Transverse spin in QCD: Radiative corrections

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In this paper we address various issues connected with transverse spin in light front QCD. The transverse spin operators, in the $A^+=0$ gauge, expressed in terms of the dynamical variables are explicitly interaction dependent unlike the helicity operator which is interaction independent in the topologically trivial sector of light-front QCD. Although it cannot be separated into an orbital and a spin part, we have shown that there exists an interesting decomposition of the transverse spin operator. We discuss the physical relevance of such a decomposition. We perform a one loop renormalization of the full transverse spin operator in light-front Hamiltonian perturbation theory for a dressed quark state. We explicitly show that all the terms dependent on the center of mass momenta get canceled in the matrix element. The entire nonvanishing contribution comes from the fermion intrinsiclike part of the transverse spin operator as a result of cancellation between the gluonic intrinsiclike and the orbital-like part of the transverse spin operator. We compare and contrast the calculations of transverse spin and helicity of a dressed quark in perturbation theory.

I. INTRODUCTION

From the early days of quantum field theory, it has been recognized that the issues associated with the spin of a composite system in an arbitrary frame are highly complex and non-trivial [1]. In equal-time quantization, the problems arise because of the fact that the Pauli-Lubanski operators, starting from which one can construct the spin operators in a moving frame, are interaction dependent for a composite object. Further, it is quite difficult to separate the center of mass and internal variables which is mandatory in the calculation of spin. Because of these difficulties, there has been rarely any attempt to study the spin of a moving composite system in the conventional equal time formulation of even simple field theoretic models, let alone quantum chromodynamics (QCD).

It is well known that in light-front field theory, in addition to the Hamiltonian, two other operators that belong to the Poincaré group, namely, $F^i(i=1,2)$, are interaction dependent. This implies interaction dependent spin operators and this complication is generally thought to be a penalty one has to pay for working with light-front dynamics. In contrast, the angular momentum operators in the familiar instant form of field theory are interaction independent. It is interesting to investigate whether one can understand better the physical origin of the interaction dependence in the light-front case.

A second problem is that, together with the light-front helicity $\mathcal{J}^3$, F^i do not obey $SU(2)$ algebra, the commutation relations obeyed by the spin operators of a massive particle. They obey $E(2)$ algebra, appropriate for a massless particle. This implies that even though F^i performs ''rotations'' about the transverse axes, they have continuous spectrum. It is, however, known how to solve this problem. In terms of the rest of the Poincaré generators, one knows [2] how to construct spin operators $\mathcal{J}^i$ that together with the helicity $\mathcal{J}^3$ obey the $SU(2)$ algebra. One observes that $\mathcal{J}^i$ is interaction dependent and has a highly nontrivial operator structure in contrast with $\mathcal{J}^3$. Further, unlike $\mathcal{J}^3$, $\mathcal{J}^i$ cannot be separated into orbital and spin parts. So far, most of the studies of the transverse spin operators in light-front field theory are restricted to free field theory [3]. Even in this case, the operators have a highly complicated structure. However, one can write these operators as a sum of orbital and spin parts, which can be achieved via a unitary transformation, called Melosh transformation [4]. In interacting theory, presumably this can be achieved order by order in a suitable expansion parameter [5] which is justifiable only in a weakly coupled theory.

Knowledge about transverse rotation operators and transverse spin operators is mandatory for addressing issues concerning Lorentz invariance in light-front theory. Unfortunately, very little is known [6] regarding the field theoretic aspects of the interaction dependent spin operators. *We emphasize that in a moving frame, the spin operators are interaction dependent irrespective of whether one considers equal-time field theory or light-front field theory*. To the best of our knowledge, in gauge field theory, the canonical structure of spin operators of a composite system in an *arbitrary* frame has never been studied.

Recently it was shown that [7], starting from the manifestly gauge invariant symmetric energy momentum tensor, in light-front QCD (the gauge $A^+=0$ and light-front variables), after the elimination of constrained variables, $\mathcal{J}^3$ becomes explicitly interaction independent and can be separated into quark and gluon orbital and spin operators. Thus

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one can write down a helicity sum rule which has a clear physical meaning. The orbital and intrinsic parts of the light-front helicity operator have also been analyzed recently in [8]. Even though $\mathcal{J}^i$ cannot be separated into orbital and spin parts and they are interaction dependent, one can still ask whether one can identify distinct operator structures in $\mathcal{J}^i$ and whether one can propose a physically interesting decomposition. Is this decomposition protected by radiative corrections? If distinct operators indeed emerge, do they have any phenomenological consequences especially in deep inelastic scattering which is a light cone dominated process?

