

## Real-time reservoir operation for irrigation

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**Abstract.** A short-term real-time reservoir operation model is developed for irrigation of multiple crops. Two major features of the model distinguish it from the earlier models which address the problem of reservoir operation for irrigation: its capability to consider interdependence of crop water allocations across time periods and its ability to provide an adaptive release policy. Integration of reservoir operation with on-farm utilization of water by crops is achieved through the use of two models: An operating policy model, which optimizes reservoir releases across time periods in a year, and an allocation model, which optimizes irrigation allocations across crops within a time period. The model is applied to a case study of an existing reservoir in India.

### Introduction

Integration of reservoir operation for irrigation with farm water allocation under hydrologic uncertainty is an important modeling problem, addressed by many researchers in recent years. Comprehensive models are now available in literature for optimizing short-, intermediate-, and long-run irrigation decisions [Dudley, 1988] and integrating the supply, demand, and delivery management [Dudley and Scott, 1993] in a stochastic environment. Some recent studies on reservoir operation for irrigation have also focused on developing long-term steady state policies, integrating reservoir operation with farm level water utilization [e.g., Dudley and Musgrave, 1988; Vedula and Mujumdar, 1992]. While the steady state models are useful in deriving policies for maximizing the long-term benefits from an irrigation system, they do not, in general, serve the purpose of providing useful policies for short-term real-time operation. The present paper discusses a model developed specifically for the short-term reservoir operation for irrigation, integrating the reservoir operation with the on-farm utilization of water by crops. The steady state operation model developed by Vedula and Mujumdar [1992] forms the main basis for the real-time operation model developed in this study. Vedula and Mujumdar [1992] is referred to as the “earlier work” in the paper to avoid repeated citation.

The model developed in the earlier work results in a steady state policy that maximizes the expected value of a measure of crop production. In that model, stochastic dynamic programming (SDP) was used as an optimization tool with the state variables defined by the reservoir storage at the beginning of a period, inflow during the period, and the average soil moisture in the command area at the beginning of the period. Inflow transition probabilities were used to derive the steady state policy. The productive value of the reservoir release was obtained by solving an allocation problem to minimize the sum of evapotranspiration deficits across the crops in a period. Solution of this model provides a steady state release policy specified through optimal end-of-the-period reservoir storages for given initial storage, inflow during the period, and the average soil moisture in the command area. Optimal crop water allocations in a period are implied in the policy.

Two major limitations of this steady state model are recognized: use of average soil moisture in place of actual soil moisture as a state variable and inability of the model to consider the interdependence of allocations to a crop across time periods. The first limitation arose because of computational considerations. In the present study, the effect of this limitation is examined. The second limitation is a more serious one, which arose because of the modeling technique used. In determining the productive value of the release in the SDP the water available for irrigation at the field level is allocated optimally across the crops in that period alone. In a steady state policy this could not be avoided because, while deriving the policy, the actual allocations made to a crop would not be known when the computations are being carried out for a period. In the present work, this limitation is overcome by introducing an additional state variable for crop production measure.

Another limitation of the steady state models developed for irrigation reservoirs arises when the policies are used for real-time operation. Since a steady state policy essentially addresses a long-term operation problem, it would be insensitive to the short-run gains from the system. When the policy is used for real-time operation over several years the policy may perform poorly in some years in terms of crop yield, but it may make up for the loss in other years thus keeping the long-term average at a reasonably high level. This behavior of the policy is undesirable from the point of view of real-time operation, especially in the context of predominantly agriculture-based developing nations. It is therefore essential that short-run yearly reservoir operation models be developed specifically for irrigation integrating the operation with the on-farm utilization of water by crops. Recent work by Dariane and Hughes [1991] provides one such model. Even though the work addresses the problem of real-time reservoir operation for irrigation it does not consider soil moisture contribution in meeting the irrigation demands and the crop yield response to a deficit water supply. The present paper addresses the problem with a more exhaustive approach.

In the present paper, a short-run real-time reservoir operation model is developed specifically for irrigation. The model overcomes some of the limitations of the steady state model of the earlier work. Many components of the model, such as the soil moisture balance, estimation of actual evapotranspiration, crop root growth with time, crop production function, and optimal allocation among crops, remain unaltered from those in the earlier work. Stress is laid in the paper therefore only on

the improvements the present model achieves over the earlier model in the context of real-time operation.

### Model Features and Formulation

The real-time operation model is formulated for solution once at the beginning of each intraseasonal period. It uses forecasted inflows for the current period (for which a decision is sought) and all subsequent periods in the year. Only the optimal decisions on release and allocations to individual crops in the current period are implemented. The state of the system is updated at the end of the period with the realized values of inflow to the reservoir, rainfall in the command area, soil moisture of each crop, and the measure of crop production. Two major features of the model distinguish it from the earlier model. One is its capability to consider interdependence of crop water allocations. Because the model is applied in real time, the productive value of previous allocations to a crop up to the beginning of the current period in real time would be known. This information is taken as an input to the model in deriving optimal releases for the subsequent periods in the year. The second feature is that the model updates the release decisions from period to period, making use of the latest available information. Actual rainfall in the command area, reservoir inflow, current production status, and the actual soil moisture of crops, all contribute to the updating of release policy for the subsequent periods.

