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Nonlinear aeroelasticity of rotating and flapping wings – a review

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Abstract

The interaction between large deflections, rotation effects and unsteady aerodynamics makes the dynamic analysis of rotating and flapping wing a nonlinear aeroelastic problem. This problem is governed by nonlinear periodic partial differential equations whose solution is needed to calculate the response and loads acting on vehicles using rotary or flapping wings for lift generation. We look at three important problems in this paper. The first problem shows the effect of nonlinear phenomenon coming from piezoelectric actuators used for helicopter vibration control. The second problem looks at the propagation on material uncertainty on the nonlinear response, vibration and aeroelastic stability of a composite helicopter rotor. The third problem considers the use of piezoelectric actuators for generating large motions in a dragonfly inspired flapping wing. These problems provide interesting insights into nonlinear aeroelasticity and show the likelihood of surprising phenomenon which needs to be considered during the design of rotary and flapping wing vehicle

(Keywords : aeroelasticity/nonlinear/rotorcraft/flapping wing/ vibration)

Introduction

Aeroelasticity is the science which studies the behavior of flexible structures when subjected to airflow. Thus, it involves fluid and structural dynamics and is an important and difficult subject in mechanics. For many physical phenomenon in fixed wing aeroelasticity, the underlying fluid and structural physics is linear, and the problems can be solved for response and flutter using the well established tools of linear aeroelasticity. However, for rotary wing aircraft, coriolis effects introduce nonlinearity in the structural dynamics, and dynamic stall and a non-uniform wake introduce nonlinearity in the fluid dynamics. Helicopter blades are long and slender, leading to moderate to large deformations creating nonlinear strain displacement relations.

Flapping wings also use the concept of moving wings to generate airflow at low speeds and provide lift. Typically, flapping wings of natural flyers undergo large motions and attempts to simulate this motion using artificial wings require large displacement actuators such as those used in piezofans. The response and stability characteristics of rotary and flapping wings need to be accurately predicted for the safe design on helicopters and flapping wing vehicles. While helicopters are established flight vehicles, flapping wing vehicles are being researched for use in dull, dirty and dangerous (D^3) environments.

The nonlinearity inherent in rotorcraft gets amplified when smart materials such as piezoceramics are used to suppress the high vibration levels through active control. The piezoceramic actuators introduce nonlinearity due to variation of piezoelectric coefficients with voltage at high electric fields and the hysteresis effects in stack actuators. These nonlinear effects are difficult to model and further complicate the rotorcraft aeroelastic problem.

In this paper, some recent work in the field of nonlinear dynamics and aeroelasticity done by the author and his students is reviewed. The objective is to bring some interesting and important problems in nonlinear aeroelasticity to the attention of other researchers. The review is done in three sections. The first section discusses the smart helicopter rotor concept and the effect of piezoceramic induced nonlinearity in its modeling. The second section discusses the effect of propagation of material uncertainty inherent in composite materials into the nonlinear response and vibratory loads of a helicopter rotor. The third section discusses the modeling of large displacement dragonfly inspired flapping wings actuated by piezoceramics.

Nonlinearity and the smart helicopter rotor

Vibration is a key problem in helicopters. The helicopter rotor is a periodic system, and steady and harmonic components of the loads act on the rotor blades. However, due to periodicity, there is a cancellation among harmonics at the rotor hub which acts as a filter. For an N-bladed helicopter rotor, the MN/rev vibratory loads ($M = 1, 2, 3, \ldots$) are transmitted to the fuselage as the main source of vibration. The notation N/rev read as N per rev is used in rotor dynamics to mean $N\Omega$ frequency where Ω is the rotation speed. For example, if the rotation speed is a typical value of 300 RPM (5 Hz), then the loads transmitted by a 4-bladed main rotor to the fuselage are at MN/rev or $4M$/rev or 4/rev, 8/rev, 12/rev etc. In terms of frequency, 4/rev corresponds to 20 Hz, 8/rev to 40 Hz, 12/rev to 60 Hz and so on. However, the 4/rev component is dominant by far which means that for a 4-bladed rotor with an RPM of 300, the vibration transmitted by the rotor to the fuselage is primarily at 20 Hz.