Another important issue concerns renormalization. In light-front QCD Hamiltonian, quark mass appears as m^2 and m terms, m^2 in the free helicity non-flip part of the Hamiltonian and m in the interaction dependent helicity flip part of the Hamiltonian. It is known that m^2 and m renormalize differently. m^2 and m also appear in $\mathcal{J}^i$. Do they undergo renormalization? Since $\mathcal{J}^i$ are interaction dependent, do they require new counterterms in addition to those necessary to renormalize the Hamiltonian?

In order to resolve the above mentioned problems and puzzles, we have undertaken an investigation of the spin of a composite system in an arbitrary reference frame in QCD. We have compared and contrasted both the instant form and front form formulations. In instant form, even though the angular momentum operators are interaction independent, they qualify as spin operators only in the rest frame of the system. In an arbitrary reference frame, the appropriate spin operators involve, in addition to angular momentum operators, also interaction dependent boost operators. Thus one puzzle is resolved, namely, the interaction dependence of the spin of a composite system in an arbitrary reference frame is not a peculiarity of light-front dynamics, it is a general feature in any formulation of quantum field theory. What is peculiar to light-front dynamics is that one can at most go only to the transverse rest frame of the particle. No frame exists in which $P^+=0$ and one is so to speak ''always in a moving frame.'' As a consequence, spin measured in any direction other than that of P^+ cannot be separated into orbital and intrinsic parts. This is to be contrasted with the light-front helicity $\mathcal{J}^3$ which is independent of interactions and further can be separated in to orbital and intrinsic parts. The situation is quite analogous to that of a light-like particle. In this case it is well known that since there is no rest frame, one can uniquely identify the spin of the particle only along the direction of motion since only along this direction one can disentangle rotation from translation for a massless particle. Also, in any direction other than the direction of motion, one cannot separate the angular momentum into orbital and intrinsic parts.

In our earlier paper [9], we have shown that even though $\mathcal{J}^i$ cannot be separated into orbital and intrinsic parts, one can still achieve a separation into three distinct operator structures. Specifically, starting from the manifestly gauge invariant symmetric energy momentum tensor in QCD, we have derived expressions for the interaction dependent transverse spin operators $\mathcal{J}^i (i=1,2)$ which are responsible for the helicity flip of the nucleon in light-front quantization. In order to construct $\mathcal{J}^i$, first we have derived expressions for the transverse rotation operators F^i. In the gauge $A^+=0$, we eliminated the constrained variables. In the completely gauge fixed sector, in terms of the dynamical variables, we have shown that one can decompose $\mathcal{J}^i=\mathcal{J}^i_I+\mathcal{J}^i_{II}+\mathcal{J}^i_{III}$ where only $\mathcal{J}^i_I$ has explicit coordinate (x^-,x^i) dependence in its integrand. The operators $\mathcal{J}^i_{II}$ and $\mathcal{J}^i_{III}$ arise from the fermionic and bosonic parts respectively of the gauge invariant energy momentum tensor. $\mathcal{J}^i_I$ is orbital-like and $\mathcal{J}^i_{II}$ and $\mathcal{J}^i_{III}$ are fermion intrinsic-like and gluon intrinsic-like spin operators respectively.

In this work, we explore the theoretical consequences of the decomposition of $\mathcal{J}^i$. We compare and contrast the consequences of this decomposition and the corresponding decomposition of the helicity operator into orbital and spin parts. Next we address the issue of radiative corrections by carrying out the calculation of the transverse spin of a dressed quark in perturbative QCD (PQCD) in the old-fashioned Hamiltonian formalism. To the best of our knowledge, this is for the first time that such a calculation has been performed in quantum field theory. This calculation is facilitated by the fact that boost is kinematical in the light-front formalism. Thus we are able to isolate the internal motion which is only physically relevant from the spurious center of mass motion. We carry out the calculations in a reference frame with arbitrary transverse momentum $P^\perp$ and explicitly verify the frame independence of our results. We find that because of cancellation between various interaction independent and dependent operator matrix elements, only one counterterm is needed. We establish the fact the mass counterterm for the renormalization of $\mathcal{J}^i$ is the same mass counterterm required for the linear mass term appearing in the interaction dependent helicity flip vertex in QCD. It is important to mention that the divergence structure and renormalization in light-front theory is entirely different from the usual equal-time theory. If one uses constituent momentum cutoff, one violates boost invariance and also encounters non-analytic behavior in the structure of counterterms [10]. In this paper, we have done one loop renormalization of the transverse spin operators by imposing cutoff on the relative transverse momenta and on the longitudinal momentum fraction. Up to one loop, we find that all infrared divergences (in the longitudinal momentum fraction) get canceled in the result. The renormalization of these operators using similarity renormalization technique [10] is to be done in future.