In the model, irrigation allocation to a crop in a period is based on (1) its current production status, which indicates the effect of water supplied to the crop (through irrigation allocations and precipitation) from the beginning of the season up to the beginning of the period, (2) available soil moisture in the root zone, and (3) competition for water with other crops. The first two conditions are introduced in the model through the use of two state variables, a crop production state variable and a soil moisture state variable, and the third condition is introduced through use of the yield factors  $K_y$  in the objective function. The state variable for crop production indicates the production potential of a crop from the current period to the end of the season. This state variable has been used earlier by *Dudley and Burt* [1973] in a SDP irrigation planning model. They used an additive form of crop production function suggested by *Burt and Stauber* [1971] that has the general form

$$\Psi(y) = h \left[ \sum \Phi_i(W_i) \right] \quad (1)$$

where  $y$  is the crop yield and  $W_i$  is a vector of climatic and cultural variables, including irrigation, to which the crop was subjected in period  $i$ . The summation is over the number of periods in the growing season. The  $h$  and  $\Phi_i$  are arbitrary functions. With this general form a single state variable that summarizes crop production potential at the beginning of period  $t + 1$  is [Dudley and Burt, 1973]

$$\Phi_1(W_1) + \Phi_2(W_2) + \Phi_3(W_3) + \cdots + \Phi_t(W_t) \quad (2)$$

A detailed discussion on the exact nature of the state variable for use in dynamic programming models is given by *Burt and Stauber* [1971]. The state variable for crop production measure used in the present paper has a similar form. Using the additive form of crop production given by *Doorenbos and Kassam* [1979], the value of the state variable  $\Psi_c^t$  for crop  $c$  at the beginning of period  $t$  is computed as

$$\Psi_c^t = \sum_{j=1}^{t-1} [1 - Ky_j^c(1 - AET_j^c/PET_j^c)] \quad (3)$$

where  $Ky_j^c$  is the yield factor for crop  $c$  in period  $j$ , and  $AET_j^c$  and  $PET_j^c$  are the actual and potential evapotranspirations of the crop  $c$  in period  $j$ , respectively. This state variable reflects the effect of the irrigation allocations to a crop up to the beginning of a period.

Keeping computational considerations in view, only those variables that influence the system operation significantly are chosen as state variables. In case of long-term operating policies using stochastic dynamic programming, generally, the inflow during the period also forms one of the state variables [e.g., *Vedula and Mujumdar*, 1992; *Karamouz and Houck*, 1987]. In the present case, however, since only the current year is of interest and forecasted inflows are used for all periods in the year, inflow to the reservoir does not form a state variable. Reservoir storage, soil moisture of each crop at the beginning of a time period, and the measure of crop production  $\Psi$  up to the beginning of the period are considered as state variables.

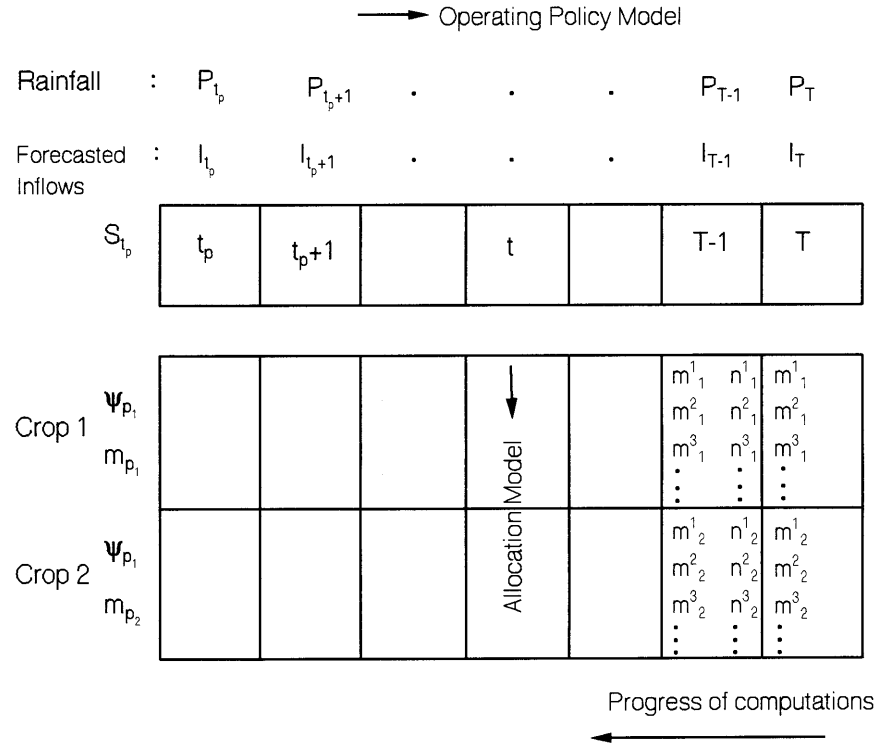
Integration of reservoir operation with the on-farm utilization of water by crops is achieved through the use of two models: an operating policy model, which optimizes releases across time periods, and an allocation model that optimizes allocations across crops in a given time period. These two models together constitute the real-time operation model. Figure 1 shows the main features of the real-time operation model. Mathematical formulations of the two models are discussed in the following paragraphs.

