GREY FUZZY MULTIOBJECTIVE OPTIMIZATION MODEL FOR RIVER WATER QUALITY MANAGEMENT

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ABSTRACT

A previously developed Fuzzy Waste Load Allocation Model (FWLAM) for a river system is extended to address uncertainty involved in fixing the membership functions for the fuzzy goals of the Pollution Control Agency (PCA) and dischargers. Decision-maker gets a range of optimal solutions as a stable interval on which some post-optimality analyses can be performed to fix the fractional removal levels of the pollutants, considering necessity of the water quality management issues. The present model maximizes the width of interval valued fractional removal levels for increasing flexibility of the decision-maker and minimizes width of the interval valued performance measure level for reducing the system uncertainty. The improvement of usefulness of optimal solutions over the previously developed Grey Fuzzy Waste Load Allocation Model (GFWLAM) is shown through the application on a hypothetical river system.

INTRODUCTION

A Waste Load Allocation (WLA) model, in general, integrates a water quality simulation model with an optimization model to provide best compromise solutions acceptable to both PCA and dischargers. Sasikumar and Mujumdar [1] developed the FWLAM for water quality management of a river system. It is capable to incorporate the conflicting and imprecise goals of the PCA and dischargers in a fuzzy optimization framework. In the solution decision-maker gets a set of optimal fractional removals (X*) of the pollutants and the maximum value of goal fulfillment level ($\lambda^{\pm}$), which is a measure of degree of fulfillment of the system goals. A major limitation in FWLAM is that the lower and upper bounds of the membership functions (membership parameters) are assumed fixed. As the model results are likely to vary considerably with change in the membership functions, uncertainty in the bounds and shape of the membership functions should be addressed in fuzzy optimization models for water quality management. Karmakar and Mujumdar [2] developed the GFWLAM, which considers the uncertainty in the bounds and shape of the membership functions in fuzzy optimization models for water quality management. The model is aimed at relaxing the membership parameters by treating them as interval grey numbers [3-5], thus provides a range of "best compromise" solutions to impart more flexibility in water quality management decisions. In GFWLAM the order of consideration of the goals of PCA and the dischargers, along with the selection of bounds of decision variables create two different situations of model formulation, which are termed as Case 1 and 2. Each case is further subdivided into two submodels to obtain two extreme values of $\lambda^{\pm}$, which give the solutions for two extreme cases encompassing all intermediate possibilities. In the present work a Flexible Fuzzy Waste Load Allocation Model (FFWLAM) is developed to maximize the width of the interval valued fractional removal levels of pollutants [i.e., $(X^{+}-X^{-})$] using grey fuzzy multiobjective optimization technique, which does not lead to complicated intermediate submodels. This enhances the flexibility and applicability in decision-making, as the decision-maker gets a wider range of optimal fractional removals for post-optimality decision making than those of solutions obtained from GFWLAM. Similar to GFWLAM, the upper and lower bounds of goal fulfillment level (i.e., λ^{+} and λ^{-}) are maximized, but in FFWLAM the system uncertainty has also been reduced by minimizing width of the degree of goal fulfillment level [i.e., $(\lambda^{+}-\lambda^{-})$].

FLEXIBLE FUZZY WASTE LOAD ALLOCATION MODEL

The FWLAM developed earlier, forms the basis for the optimization model developed in the present work. The system consists of a set of dischargers, who are allowed to release pollutants into the river after removing some fraction of the pollutants. Acceptable water quality condition is ensured by checking the water quality in terms of water quality indicators (e.g., DO-deficit, hardness, nitrate-nitrogen concentration) at a finite number of locations, which are referred to as checkpoints. Goals of the PCA and dischargers are expressed as fuzzy goals. The fuzzy membership functions themselves being imprecisely stated. A terminology “imprecise membership function” is used to represent the membership functions with uncertain membership parameters, expressed as interval grey numbers. An interval grey number ($x^{\pm}$) is defined as an interval with known lower (x^{-}) and upper (x^{+}) bounds but unknown distribution information for x [5].