The plan of the paper is as follows. In Sec. II, first, we briefly review the complexities associated with the description of the spin of a composite system in a moving frame in the conventional equal time quantization. Then we give the explicit form of transverse rotation operators in light-front QCD. In Sec. III, we discuss the physical relevance of the decomposition of the transverse spin operator and also compare and contrast it with the helicity operator. In Sec. IV, we present the calculation of the transverse spin for a dressed quark state up to $O(\alpha_s)$ in perturbation theory. Discussion and conclusions are given in Sec. V. The explicit forms of the kinematical operators and the Hamiltonian in light-front QCD starting from the gauge invariant symmetric interaction dependent energy momentum tensor are derived in Appendix

A. The evaluation of the transverse spin of a system of two free fermions is given in Appendix B. The detailed derivation of the transverse rotation operators in QCD, which are needed for the construction of the transverse spin operators, is given in Appendix C. The full evaluation of the transverse spin operator for a dressed quark in an arbitrary reference frame is given in Appendix D. There we also show the manifest cancellation of all the center of mass momentum dependent terms. Some details of the calculation are provided in Appendix E.

II. THE TRANSVERSE SPIN OPERATORS IN QCD

In this section we first discuss the complexities associated with the spin operators for a composite system in equal-time formulation and also compare with the light-front case. Then we give the expressions for interaction dependent transverse rotation operators in light-front QCD starting from the manifestly gauge invariant energy momentum tensor.

The angular momentum density

$$\mathcal{M}^{\alpha\mu\nu}=x^{\mu}\Theta^{\alpha\nu}-x^{\nu}\Theta^{\alpha\mu}. \tag{2.1}$$

In equal time theory, generalized angular momentum

$$M^{\mu\nu}=\int d^3x\mathcal{M}^{0\mu\nu}. \tag{2.2}$$

The rotation operators are $J^i=\epsilon^{ijk}M^{jk}$. Thus in a non-gauge theory, all the three components of the rotation operators are manifestly interaction independent. However, the spin operators S^i for a composite system in a moving frame involves, in addition to J^i, the boost operators $K^i=M^{0i}$ which are interaction dependent. Intrinsic spin operators in an arbitrary reference frame in equal-time quantization are given [11] in terms of the Poincaré generators by,

$$\begin{aligned}\mathbf{S}&=\frac{1}{M}\left[\mathbf{W}-\frac{\mathbf{P}W^0}{M+H}\right]\\&=\mathbf{J}\ \frac{P^0}{M}-\mathbf{K}\times\frac{\mathbf{P}}{M}-\frac{(\mathbf{J}\cdot\mathbf{P})}{M+P^0}\frac{\mathbf{P}}{M}\end{aligned} \tag{2.3}$$

where $\mathbf{W}$ are the space components of the Pauli-Lubanski operator, $W^{\mu}=-\frac{1}{2}\epsilon^{\mu\nu\rho\lambda}M_{\nu\rho}P_{\lambda}$. H, $\vec{P}$ are equal time Hamiltonian and momentum operators respectively obtained by integrating the energy momentum tensor over a spacelike surface and $\vec{J}$ and $\vec{K}$ are the equal time rotation and boost generators respectively, which are obtained by integrating the angular momentum density over a spacelike surface. Since boost $\mathbf{K}$ is dynamical, *all the three components of* $\mathbf{S}$ *are interaction dependent* in the equal time quantization. Nevertheless, the component of $\mathbf{S}$ along $\mathbf{P}$ remains kinematical. This is to be compared with light-front quantization where *the third component of the light-front spin operator* $\mathcal{J}^3$ *is kinematical*. This arises from the facts that boost operators are kinematical on the light front, the interaction dependence of light-front spin operators $\mathcal{J}^i$ arises solely from the rotation operators, and the third component of the rotation operator J^3 is kinematical on the light front.