### Formulation of the Operating Policy Model

The operating policy model is formulated to obtain the optimal release decisions across time periods in a year, starting from the current period in real time to the last period in the year. The model employs a backward recursive dynamic programming algorithm. Forecasted inflows for the current and all subsequent periods in the year, soil moisture of each crop, crop production measure, and reservoir storage at the beginning of the current period form the inputs to this model. The state space is composed of the soil moisture state of each crop  $\mathbf{M} = \{m_1, m_2, \dots, m_c\}$  a measure of crop production  $\Psi$ , and reservoir storage at the beginning of each period subsequent to the current period. Values of performance measure required by this model are supplied by the allocation model for a given state of the system during a period. The system performance measure  $\Phi(k, l, \mathbf{M}, \Psi, t)$  is a function of release  $R_{k,l,t}$ , soil moisture state  $\mathbf{M}$ , crop parameters during the period  $t$ , and the effect of the water allocated to the crops up to the beginning of the period  $t$  reflected through crop production measure  $\Psi$ . The objective function value obtained in the allocation model when water available for irrigation  $X_{k,l,t}$  is optimally allocated among the crops for a given  $\mathbf{M}$  and  $\Psi$  defines the value of  $\Phi(k, l, \mathbf{M}, \Psi, t)$ . Each value of  $\Phi(k, l, \mathbf{M}, \Psi, t)$  is associated with a set of optimal crop water allocations and the end-of-the-period soil moistures, denoted by soil moisture vector  $\mathbf{N}$ .

Following backward recursion in dynamic programming, the recursive relationship for any period  $t$  between  $t_p + 1$  and  $T$  with  $j$  periods to go until the end of year is written as

$$f_i^j(k, \mathbf{M}, \Psi_b) = \underset{\text{feasible } l}{\text{Max}} [\Phi(k, l, \mathbf{M}, \Psi_b, t) + f_{i+1}^{j-1}(l, \mathbf{N}, \Psi_c)] \quad \forall k, \mathbf{M} \quad (4)$$



**Figure 1.** Features of the real-time operation model:  $m_c^r$ ,  $r$ th soil moisture class of crop  $c$  at the beginning of a period;  $n_c^r$ ,  $r$ th soil moisture class of crop  $c$  at the end of a period (not necessarily resulting from  $m_c^r$ );  $m_{p_c}$ , known soil moisture of crop  $c$  at the beginning of the current period;  $S_{t_p}$ , known reservoir storage at the beginning of the current period;  $\Psi_{p_c}$ , known crop production measure of crop  $c$  at the beginning of the current period.

Here  $\mathbf{N} = \{n_1, n_2, \dots, n_{NC}\}$ , with  $n_i$  representing the soil moisture state of the  $i$ th crop at the end of the time period. The transformation of the crop production state variable from  $\Psi_b$  at the beginning of the period to  $\Psi_e$  at the end of the period is a function of the evapotranspiration deficit  $(1 - AET/PET)$  during the period. The  $f_t^j(k, \mathbf{M}, \Psi)$  represents the total value of the performance measure in period  $t$  with  $j$  periods to go until the end of the year, for given  $k, \mathbf{M}$ , and  $\Psi$ .

For the current period  $t_p$ , the reservoir storage state  $k_p$ , soil moisture state  $\mathbf{M}_p$ , and the crop production measure  $\Psi_p$  are known. The relationship for the current period  $t_p$  is given by

$$f_{t_p}^j(k_p, \mathbf{M}_p, \Psi_p) = \underset{\text{feasible } l}{\text{Max}} [\Phi(k_p, l, \mathbf{M}_p, \Psi_p, t_p) + f_{t_p+1}^{j-1}(l, \mathbf{N}, \Psi_e)] \quad (5)$$

Reservoir mass balance is maintained through use of forecasted inflows,  $I_{t_p}, I_{t_p+1}, \dots, I_T$ . The change in the soil moisture from period to period is accounted through soil moisture balance considering the crop root growth. Starting with a known initial soil moisture  $Sm_t^c$  for crop  $c$  in period  $t$ , the soil moisture at the end of the period  $Sm_{t+1}^c$  is computed accounting for the evapotranspiration, deep percolation, rainfall, irrigation application, and the soil moisture in the zone into which the crop root extends during the period. The soil moisture balance equation is given by

$$Sm_{t+1}^c D_{t+1}^c = Sm_t^c D_t^c + x_t^c - AET_t^c + P_t + So_t^c (D_{t+1}^c - D_t^c) - Dp_t^c \quad (6)$$

where

- $Sm_t^c$  soil moisture for crop  $c$  at the beginning of the time period  $t$ , mm/cm;
- $x_t^c$  irrigation allocation to crop  $c$  associated with the release  $R_{k,l,t}$  during time period  $t$ , mm;
- $AET_t^c$  actual evapotranspiration for crop  $c$  resulting from the allocation  $x_t^c$  in time period  $t$ , mm;
- $P_t$  rainfall in the time period  $t$ , mm;
- $D_t^c$  average root depth for crop  $c$  during time period  $t$ , cm;
- $Dp_t^c$  deep percolation for crop  $c$  during time period  $t$ , mm;
- $So_t^c$  initial soil moisture of the layer of soil added to the root zone, mm/cm.