$$x^{\pm} = [x^{-}, x^{+}] = [t \in x \mid x^{-} \le t \le x^{+}] \tag{1}$$

$x^{\pm}$ becomes a “deterministic number” or “white number” when, $x^{\pm} = x^{-} = x^{+}$. The formulation of GFWLAM is represented as:

$$Max \;\; \lambda^{\pm} \tag{2}$$

$$Subject\ to \quad \mu^{\pm}_{E^{\pm}_{jl}}(c^{\pm}_{jl}) = [(c^{H\pm}_{jl} - c^{\pm}_{jl})/(c^{H\pm}_{jl} - c^{D\pm}_{jl})]^{\gamma_{jl}} \ge \lambda^{\pm} \qquad \forall\ j,l \tag{3}$$

$$\mu^{\pm}_{F^{\pm}_{jmn}}(x^{\pm}_{jmn}) = [(x^{M\pm}_{mn} - x^{\pm}_{jmn})/(x^{M\pm}_{mn} - x^{L\pm}_{mn})]^{\beta_{jmn}} \ge \lambda^{\pm} \qquad \forall\ j,m,n \tag{4}$$

$$c^{D\pm}_{jl} \le c^{\pm}_{jl} \le c^{H\pm}_{jl} \qquad \forall\ j,l \tag{5}$$

$$x^{L\pm}_{mn} \le x^{\pm}_{jmn} \le x^{M\pm}_{mn} \qquad \forall\ j,m,n \tag{6}$$

$$0 \le \lambda^{\pm} \le 1 \qquad (7)$$

The PCA sets the desirable concentration level ($c^{D\pm}_{jl}$) and maximum permissible concentration level ($c^{H\pm}_{jl}$) of the water quality indicator j (e.g., DO-deficit, hardness, nitrate-nitrogen concentration) at the water quality checkpoint l ($c^{D\pm}_{jl} \le c^{H\pm}_{jl}$). The grey fuzzy goal of the PCA ($E^{\pm}_{jl}$) is to make the concentration level ($c^{\pm}_{jl}$) of water quality indicator j at the checkpoint l as close as possible to the desirable level, $c^{D\pm}_{jl}$. For example, if DO-deficit is the water quality indicator, a non-increasing membership function suitably reflects goals of the PCA with respect to DO-deficit at a checkpoint. The uncertainty associated with membership parameters ($c^{D\pm}_{jl}$ and $c^{H\pm}_{jl}$) is addressed using interval grey numbers. Using non-increasing imprecise membership functions, the grey fuzzy goals of PCA are expressed as in constraint (3), which defines the minimum goal fulfillment level ($\lambda^{\pm}$). The exponent γ_{jl} is nonzero positive real number. Assignment of numerical value to this exponent is subject to the desired shape of the membership functions. A value of $\gamma_{jl} =$ 1 leads to linear imprecise membership function. The grey fuzzy goals of the dischargers ($F^{\pm}_{jmn}$) are similarly expressed as in constraint (4), where the aspiration level and maximum acceptable level of fractional removal of the pollutant n at discharger m are represented as $x^{L\pm}_{mn}$ and $x^{M\pm}_{mn}$, respectively ($x^{L\pm}_{mn} \le x^{M\pm}_{mn}$). Similar to the exponent γ_{jl} in constraint (3), β_{jmn} is nonzero positive real number. The goal of the dischargers is to make the fractional removal level ($x^{\pm}_{jmn}$) as close as possible to $x^{L\pm}_{mn}$, to minimize the waste treatment cost for pollutant n. The conflict between the imprecise goals is modeled using fuzzy decision [6]. But introduction of interval grey numbers as the membership parameters, the fuzzy decision leads to a imprecise fuzzy decision [2]. The crisp constraints (5) and (6) are based on the water quality requirements set by the PCA, and acceptable fractional removal levels by the dischargers, respectively. Constraint (7) represents the bounds on the parameter $\lambda^{\pm}$.