A further complication arises in equal time quantization. In order to describe the intrinsic spin of a composite system, one should be able to separate the center of mass motion from the internal motion. Even in free field theory, this turns out to be quite involved (see Ref. [12] and references therein). On the other hand, in light-front theory, since transverse boosts are simply Galilean boosts, separation of center of mass motion and internal motion is as simple as in nonrelativistic theory (see Appendix D).

A gauge invariant separation of the nucleon angular momentum is performed in Ref. [13]. However, as far the spin operator in an arbitrary reference frame is concerned, the analysis of this reference is valid only in the rest frame where spin coincides with total angular momentum operator. Further, there is no mention of the complications in the equal time theory, which arise from the need to project out the center of mass motion in an arbitrary reference frame. Moreover, the distinction between the longitudinal and transverse components of the spin is not made. It is crucial to make this distinction since physically the longitudinal and transverse components of the spin carry quite distinct information (as is clear, for example, from the spin of a massless particle). Moreover, even for the third component of the spin of a composite system in a moving frame, there is crucial difference between equal time and light front cases. $\mathcal{J}^3$ (helicity) is interaction independent whereas S^3 is interaction dependent in general except when measured along the direction of $\mathbf{P}$.

In light-front theory, generalized angular momentum

$$M^{\mu\nu}=\frac{1}{2}\int dx^-d^2x^{\perp}\mathcal{M}^{+\mu\nu}. \tag{2.4}$$

J^3 which is related to the helicity is given by

$$J^3=M^{12}=\frac{1}{2}\int dx^1d^2x^{\perp}[x^1\Theta^{+2}-x^2\Theta^{+1}] \tag{2.5}$$

and is interaction independent. On the other hand, the transverse rotation operators which are related to the transverse spin are given by

$$F^i=M^{-i}=\frac{1}{2}\int dx^-d^2x^{\perp}[x^-\Theta^{+i}-x^i\Theta^{+-}].$$

They are interaction dependent even in a non-gauge theory since Θ^{+-} is the Hamiltonian density.

For a massive particle, the transverse spin operators [2] $\mathcal{J}^i$ in light-front theory are given in terms of Poincaré generators by

$$M\mathcal{J}^1=W^1-P^1\mathcal{J}^3=\frac{1}{2}F^2P^++K^3P^2-\frac{1}{2}E^2P^--P^1\mathcal{J}^3, \tag{2.6}$$

$$M\mathcal{J}^2=W^2-P^2\mathcal{J}^3=-\frac{1}{2}F^1P^+-K^3P^1+\frac{1}{2}E^1P^--P^2\mathcal{J}^3. \tag{2.7}$$

The first term in Eqs. (2.6) and (2.7) contains both center of mass motion and internal motion and the next three terms in these equations serve to remove the center of mass motion.

The helicity operator is given by

$$\mathcal{J}^3 = \frac{W^+}{P^+} = J^3 + \frac{1}{P^+}(E^1P^2 - E^2P^1). \qquad (2.8)$$

Here, J^3 contain both center of mass motion and internal motion and the other two terms serve to remove the center of mass motion. The operators $\mathcal{J}^i$ obey the angular momentum commutation relations

$$[\mathcal{J}^i, \mathcal{J}^j] = i\epsilon^{ijk}\mathcal{J}^k. \qquad (2.9)$$

In order to calculate the transverse spin operators, first we need to construct the Poincaré generators P^+, P^i, P^-, K^3, E^i, J^3 and F^i in light-front QCD. The explicit form of the operator J^3 is given Ref. [7]. The construction of F^i which is algebraically quite involved is carried out in Appendix C. The final form of F^i is also given in Ref. [9]. The construction of the rest of the kinematical operators is given in Appendix A. In this Appendix we have presented all the operators in the manifestly Hermitian form, which is necessary, as we shall see later.

In order to have a physical picture of the complicated situation at hand, it is instructive to calculate the spin operator in free field theory. The case of two free massive fermions is carried out in Appendix B.