Details of the various components of (6) are discussed by Vedula and Mujumdar [1992].

#### Formulation of Allocation Model

The allocation model is adopted from the section Allocation Problem of the earlier work [Vedula and Mujumdar, 1992]. The model is formulated to optimally allocate a given amount of available water ( $X_{k,l,t}$ ) among the crops in a given time period  $t$  for known initial soil moistures and production status of the crops. The available water  $X_{k,l,t}$  is obtained from the release  $R_{k,l,t}$  corresponding to the reservoir storage state  $k$  at the beginning of the period and the storage state  $l$  at the end of the period after accounting for conveyance and other losses. The problem is formulated using backward recursion of dynamic

programming (DP). The objective function for the allocation problem is given by

$$\text{Max} \sum_{c=1}^{NC} \Psi_c + [1 - Ky_t^c(1 - AET_t^c/PET_t^c)] \quad (7)$$

where  $c$  is the crop index,  $\Psi_c$  is the state of the crop production measure for the crop  $c$  up to the beginning of the period  $t$ , and  $NC$  is the number of crops in the time period  $t$ . The state variable in the allocation model is the amount of water available at a given stage for allocation among all the stages up to and including that stage. The additive form of the production function [Doorenbos and Kassam, 1979] forms the basis for the objective function, (7). The recursive relationship for the problem may be written with a fairly simple modification to include  $\Psi_c$  in the formulation of the allocation problem discussed in the earlier work.

The role of the allocation model, in the real-time reservoir operation model may be stated as follows:

1. Water available for irrigation  $X_{k,l,t}$  in time period  $t$  as determined in the operating policy model for the storage states  $k$  and  $l$  is optimally allocated among the crops in that period, with respect to known initial soil moisture and the measure of crop production up to the beginning of the period of individual crops.

2. The optimal value of the objective function, optimal allocations during the period to individual crops, and the soil moistures and crop production values of the crops at the beginning of the time period  $t + 1$  resulting from the optimal allocations are transferred to the operating policy model.

### Real-Time Operation

In real time the operating policy model is solved from the current period  $t_p$  to the last period  $T$  in the year to obtain the optimal operating policy. The allocation model is solved for each discrete feasible release sequence to provide the input required for the operating policy model. In doing so, the soil moisture continuity over the periods is maintained with optimal allocations arising out of the release sequence being examined. Since a backward algorithm of dynamic programming is used for solution of the operating policy model, the last period  $T$  corresponds to the first stage (Figure 1). For each period  $t$  within the time horizon between current period  $t_p$  to the last period  $T$ , release sequences are examined starting from the last period and moving backward to that period  $t$ . Discrete releases are obtained based on the reservoir storage state  $k$  at the beginning of the period and the reservoir storage state  $l$  at the end of the time period, if feasible, for a given set of forecasted inflows. For all possible discrete values of the crop production measure  $\Psi$  the performance value  $\Phi(k, l, \mathbf{M}, \Psi, t)$  of each discrete feasible release  $R_{k,l,t}$  is determined for each of the soil moisture states  $\mathbf{M} = \{m_1, m_2, \dots, m_c\}$ . Here  $m_i$  represents the soil moisture state of  $i$ th crop at the beginning of the period  $t$ . For the combinations of  $k, l, \Psi$ , and  $\mathbf{M}$  the performance value  $\Phi(k, l, \mathbf{M}, \Psi, t)$  is obtained from the allocation model. The soil moisture of a crop at the end of the period is obtained by soil moisture balance using the optimal allocation to that crop. For the current period the set of soil moistures of the crops  $\mathbf{M}_p$ , the reservoir storage state at the beginning of the period  $k_p$ , and the measure of crop production are known. Therefore the search is made with these initial values of  $\mathbf{M}_p, k_p$ , and  $\Psi$  for the current period  $t_p$ . Once

the performance values of all discrete feasible releases are estimated the DP algorithm tracks out the optimal releases for the periods starting from current period  $t_p$  to the last period  $T$  in the year. When operated in real time only the current period's decision is implemented. For the computations to continue for the subsequent periods the state of the system is updated with the storage, actual soil moisture, and the crop production measure obtained up to that period. A new optimal operating policy is derived for the subsequent periods. This procedure is continued until the end of the last period in the year.

