 1–9.

Vedula, S. and D. Nagesh Kumar. 1996. "An Integrated Model for Optimal Reservoir Operation for Irrigation of Multiple Crops." *Water Resources Research*. 32, No. 4: 1101–1108.

Wardlaw, R. and J. Barnes. 1999. "Optimal Allocation of Irrigation Water Supplies in Real Time." *Journal of Irrigation and Drainage Engineering*125, No. 6: 345–353.

Yin, Y.Y., G.H. Hang, and K.W. Hipel. 1999. "Fuzzy Relation Analysis for Multicriteria Water Resources Management." *Journal of Water Resources Planning and Management* 125, No. 1: 41–47.

Zimmermann, H.J. 1985. "Application of Fuzzy Set Theory to Mathematical Programming." *Information Sciences* 36: 29–58.