In light-front theory we set the gauge $A^+ = 0$ and eliminate the dependent variables ψ^- and A^- using the equations of constraint. We have shown that [9] (for details of the derivation see Appendix C), in the topologically trivial sector of the theory one can write the transverse rotation operator as

$$F^2 = F_I^2 + F_{II}^2 + F_{III}^2, \qquad (2.10)$$

where

$$F_I^2 = \frac{1}{2}\int dx^- d^2x^\perp [x^- \mathcal{P}_0^2 - x^2(\mathcal{H}_0 + \mathcal{V})], \qquad (2.11)$$

$$F_{II}^2 = \frac{1}{2}\int dx^- d^2x^\perp \left[\xi^\dagger[\sigma^3\partial^1 + i\partial^2]\frac{1}{\partial^+}\xi + \left[\frac{1}{\partial^+}(\partial^1\xi^\dagger\sigma^3 - i\partial^2\xi^\dagger)\right]\xi\right] + \frac{1}{2}\int dx^- d^2x^\perp m\left[\xi^\dagger\left[\frac{\sigma^1}{i\partial^+}\xi\right] - \left[\frac{1}{i\partial^+}\xi^\dagger\sigma^1\right]\xi\right]$$
$$+ \frac{1}{2}\int dx^- d^2x^\perp g\left[\xi^\dagger\frac{1}{\partial^+}[(-i\sigma^3A^1 + A^2)\xi] + \frac{1}{\partial^+}[\xi^\dagger(i\sigma^3A^1 + A^2)]\xi\right], \qquad (2.12)$$

$$F_{III}^2 = -\int dx^- d^2x^\perp 2(\partial^1A^1)A^2 - \frac{1}{2}\int dx^- d^2x^\perp g\frac{4}{\partial^+}(\xi^\dagger T^a\xi)A^{2a} - \frac{1}{2}\int dx^- d^2x^\perp g f^{abc}\frac{2}{\partial^+}(A^{ib}\partial^+A^{ic})A^{2a}. \qquad (2.13)$$

Here $\mathcal{P}_0^i$ is the free momentum density, $\mathcal{H}_o$ is the free Hamiltonian density and $\mathcal{V}$ are the interaction terms in the Hamiltonian in manifestly Hermitian form (see Appendix A). The operators F_{II}^2 and F_{III}^2 whose integrands do not explicitly depend upon coordinates arise from the fermionic and bosonic parts respectively of the gauge invariant symmetric energy momentum tensor in QCD. The above separation is slightly different from that in [9]. From Eq. (2.6) in Sec. II it follows that the transverse spin operators $\mathcal{J}^i$, ($i = 1,2$) can also be written as the sum of three parts, $\mathcal{J}_I^i$ whose integrand has explicit coordinate dependence, $\mathcal{J}_{II}^i$ which arises from the fermionic part, and $\mathcal{J}_{III}^i$ which arises from the bosonic part of the energy momentum tensor.

In the next section, we propose a decomposition of transverse spin in analogy with the helicity case and compare and contrast the two cases.

III. THE DECOMPOSITION OF TRANSVERSE SPIN

The transverse spin operators $\mathcal{J}^i$ in light-front theory for a massive particle can be given in terms of Poincaré generators by Eq. (2.6). In [7] it has been shown explicitly that the helicity operator $\mathcal{J}^3$ in the light-front gauge, in terms of the dynamical fields in the topologically trivial sector of QCD can be written as

$$\mathcal{J}^3 = \mathcal{J}_{fi}^3 + \mathcal{J}_{fo}^3 + \mathcal{J}_{go}^3 + \mathcal{J}_{gi}^3 \qquad (3.1)$$

where $\mathcal{J}_{fi}^3$ is the fermion intrinsic part, $\mathcal{J}_{fo}^3$ is the fermion orbital part, $\mathcal{J}_{go}^3$ is the gluon orbital part and $\mathcal{J}_{gi}^3$ is the gluon intrinsic part. The helicity sum rule is given by, for a longitudinally polarized fermion state,

$$\frac{1}{\mathcal{N}}\langle PS^\parallel|\mathcal{J}_{fi}^3 + \mathcal{J}_{fo}^3 + \mathcal{J}_{go}^3 + \mathcal{J}_{gi}^3|PS^\parallel\rangle = \pm\frac{1}{2}. \qquad (3.2)$$