### Model Application to a Case Study

The real-time operation model developed is applied to an existing reservoir system in Karnataka, India. The details of the case study are given in the earlier work. For demonstration of model application the computations are carried out with intraseasonal time periods of 10 days each, 15 storage class intervals, and 5 soil moisture class intervals. The maximum number of crops in any period is five, and the number of intraseasonal periods within a year is 36. A year is composed of two seasons, namely, kharif (June to October) and rabi (October to July). An autoregressive model AR(1), identified for the reservoir site by Mujumdar and Nagesh Kumar [1990], is used for forecasting inflows. Real-time simulation of the reservoir operation and crop response is carried out updating the state with actual inflow and rainfall data at the beginning of every period.

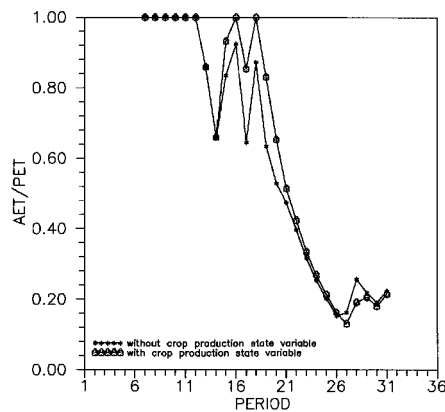
The optimal release from the reservoir during a period is obtained from the optimal operating policy for the known conditions of initial reservoir storage, forecasted inflows, and the soil moisture of each crop at the beginning of the period. The soil moisture at the beginning of the next period in simulation is obtained by soil moisture balance, based on the optimal irrigation allocations made and actual rainfall realized.

The following aspects of modeling interest have been studied through the case study: (1) effect of inclusion of crop production state variable, (2) performance of the real-time operation model against the steady state model, (3) sensitivity of the real-time operating policy to the soil moisture state variable, and (4) performance of policies in standard and adaptive modes. Minor modifications were necessary in the model to study each of these aspects.

### Inclusion of Crop Production State Variable

One state variable is needed for the crop production measure for each crop. For the case study, with a maximum of five crops in any period, this would mean an additional five state variables. Even with only one soil moisture state variable (being the average soil moisture) a total of seven state variables are needed for the case study. This renders the problem computationally intractable. To study the sensitivity of model results with the crop production state variable, therefore the model was solved with only two crops in the entire command area: the two seasonal crop cotton, denoted as crop 1 and kharif maize and rabi maize, both denoted as crop 2.

A low-inflow year was chosen for the real-time simulation, with the reservoir empty at the beginning of the year. Figures 2 and 3 show the  $AET/PET$  ratios for the two crops in the intraseason periods. In the kharif season (periods 1–16), crop 2 extended from period 1 to period 12, and its water requirement was fully met, making the actual evapotranspiration  $AET$

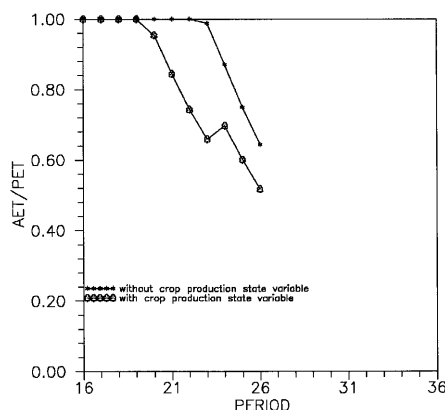


**Figure 2.** Response of the biseasonal crop to operating policies.

equal to potential evapotranspiration  $PET$  during all those periods, both in simulations with and without the crop production state variable. Although the reservoir releases and crop water allocations were different for the two simulations, there was only a slight change in the relative yields of the two crops. Simulation without the crop production measure resulted in relative yields of 82.1 and 97.9% for the two crops, and when the state variable was included, the relative yields were 83.8 and 93.3%, for the two crops, respectively. The water allocation to the two crops in the two cases was in the approximate proportion, 70:30 and 82:18, respectively. Optimal crop water allocations were significantly different in the two cases. These results, however, depend on the types of crops chosen, areas of cultivation, and extent of water deficit. A more detailed study, including more number of crops is necessary for generalization.

#### Steady State and Short-Term Policies

An implication of a steady state operation of a system is that the value of the system performance measure will be maximized when the system is operated over a long period of time (spanning over at least a few years). When only the current year is of interest the short-term policy should perform better in case of a critical year. To examine this, simulations were carried out with the steady state policy of Vedula and Mujumdar [1992] and the real-time operating policy of the present work for three different operating horizons: 1, 5, and 28 years.



**Figure 3.** Response of the rabi crop to operating policies.

**Table 1.** Performance of Policies (28 Year Simulation)

$\lambda$	Productivity Index	
	Real-Time Policy	Steady State Policy
0.965	1.000	1.000
0.975	0.893	0.893
0.978	0.714	0.786
0.980	0.536	0.715
0.982	0.214	0.679
0.986	0.036	0.357

The 5 year period was chosen from the most critical (low-inflow) period in the historical data. Productivity index [Mujumdar and Vedula, 1992] was used as a performance measure for the 5 and 28 year simulations, and average relative yield was used for the 1 year simulation. Productivity index is defined as the probability that the average relative yield over all the crops over the period of simulation is greater than a specified value  $\lambda$ . Tables 1 and 2 give the summary results of the 28 and 5 year simulations. For the 1 year simulation the average relative yield resulting from the real-time policy was 0.975, and from the steady state policy it was 0.968. Performance of both the steady state and the short-term policies depend on the operating horizon, inflow and rainfall sequences, initial reservoir storage, and the inflow forecasts. It is observed that for short operating horizons with low inflows the short-term policy performs better than the steady state policy. Note that even a small change in the value of  $\lambda$  could mean a significant change in the crop yield, since  $\lambda$  indicates the average relative yield.