The grey fuzzy optimization model given in (2) to (7) forms the basis of FFWLAM. The fuzzy inequality constraints (3) and (4) addressing the goals of the PCA and dischargers are the order relations (e.g., the relations "greater than or equal to" or “less than or equal to”) containing interval grey numbers in both the sides. Determination of meaningful ranking between two partially or fully overlapping intervals in the order relations is a potential research area [3, 7]. Recently, Sengupta et al. [7] proposed a satisfactory deterministic equivalent form of inequality constraints containing interval grey numbers by using the acceptability index (***A***). A premise $a^{\pm}$ (<) $b^{\pm}$ is formed to imply that $a^{\pm}$ is inferior to $b^{\pm}$. Here, the term “inferior to” (“superior to”) is analogous to “less than” (“greater than”). Let I be the set of all closed intervals on the real line ***R***. The acceptability index (***A***) is defined as ***A***: $I \times I \rightarrow [0, \propto)$ such that,

$$\boldsymbol{A}\,[a^{\pm}\ (<)\ b^{\pm}] = [m(b^{\pm}) - m(a^{\pm})]/[w(b^{\pm}) + w(a^{\pm})] \qquad (8)$$

where $[w(b^{\pm}) + w(a^{\pm})] \neq 0$; $w(a^{\pm}) = ½\,(a^{+} - a^{-})$; $m(a^{\pm}) = ½\,(a^{-} + a^{+})$. Notations are similarly defined for the interval grey number $b^{\pm}$. The grade of acceptability of $a^{\pm}$ (<) $b^{\pm}$ may be classified and interpreted further on the basis of comparative position of mean and half-width of interval $b^{\pm}$ with respect to those of interval $a^{\pm}$. Let us consider an interval inequality relation $a^{\pm}x \geq b^{\pm}$, where x is a deterministic variable. A satisfactory deterministic equivalent form of interval inequality relation $a^{\pm}x \geq b^{\pm}$, is proposed as:

$$a^{\pm}x \geq b^{\pm} \Rightarrow \{ a^{-}x \geq b^{-} \text{ and } \boldsymbol{A}\, [a^{\pm}x\,(<)\, b^{\pm}] \leq \alpha \in [0, 1]\} \tag{9}$$

where α is interpreted as an optimistic threshold assumed and fixed by the decision-maker. At $\alpha = 0$, the solutions at no optimism level are obtained.

Model Formulation

The deterministic equivalent of the grey fuzzy optimization model given in (2) to (7) is formulated using the expression given in Eq. (9), is as follows:

$$Max\ \lambda^{+} \tag{10}$$

$$Max\ \lambda^{-} \tag{11}$$

$$Min\ [(\lambda^{+} - \lambda^{-})/(\lambda^{+} + \lambda^{-})] \tag{12}$$

$$subject\ to\ \mu^{-}_{\mathrm{E}^{\pm}_{\mathrm{jl}}}(c^{\pm}_{jl}) = [(c^{H-}_{jl} - c^{+}_{jl})/(c^{H+}_{jl} - c^{D-}_{jl})] \geq \lambda^{-} \qquad \forall\ j, l \tag{13}$$

$$\mu^{-}_{\mathrm{F}^{\pm}_{\mathrm{jmn}}}(x^{\pm}_{jmn}) = [(x^{M-}_{mn} - x^{+}_{jmn})/(x^{M+}_{mn} - x^{L-}_{mn})] \geq \lambda^{-} \qquad \forall\ j, m, n \tag{14}$$

$$\boldsymbol{A}\, [(c^{H\pm}_{jl} - c^{\pm}_{jl})/(c^{H\pm}_{jl} - c^{D\pm}_{jl})\,(<)\, \lambda^{\pm}] \leq \alpha_1 \in [0, 1] \tag{15}$$

$$\boldsymbol{A}\, [(\mathrm{x}^{\mathrm{M}\pm}_{\mathrm{jmn}} - \mathrm{x}^{\pm}_{\mathrm{jmn}})/(\mathrm{x}^{\mathrm{M}\pm}_{\mathrm{jmn}} - \mathrm{x}^{\mathrm{L}\pm}_{\mathrm{jmn}})\,(<)\, \lambda^{\pm}] \leq \alpha_2 \in [0, 1] \qquad \forall\ \mathrm{j, m, n} \tag{16}$$

$$c^{D-}_{jl} \leq c^{+}_{jl} \leq c^{H+}_{jl}; \quad c^{D-}_{jl} \leq c^{-}_{jl} \leq c^{H+}_{jl} \qquad \forall\ j, l \tag{17}$$

$$x^{L-}_{mn} \leq x^{+}_{jmn} \leq x^{M+}_{mn}; \quad x^{L-}_{mn} \leq x^{-}_{jmn} \leq x^{M+}_{mn} \qquad \forall\ j, m, n \tag{18}$$