In the transverse rest frame ($P^\perp = 0$), the helicity sum rule takes the form

$$\frac{1}{\mathcal{N}}\langle PS^\parallel|J_{fi}^3 + J_{fo}^3 + J_{go}^3 + J_{gi}^3|PS^\parallel\rangle = \pm\frac{1}{2}. \qquad (3.3)$$

For a boson state, RHS of the above equation should be replaced with the corresponding helicity. Here, $\mathcal{N}$ is the normalization constant of the state. Unlike the helicity operator, which can be separated into orbital and spin parts, the transverse spin operators cannot be written as a sum of orbital and spin contributions. Only in the free theory, one can write them as a sum of orbital and spin parts by a unitary transformation called Melosh transformation. However, we have shown that they can be separated into three distinct components. At this point, we would also like to contrast our work with Ref. [13], where a gauge invariant decomposition of nucleon spin has been done. The analysis in Ref. [13] has been performed in the rest frame of the hadron and no distinction is made between helicity and transverse spin, whereas, we have worked in the gauge fixed theory in an arbitrary reference frame.

In analogy with the helicity sum rule, we propose a decomposition of the transverse spin, which can be written as

$$\frac{1}{\mathcal{N}}\langle PS^{\perp}|\mathcal{J}^i_I+\mathcal{J}^i_{II}+\mathcal{J}^i_{III}|PS^{\perp}\rangle=\pm\frac{1}{2} \tag{3.4}$$

for a fermion state polarized in the transverse direction. For a bosonic state, RHS will be replaced with the corresponding transverse component of spin.

What is the physical relevance of such a decomposition of the transverse spin operator? The fermion intrinsic part of the helicity operator can be related to the first moment of the quark helicity distribution measured in longitudinally polarized deep inelastic scattering. In the case of the transverse spin operator, we have shown [9] that there exists a direct connection between the hadron expectation value of the fermionic intrinsic-like part of the transverse spin operator $\mathcal{J}^i_{II}$ and the integral of the quark distribution function g_T that appear in transversely polarized deep inelastic scattering. Also we can identify [9] the operators that are present in the hadron expectation value of $\mathcal{J}^i_{III}$ with the operator structures that are present in the integral of the gluon distribution function that appear in transverse polarized hard scattering. The physical relevance of the decomposition is made clear from the identification. Our results show the intimate connection between transverse spin in light-front QCD and transverse polarized deep inelastic scattering. As far as we know, such connections are not established so far in instant form of field theory and this is the first time that the first moment of g_T is related to a conserved quantity. It is already known that the interaction independent light-front helicity operator $\mathcal{J}^3$ can be separated as $\mathcal{J}^3=\mathcal{J}^3_{q(i)}+\mathcal{J}^3_{q(o)}+\mathcal{J}^3_{g(i)}+\mathcal{J}^3_{g(o)}$ and further, hadron expectation value of $\mathcal{J}^3_{q(i)}$ is directly related to the integral of the deep inelastic helicity structure function g_1. Thus we find natural physical explanation for the simplicity and complexity of operator structures appearing in the structure functions g_1 and g_T respectively. Another important point is that in perturbation theory, the helicity flip interactions which are proportional to mass play a crucial role both in g_T and in the transverse spin operator whereas they are not important in the case of the helicity operator.

Because the transverse spin operators are interaction dependent, they acquire divergences in perturbation theory. One has to regularize them by imposing momentum cutoffs and in the regularized theory the Poincaré algebra as well as the commutation relation obeyed by the spin operators are violated [14]. One has to introduce appropriate counterterms to restore the algebra. In the next section, we perform the renormalization of the full transverse spin operator up to $O(\alpha_s)$ in light-front Hamiltonian perturbation theory by evaluating the matrix element for a quark state dressed with one gluon. This calculation also verifies the relation (3.4) up to $O(\alpha_s)$ in perturbation theory.

IV. TRANSVERSE SPIN OF A DRESSED QUARK IN PERTURBATION THEORY

In this section, we evaluate the expectation value of the transverse spin operator in perturbative QCD for a dressed quark state.