#### Effect of Average Soil Moisture

The effect on the crop yield of the approximation of using the average soil moisture as a state variable in place of individual soil moistures is studied. To facilitate comparison, the original model is modified to use the average soil moisture of the crops at the beginning of a period instead of soil moistures of each crop as a state variable. The original model with one soil moisture state variable for each crop is referred to as model 1, and the modified model with a single soil moisture state variable (being the average soil moisture) is referred to as model 2. Simulation is carried out for 1 critical year, and the performances are compared. The values of cumulative average relative yield are given in Figure 4. These values indicate that the original model with soil moisture of each crop as state variable showed little improvement over the modified model. Figure 4 also shows the average relative yields of crops obtained from both the models. It is concluded that both the models, model 1 and model 2, performed equally well. One reason for this is that although the individual soil moistures resulting from the two models were quite different, the corresponding actual evapotranspirations were not significantly different. This is because whenever the available soil moisture is

**Table 2.** Performance of Policies (5 Year Simulation)

$\lambda$	Productivity Index	
	Real-Time Policy	Steady State Policy
0.969	1.0	0.8
0.973	1.0	0.6
0.975	0.6	0.4

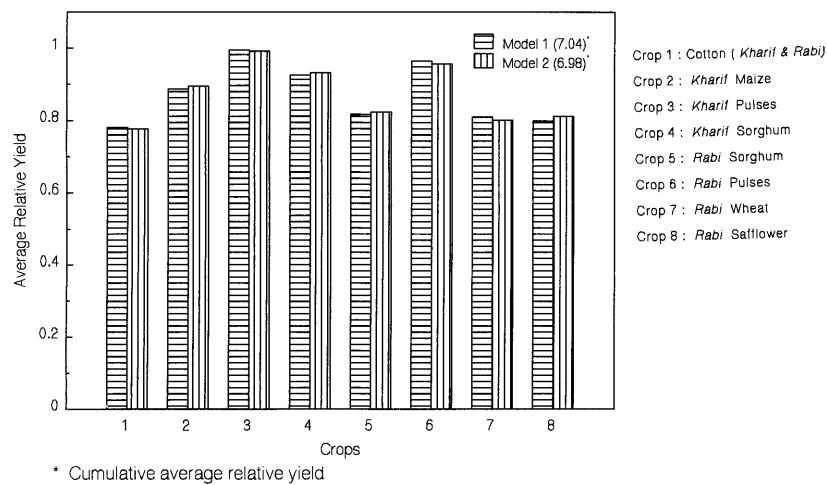


Figure 4. Average relative yields for the two models.

above a critical level, the actual evapotranspiration would be equal to the potential evapotranspiration. This critical level of available soil moisture varies mainly with the moisture holding properties of the soil, and, to a lesser extent, with the type of crop and may be as low as 50% of the maximum available soil moisture. With all crops grown on the same type of soil having uniform moisture holding properties, in the case study, the resulting actual evapotranspirations and crop yields from the two models were nearly the same. In general, the approximation of using a single, average soil moisture state variable for all the crops in place of one soil moisture state variable for each crop in a reservoir operation model may not significantly affect the results when the moisture holding properties of the soil are uniform over the entire irrigated area. End-of-the-period reservoir storages for both the models are shown in Figure 5. It is seen that both the models resulted in the same storages for most of the periods in simulation. One major drawback of model 1, which limits its usage, is the enormous computational time required to solve it for 1 year. The fact that both the models performed equally well in simulation suggests that the model 2 can be used instead of model 1 without losing much accuracy.

#### Standard and Adaptive Operation

The real-time reservoir operation model developed is studied for performance when operated in two modes of operation: a standard real-time operation and an adaptive real-time operation. In standard real-time operation the model is solved only once at the beginning of the year with forecasted values of inflows for all the periods in the year, starting with a known initial reservoir storage and known soil moisture of each of the crops. Optimal release decisions are implemented to the extent possible, and the reservoir storage and soil moisture are updated based on the actual inflow and rainfall realized. The standard mode of operation obviously does not have any practical utility in operation. It is used here only to examine the effect of updating the inflow forecasts and release policy on the performance of the system. When the model is solved at the beginning of the year, it specifies the release decisions for each period in the year, as a function of the reservoir storage and the average soil moisture at the beginning of the period. These decisions are derived for single-valued inflow forecasts. In the standard mode of operation this release policy remains unal-

tered throughout the year. Corrective measures are applied only through updating reservoir storage and soil moisture based on actual inflow and rainfall, respectively. In case of adaptive real-time operation, on the other hand, the model is solved once at the beginning of every period. Only the current period's decision is implemented and the reservoir storage and soil moisture of each crop are updated based on the actual inflow and actual rainfall in the command area. For the computations to proceed for the subsequent periods the model is rerun for the remaining periods with a new set of inflow forecasts to obtain a new operating policy.