$$c^{-}_{jl} \leq c^{+}_{jl} \qquad \forall\ j, l \tag{19}$$

$$x^{-}_{jmn} \leq x^{+}_{jmn} \qquad \forall\ j, m, n \tag{20}$$

$$0 \leq \lambda^{+} \leq 1; \quad 0 \leq \lambda^{-} \leq 1 \tag{21}$$

$$\lambda^{-} \leq \lambda^{+} \tag{22}$$

The constraints (13) – (16) are expressed as deterministic equivalent of the constraints (3) - (4) using the acceptability index. The goals of the PCA and dischargers are represented by linear imprecise membership functions (i.e., α_{jl}, β_{jmn} = 1), as the Eq. (9) with acceptability index for ranking the interval grey numbers in the inequality constraints is applicable only for linear programming problems [7]. The objective (12) minimizes the system uncertainty by minimizing width of the goal fulfillment level $[(\lambda^{+}-\lambda^{-})]$[5]. In FFWLAM, the three objectives are optimized individually in three separate sub-problems along with the contraints (13) – (22) to obtain the maximum and minimum possible values of λ^{+}, λ^{-} and $[(\lambda^{+}-\lambda^{-})/(\lambda^{+}+\lambda^{-})]$ (i. e., ideal points and worst possible values of fuzzy goal programming [8]), respectively. As discussed earlier, another objective of the river water quality management is to permit more flexibility (i.e., more width of the interval) in the optimal fractional removal level ($x^{\pm *}_{jmn}$). Thus maximization of the grey degree of $x^{\pm}_{jmn}$ is considered as another objective along with objectives (10) to (12). The maximum and minimum values of grey degree of $x^{\pm}_{jmn}$ are determined from the three sub-problems. All the objectives are quantified by using appropriate membership functions according to fuzzy goal programming technique [8]. The fuzzy decision concept with minimum operator is applied to aggregate the membership functions of the objectives (10) – (12) along with other objectives for minimizing Gd ($x^{\pm}_{jmn}$) for the dischargers. In the next section the FFWLAM for water quality management is applied to a hypothetical river system shown in Fig. 1.

MODEL APPLICATION

The water quality indicator of interest is the DO-deficit at 18 number of checkpoints in the river system due to the point sources of BOD. The saturation DO concentration is taken as 10mg/L for all the reaches. A deterministic value of river flow as 7 Mcum/day is considered. The details of the effluent flow and imprecise membership functions are given in Table 1 and 2, respectively.

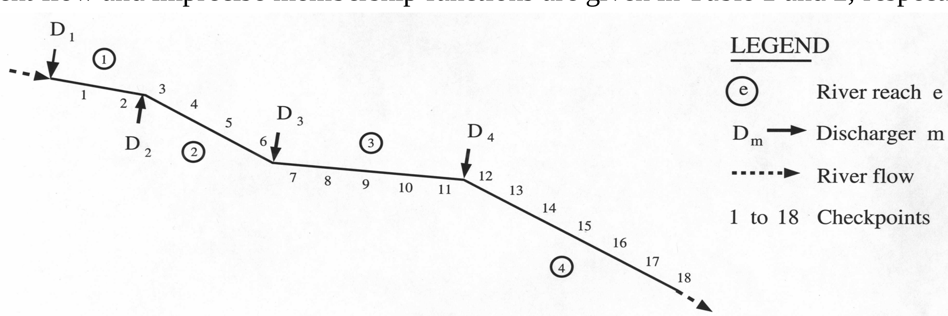

Fig. 1 A Hypothetical River System

In constraints (15) and (16), α_1 and α_2 are optimistic thresholds assumed and fixed equal to zero. The mean values of both the intervals become equal to the mean of $\lambda^{\pm}$, thus a conservative optimal solution or solution at no optimism level is obtained. The decision-maker may select the values of α_1 and α_2 equal to zero when the water quality management issues in the river system are too critical and important. In model (10)-(22); the concentration level $c^{\pm}_{jl}$ may be mathematically expressed as a function of $x^{\pm}_{jmn}$, using the transfer function. The transfer function can be evaluated using appropriate mathematical models that determine spatial distribution of the water quality indicator due to pollutant discharge into the river system from point sources [1]. For most water quality indicators, a high level of fractional removal of pollutants (e.g., BOD loading, toxic pollutant concentration, etc.) results in a low level of water quality indicator (e.g., DO-deficit, nitrate-nitrogen concentration, etc.). The lower bound of water quality indicator (c^-_{jl}) is therefore expressed in terms of upper bound of fractional removal level ($x^{\pm}_{jmn}$) and similarly, c^+_{jl} is expressed in terms of x^-_{jmn}, which results $x^{\pm}_{jmn}$ as the only decision variables in the optimization model (10) – (22).