The dressed quark state with fixed helicity σ can be expanded in Fock space as

$$\begin{aligned}|P,\sigma\rangle=&\phi_1^{\lambda}b^{\dagger}(P,\sigma)|0\rangle\\&+\sum_{\sigma_1,\lambda_2}\int\frac{dk_1^+d^2k_1^{\perp}}{\sqrt{2(2\pi)^3k_1^+}}\int\frac{dk_2^+d^2k_2^{\perp}}{\sqrt{2(2\pi)^3k_2^+}}\\&\times\sqrt{2(2\pi)^3P^+}\,\delta^3(P-k_1-k_2)\\&\times\phi^{\sigma}_{\sigma_1,\lambda_2}(P,|k_1,;k_2)b^{\dagger}(k_1,\sigma_1)a^{\dagger}(k_2,\lambda_2)|0\rangle.\end{aligned} \tag{4.1}$$

We are considering dressing with one gluon since we shall evaluate the expectation value up to $O(g^2)$. The normalization of the state is given by

$$\langle k',\lambda'|k,\lambda\rangle=2(2\pi)^3k^+\delta_{\lambda\lambda'}\delta(k^+-k'^+)\,\delta(k^{\perp}-k'^{\perp}). \tag{4.2}$$

The quark target transversely polarized in the x direction can be expressed in terms of helicity up and down states by

$$|k^+,k^{\perp},s^1\rangle=\frac{1}{\sqrt{2}}(|k^+,k^{\perp},\uparrow\rangle\pm|k^+,k^{\perp},\downarrow\rangle) \tag{4.3}$$

with $s^1=\pm m_R$, where m_R is the renormalized mass of the quark.

We introduce the boost invariant amplitudes Φ_1^{λ} and $\Phi^{\lambda}_{\sigma_1\lambda_2}(x,q^{\perp})$ respectively by $\phi^{\lambda}(k)=\Phi_1^{\lambda}$ and $\phi^{\lambda}_{\lambda_1\lambda_2}(k;k_1,k_2)=(1/\sqrt{k^+})\Phi^{\lambda}_{\lambda_1\lambda_2}(x,q^{\perp})$, where $x=k_1^+/P^+$ and $q^{\perp}=k_1^{\perp}-xP^{\perp}$. From the light-front QCD Hamiltonian, to lowest order in perturbative QCD, we have

$$\Phi^{\lambda}_{\sigma_1,\sigma_2}(x,q^{\perp}) = -\frac{x(1-x)}{(q^{\perp})^2+m^2(1-x)^2}\frac{1}{\sqrt{1-x}} \times \frac{g}{\sqrt{2(2\pi)^3}} T^a \chi^{\dagger}_{\sigma_1}\left[2\frac{q^{\perp}}{1-x}+\frac{\tilde{\sigma}^{\perp}\cdot q^{\perp}}{x}\tilde{\sigma}^{\perp} - \tilde{\sigma}^{\perp} im \frac{1-x}{x}\right]\chi_{\lambda}\cdot(\epsilon^{\perp}_{\sigma_2})^{*}\Phi^{\lambda}_{1}. \tag{4.4}$$

Here m is the quark mass and x is the longitudinal momentum carried by the quark. Also, $\tilde{\sigma}^1=\sigma^2$ and $\tilde{\sigma}^2=-\sigma^1$. It is to be noted that the m dependence in the above wave function arises from the helicity flip part of the light-front QCD Hamiltonian. This term plays a very important role in the case of transversely polarized target states.

For simplicity, in this section, we calculate the matrix element of the transverse spin operator for a dressed quark state in a frame where the transverse momentum of the quark is zero. It can be seen from Eq. (2.6) that the sole contribution in this case comes from the first term in the RHS, namely the transverse rotation operator. A detailed calculation of the matrix elements of the transverse spin operator in an arbitrary reference frame is given in Appendix D where we have explicitly shown that all the terms depending on $P^{\perp}$ get canceled.

The matrix elements presented below have been evaluated between states of different helicities, namely σ and σ'. Since the transversely polarized state can be expressed in terms of the longitudinally polarized (helicity) states by Eq. (4.3), the matrix elements of these operators between transversely polarized states can be easily obtained from these expressions.

Here, we have used the manifest Hermitian form of all the operators. It is necessary to keep manifest Hermiticity at each intermediate step to cancel terms containing derivative of delta function.