Two critical years were used for simulation, one with very high actual inflows and another with very low actual inflows. Forecasted and actual inflow values for the year in which high actual inflows are realized are shown in Figures 6 and 7 for both the operations. Figure 8 gives the optimal release decisions obtained by both the operations for the year in which high inflows were realized. Since the model restricts the release to the irrigation demand in a period, it is seen from Figure 8 that the adaptive real-time operation performed better than the standard real-time operation. The aggregate reservoir release over the year in the case of adaptive mode is nearly twice that in the case of standard mode, indicating a near-100% improvement in performance. In case of standard real-time operation the poor inflow forecasts would lead to very low release decisions, even though the actual inflows realized are

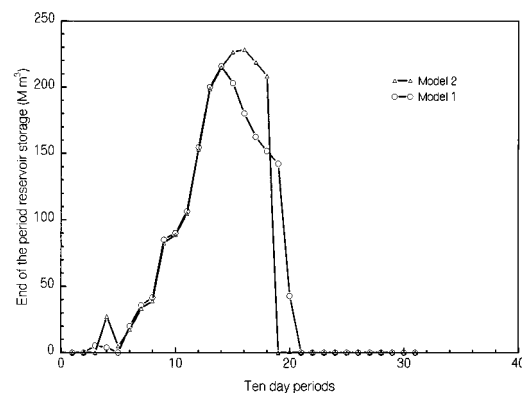


Figure 5. End-of-the-period reservoir storages.

very high. Since the AR(1) model used in the present study provides a better one step ahead forecast, the adaptive real-time operation performs much better in comparison with the standard real-time operation. In case of simulation for a year with low actual inflows, both standard real-time operation and adaptive real-time operation performed equally well. This can be attributed to a very low difference between the original and the updated forecasts. Since single-valued inflow forecasts are used in the decision making, decision errors due to inaccurate forecasts can not be avoided. The adaptive operation, by updating both the inflow forecasts and release policies at the beginning of every period, ensures that the errors are not accumulated over time. It is obvious that if perfect forecasts are available, then both modes of operation would perform equally well.

### General Remarks

The real-time operating policy model developed in this paper addresses the problem of short-term yearly operation of an irrigation reservoir. The operating horizon of 1 year used in the model is appropriate in situations where annual cyclicity of reservoir inflows is well pronounced, as it is in the case of most monsoon climates. In cases where the system is planned and designed with a clearly defined long-term objective, targets such as the end-of-year reservoir storage may be specified from a long-term policy. Given these targets, the short-term policy model may be used to optimally distribute the water within the year among intraseasonal periods in a deterministic framework, with the inflow uncertainty addressed through short-term inflow forecasts that are updated at the beginning of every intraseasonal period. Although inclusion of overyear storage is not explicitly addressed in the model formulation, its inclusion in the model is fairly straightforward. Referring to Figure 1, the reservoir storage at the end of the last period  $T$  in the year may be fixed at the specified overyear storage. When the computations begin with the last period in a backward algorithm only those releases that result in the specified overyear storage would be considered feasible.

A limitation of the real-time operation model presented in this paper is its inability to handle inflow uncertainty explicitly. The model is formulated in a deterministic framework. Since single-valued inflow forecasts are used in the model, the uncertainties associated with the inflow forecasts are still present in the model. This limitation in the model is conceded because

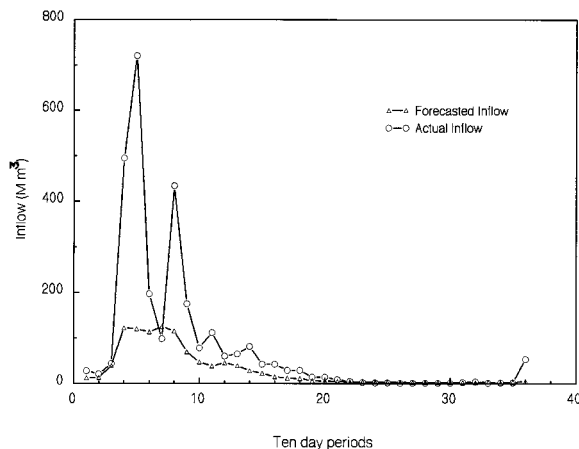


Figure 6. Inflow forecasts in the standard operation.

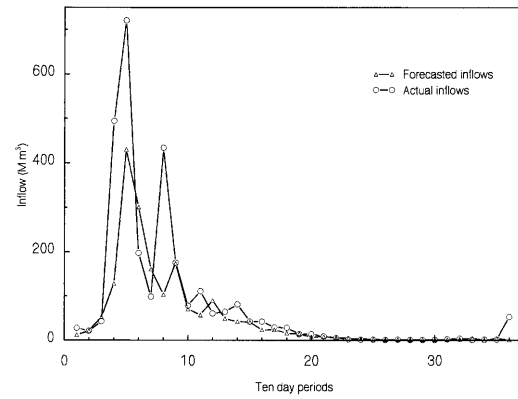


Figure 7. Inflow forecasts in the adaptive operation.

use of the inflow forecasts, with the consequent elimination of the inflow state variable in the DP model ensures a significant computational advantage, especially in view of the increased computational burden caused by the inclusion of soil moisture and crop production state variables.