Table 1. Effluent Flow Data [9]

Dis-Charger	Effluent Flow Rate ($10^4 m^3$/day)	BOD Concentration (mg/L)	DO Concentration (mg/L)
1	2.134	1250	1.230
2	6.321	1415	2.400
3	7.554	1040	1.700
4	5.180	935	2.160

Table 2. Details of Imprecise Membership Functions

River Reach	Check-points	c^D_l (mg/L)		c^H_l (mg/L)		x^L_m		x^M_m	
		–	+	–	+	–	+	–	+
1	1-2	(0.00)		(3.00)		(0.30)		(0.85)	
		0.00	0.00	2.70	3.20	0.25	0.35	0.80	0.90
2	3-6	(0.10)		(3.00)		(0.30)		(0.85)	
		0.00	0.10	2.70	3.20	0.25	0.35	0.80	0.90
3	7 –11	(0.20)		(3.50)		(0.35)		(0.85)	
		0.17	0.22	3.30	3.70	0.30	0.40	0.80	0.90
4	12 –18	(0.20)		(3.50)		(0.35)		(0.85)	
		0.17	0.22	3.30	3.70	0.30	0.40	0.80	0.90

( ) : Deterministic case, '–' : Lower bound, '+' : Upper bound

RESULTS AND DISCUSSION

In Table 3, columns 1–3 show the results obtained from Sub-problems 1–3, i.e., maximization of λ^+, maximization of λ^- and minimization of the width of λ^+, λ^- and (λ^+-λ^-) (equivalent to minimizing [(λ^+-λ^-)/(λ^++λ^-)]), respectively. The minimum and maximum values of λ^+, λ^-, [(λ^+-λ^-)/(λ^++λ^-)] and Gd($x^{\pm}_1$), … , Gd($x^{\pm}_4$) are taken from the columns 1–3; rows 6, 5, 8, and 9 – 12, respectively. For example, columns 1–3, row 5 show the values of λ^- obtained from Sub-problem 1–3, respectively. The maximum value of λ^- (i.e., 0.3121) is obtained from Sub-problem 2 and the minimum value (i.e., 0.0006) is obtained from Sub-problem 1. The fuzzy requirements of all the objectives are quantified by eliciting linear membership functions considering the minimum and maximum values of the objectives as membership parameters, according to the fuzzy goal programming technique [8]. Column 4, rows 1-6 show the optimal fractional – removal levels of the pollutants by different dischargers ($X^{\pm*}$) and corresponding $\lambda^{\pm*}$-values. To evaluate the quality of input or output uncertain information, a measure of "Grey degree" is used [5]. The expression

Table 3. Details of Fractional Removal Levels using Fuzzy Multiobjective Optimization Technique