The operator $\frac{1}{2}F^2P^+$ can be separated into three parts [15],

$$\frac{1}{2}F^2P^+ = \frac{1}{2}F^2_I P^+ + \frac{1}{2}F^2_{II}P^+ + \frac{1}{2}F^2_{III}P^+, \tag{4.5}$$

where F^2_I, F^2_{II} and F^2_{III} have been defined earlier. The matrix elements of the different parts of these for a dressed quark state are given below. The evaluation of the matrix element of $\frac{1}{2}F^2_I P^+$ is quite complicated since it involves derivatives of delta functions. A part of this calculation has been given in some detail in Appendix E. The operator

$$\frac{1}{2}F^2_I P^+ = \frac{1}{2}F^2_I(1)P^+ - \frac{1}{2}F^2_I(2)P^+ - \frac{1}{2}F^2_I(3)P^+. \tag{4.6}$$

The first term contains the momentum density, the second and the third terms contain the free and the interaction parts of the Hamiltonian density respectively. The matrix elements are given by

$$\begin{aligned}\langle P,\sigma|\frac{1}{2}F^2_I(1)P^+|P,\sigma'\rangle &= \langle P,\sigma|\frac{1}{2}\int dx d^2q^{\perp} x^- P^2_0 \frac{1}{2}P^+|P,\sigma'\rangle \\ &= -\frac{i}{2}\sum_{spin}\int dx d^2q^{\perp} q^2 \Phi^{*\sigma}_{\sigma_1\lambda}\frac{\partial \Phi^{*\sigma'}_{\sigma'_1\lambda'}}{\partial x} + \text{H.c.}\end{aligned} \tag{4.7}$$

$$\begin{aligned}\langle P,\sigma|\frac{1}{2}F^2_I(2)P^+|P,\sigma'\rangle &= \langle P,\sigma|\frac{1}{2}\int dx d^2q^{\perp} x^2 P^-_0 \frac{1}{2}P^+|P,\sigma'\rangle \\ &= \frac{i}{4}\sum_{spin}\int dx d^2q^{\perp}\Phi^{*\sigma}_{\sigma_1\lambda}\frac{\partial\Phi^{\sigma'}_{\sigma'_1\lambda'}}{\partial q^2}(q^{\perp})^2 \times\left(\frac{1-x}{x}-\frac{x}{1-x}\right) + \frac{i}{4}\sum_{spin}\int dx d^2q^{\perp} m^2 \\ &\quad \times \frac{1-x}{x}\Phi^{*\sigma}_{\sigma_1\lambda}\frac{\partial\Phi^{\sigma'}_{\sigma'_1\lambda'}}{\partial q^2} + \text{H.c.}\end{aligned} \tag{4.8}$$

In the above two equations, both the single particle and two particle diagonal matrix elements contribute. Here, H.c. is the Hermitian conjugate, Σ_{spin} is the summation over $\sigma_1,\sigma'_1,\lambda_1,\lambda'_1$. P^-_0 is the free part of the Hamiltonian density:

$$\begin{aligned}\langle P,\sigma|\frac{1}{2}F^2_I(3)P^+|P,\sigma'\rangle &= \langle P,\sigma|\frac{1}{2}\int dx d^2q^{\perp} x^2 P^-_{int}\frac{1}{2}P^+|P,\sigma'\rangle \\ &= \frac{g}{\sqrt{2(2\pi)^3}}\sum_{spin}\int dx d^2q^{\perp}\frac{1}{\sqrt{1-x}}\Bigg(-\frac{i}{4}\Phi^{*\sigma}_1\chi^{\dagger}_{\sigma} \\ &\quad \times\left[\tilde{\sigma}^2(\tilde{\sigma}^{\perp}\cdot\epsilon^{\perp})+\frac{(\tilde{\sigma}^{\perp}\cdot\epsilon^{\perp})\tilde{\sigma}^2}{x}\right]\chi_{\sigma_1}\Phi^{\sigma'}_{\sigma_1\lambda}+\text{H.c.}\Bigg).\end{aligned} \tag{4.9}$$

P^-_{int} is the interaction part of the light-front QCD Hamiltonian density. Only the qqg part of it contributes to the dressed quark matrix element.

The operator $\frac{1}{2}F^2_{II}P^+$ which originates from the fermionic part of the energy momentum tensor, can be separated into three parts:

$$\frac{1}{2}F^2_{II}P^+ = \frac{1}{2}F^2_{mII}P^+ + \frac{1}{2}F^2_{q^{\perp}II}P^+ + \frac{1}{2}F^2_{gII}P^+ \tag{4.10}$$