### Conclusion

A real-time reservoir operation model has been developed to provide optimal reservoir releases and crop water allocations. The reservoir storage, soil moistures of individual crops, and a crop production measure constitute the state space for the dynamic programming model developed. The model overcomes some of the limitations of the steady state model developed in an earlier work [Vedula and Mujumdar, 1992]. The model has been applied to the Malaprabha irrigation reservoir in Karnataka, India. Simulation with steady state and the short-term real-time operation model showed that for shorter operating horizons, the real-time adaptive policy performs better in case of critical low-flow years.

### Notation

$AET$	actual evapotranspiration, mm.
$AET_t^c$	actual evapotranspiration of crop $c$ in time period $t$ , mm.

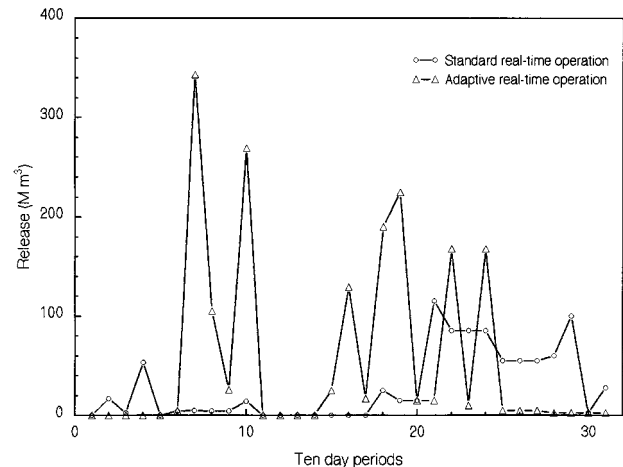


Figure 8. Optimal releases in real-time simulation of a critical year.

- $c$  crop index.  
 $D_t^c$  average root depth for crop  $c$  during time period  $t$ , mm/cm.  
 $Dp_t^c$  deep percolation for crop  $c$  during time period  $t$ , mm.  
 $f_t^j(k, \mathbf{M}, \Psi)$  total value of performance measure with  $j$  periods to go till the end of the year for given  $k$ ,  $\mathbf{M}$ , and  $\Psi$ .  
 $I_t$  forecasted inflow during time period  $t$ ,  $M \text{ m}^3$ .  
 $k$  reservoir storage state at the beginning of time period  $t$ .  
 $ky_t^c$  yield factor for crop  $c$  in the period  $t$ .  
 $l$  reservoir storage state at the end of time period  $t$ .  
 $m_i$  soil moisture state of  $i$ th crop at the beginning of a period.  
 $\mathbf{M}$  vector of soil moistures at the beginning of time period  $t$ .  
 $\mathbf{M}_p$  vector of soil moistures at the beginning of the current period  $t_p$ .  
 $\mathbf{N}$  vector of soil moistures at the end of time period  $t$ .  
 $NC$  number of crops.  
 $PET$  potential evapotranspiration, mm.  
 $PET_t^c$  potential evapotranspiration for crop  $c$  in time period  $t$ , mm.  
 $P_t$  rainfall in the command area during time period  $t$ , mm.  
 $R_{k,l,t}$  release from the reservoir corresponding to reservoir storage at the beginning of time period  $t$ ,  $S_t^k$  and storage at the beginning of time period  $t + 1$ ,  $S_{t+1}^l$ ,  $M \text{ m}^3$ .  
 $S_t^k$  representative value of reservoir storage at the beginning of the time period  $t$ .  
 $Sm_t^c$  soil moisture for the crop  $c$  at the beginning of time period  $t$ , mm/cm.  
 $So_t^c$  initial soil moisture of the layer of soil added to the root zone for crop  $c$  during time period  $t$ , mm/cm.  
 $T$  last time period in the year.  
 $t$  time period index.  
 $t_p$  current time period.  
 $x_t^c$  irrigation allocation to crop  $c$  during time period  $t$ , mm.  
 $X_{k,l,t}$  amount of water available for irrigation for given  $k$  and  $l$  in time period  $t$ ,  $M \text{ m}^3$ .  
 $\Phi_i$  functional notation in the crop production function.  
 $\Phi(k, l, \mathbf{M}, \Psi, t)$  a measure of system performance corresponding to storage states  $k$  and  $l$ , soil moisture state  $\mathbf{M}$ , and the crop production measure  $\Psi$  in time period  $t$ .  
 $\Psi$  crop production measure.

**Acknowledgment.** The authors sincerely thank the anonymous reviewers for their objective criticism of the paper.

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(Received May 2, 1996; revised October 14, 1996; accepted December 16, 1996.)