	Sub-problem-1 (Max. λ^+) (1)	**Sub-problem -2 (Max. λ^-) (2)**	**Sub-problem -3 Min. $(\lambda^+-\lambda^-)/(\lambda^++\lambda^-)$ (3)**	**FFWLAM (4)**	**FWLAM (5)**	**GFWLAM (6)**
$x_1^{\pm}$	[0.4845, 0.7751]	[0.5955, 0.5964]	[0.6060, 0.6531]	[0.5268, 0.6652]	[0.6150, 0.6150]	[0.5970, 0.6410]
$x_2^{\pm}$	[0.4682, 0.7987]	[0.5967, 0.5970]	[0.4301, 0.6578]	[0.5302, 0.6656]	[0.6150, 0.6150]	[0.5970, 0.6410]
$x_3^{\pm}$	[0.5190, 0.7951]	[0.6124, 0.6126]	[0.5517, 0.5679]	[0.5367, 0.6757]	[0.6360, 0.6360]	[0.6120, 0.6700]
$x_4^{\pm}$	[0.5172, 0.7970]	[0.6121, 0.6127]	[0.6324, 0.6517]	[0.5357, 0.6747]	[0.6360, 0.6360]	[0.6120, 0.6700]
λ^-	0.0006	0.3121	0.1052	0.2066	0.4277	0.3126
λ^+	0.9592	0.3618	0.1052	0.5064	0.4277	0.5745
Gd ($\lambda^{\pm}$)	- -	- -	- -	0.8411	0.0000	59.0330
$(\lambda^+-\lambda^-)/(\lambda^++\lambda^-)$	0.9987	0.0737	0.0000	- -	- -	- -
Gd($x^{\pm}_1$)	0.4615	0.0015	0.0748	0.2322	0.0000	0.0711
Gd($x^{\pm}_2$)	0.5217	0.0004	0.4186	0.2265	0.0000	0.0711
Gd($x^{\pm}_3$)	0.4202	0.0003	0.0288	0.2293	0.0000	0.0905
Gd($x^{\pm}_4$)	0.4259	0.0010	0.0300	0.2297	0.0000	0.0905
Avg. Gd($X^{\pm}$)	- -	- -	- -	0.2294	0.0000	0.0812

for grey degree of interval grey number ($x^{\pm}$) is $Gd(x^{\pm}) = (x_{\omega} / x_m) \times 100\%$, where $x_m = ½ [x^- + x^+]$, and $x_{\omega} = [x^+ - x^-]$. As the grey degree of optimal value of objective function decreases, the effectiveness of the grey model increases with decreasing system uncertainties. In column 5 the deterministic case is presented, for which average value of grey degree of input parameters is zero. In column 6, the grey uncertain case is presented as obtained from GFWLAM, for which the average value of grey degree of input parameters (i.e., 31.304%) is the same as in the FFWLAM given in Table 2. Comparing the results shown in column 4 and column 6, rows 1-4, it can be concluded that the widths of optimal fractional removal levels of BOD are more than those of GFWLAM, which allows a wider choice to the decision-maker for post-optimality decision making.

The results for α_1=0, α_2=0 (conservative solution); α_1=1, α_2=1 (optimistic solution) and for some other intermediate values of α_1 and α_2 are also determined to analyze the sensitivity of the optimistic thresholds, but are not presented here.

CONCLUSIONS

The model demonstrates the modeling aspect of uncertain membership functions and shows the usefulness of solutions with a simplified hypothetical river system. The membership functions themselves are subjective statements of the perceptions of the decision-makers. More over, in practical situations for a water quality indicator, different water quality standards on surface water intakes are used for different uses, which results in an uncertainty in the membership parameters for goals of PCA. Consideration of imprecise membership parameters in the fuzzy optimization model imparts flexibility in the solutions as the fractional removal levels are obtained as interval grey numbers. The decision-maker can perform some post-optimality analysis over the range of optimal fractional levels for fixing the final decision, considering the technical and economic feasibility of a particular level of fractional removal for a discharger.

NOMENCLATURE

A	=	Acceptability index;
$c^{\pm}_{jl}$	=	Conc. level of water quality indicator j at the checkpoint l;
$c^{D\pm}_{jl}$, $c^{H\pm}_{jl}$	=	Desirable and max. permissible conc. levels of quality indicator;
$E^{\pm}_{jl}$	=	Grey fuzzy goal of the PCA;
$F^{\pm}_{jmn}$	=	Grey fuzzy goal of the discharger;
$Gd(x^{\pm})$	=	Grey degree of the interval grey number $x^{\pm}$;
$x^{\pm}_{jmn}$	=	Fractional removal level of the pollutant n influencing water quality indicator j by the discharger m;
$X^{\pm}$	=	Vector of fractional removal levels;
$x^{L\pm}_{mn}$, $x^{M\pm}_{mn}$	=	Aspiration and max. acceptable fractional removal levels;
$\lambda^{\pm}$	=	Optimal degree of goal fulfillment level (dimension less) ;

γ_{jl}, β_{jmn} = Nonzero positive real numbers (dimension less) ; and

α = Optimistic threshold (dimension less) ;

Subscripts / Superscripts

J = Water quality indicator;

L = Checkpoint;

M = Discharger;

N = Pollutant;

$\pm$ = Interval valued quantity;

+, – = Upper and lower bound of any interval grey number; and

* = Optimal value;

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