Therefore, reducing vibration at the main rotor is the most effective way for reducing helicopter vibration. Researchers have investigated both active and passive approaches to alleviate the vibration problem[1,2]. The passive approaches involve the use of vibration absorbers and isolators or designing the rotor blade to have inherently low vibrations. However, passive approaches suffer from weight penalty and performance degradation away from the tuned flight condition. In recent years, there has been a great interest in active control of helicopter vibration. The popular active approach involves the use of individual blade control (IBC) where the blades are excited at higher harmonics of the rotor speed to cancel the existing unsteady loads which are the main cause of helicopter vibration. The advent of smart materials such as piezoceramics has lead to two active approaches becoming most popular. These are the trailing edge flaps[3-5] and the active rotor twist[6-8]. These two approaches are discussed below.

Trailing edge flaps for vibration control

Trailing edge flaps placed near the blade tip are actuated at higher harmonics of the rotor speed to reduce the vibration at its source: the main rotor. For a 4-bladed rotor, actuation at 2, 3, 4 and 5/rev can reduce vibration and noise, and also improve performance. Early research of trailing edge flaps focused on the development of aerodynamic modeling for the unsteady aerodynamic loads developed by such flaps and control algorithms which resulted in reductions in vibration while restricting the flap inputs to small values such as 2 degrees which were possible to obtain using piezoelectric actuators[3]. The large displacement needed to actuate the flaps and the bandwidth required makes piezoceramic stack actuators the best choice for the vibration control problem. However, piezostacks lead to considerable hysteresis nonlinearity which gets further amplified by the nonlinearity due to mechanical amplification which is needed to convert the linear motion to rotary motion.

A major limitation of piezostack actuators is their lack of accuracy due to hysteresis and drift. Hysteresis in piezoelectric materials is a form of nonlinearity with memory. Thus, the piezoelectric expansion depends not only on the current voltage excitation but also on the history of the excitation. Not modeling hysteresis in the piezoceramic actuator can lead to inaccuracy in open-loop control and can lead to amplitude dependant phase shifts. Macroscopic models such as the classical Preisach model (CPM) are commonly used to characterize piezoceramic hysteresis due to their simplicity and ease of incorporation in controller designs. A disadvantage of these phenomenonological models is that it is difficult to update the model parameters due to variations in operating conditions. Though hysteresis is very important in control of smart structures, helicopter vibration analysis using smart materials neglected these effects for quite some time.

Addressing this issue, Viswamurthy and Ganguli[9] investigated the use of piezostack actuators for moving the trailing edge flaps. A harmonic controller was devised and the effect of static hysteresis on the controller was analyzed. The static hysteresis data was obtained from Hall and Prechtl[10] and a static model based on CPM for actuator hysteresis was created. Here the term static means that the system output depends only on the past extremum values of input, while the speed of the input variation between the extremum points has no influence on the output. The earlier study[9] clearly showed that even the simple static hysteresis model resulted in considerable phase difference between the voltage applied to the piezostack actuators and the motion of the flaps. Controller performance deteriorated in the presence of actuator hysteresis. It was shown that the control input for achieving the optimal flap motion is different in the case of a hysteretic actuator as compared to an ideal-linear actuator.

The classical Preisach model used[9], though powerful, has several limitations. The CPM is static and rate independent in nature and is limited in describing dynamic hysteresis phenomenon. Thus, the hysteresis depends on the frequency of actuation and the actuator needs to be characterized experimentally for the frequencies under consideration. The typical rotation speed for a helicopter is about 5 Hz. The flaps are actuated at 2, 3, 4 and 5 times the rotation speeds which lead to rates of 10 Hz to 25 Hz. The hysteresis model should be able to account for the behavior of the actuator at these frequencies.

Viswamurthy and Ganguli[11] used a dynamic hysteresis model based on extending the CPM. In this dynamic hysteresis model, the static property is relaxed by introducing the dependence of the Preisach distribution function on the speed of output variations. Experiments were conducted to study the APA500L actuators obtained from CEDRAT and the data was used to create dynamic hysteresis models. The actuator is shown in Fig. 1 and the experimental setup is shown in Fig. 2. The APA500L is a large stroke, amplified piezoelectric actuator which is suitable for actuation of a full scale trailing edge flap (voltage range: –20 to + 150 V). The DSPACE controldesk was used to send command to the high voltage amplifier required to power the APA500L actuator. An LVDT sensor was used to measure the displacement of the actuator and the data was collected using DSPACE for postprocessing. The actuator displacement is converted to equivalent flap deflection by multiplying it with a known gain factor. The factor can be thought of as the constant gain of an ideal-linear amplification mechanism with zero friction or nonlinearities.

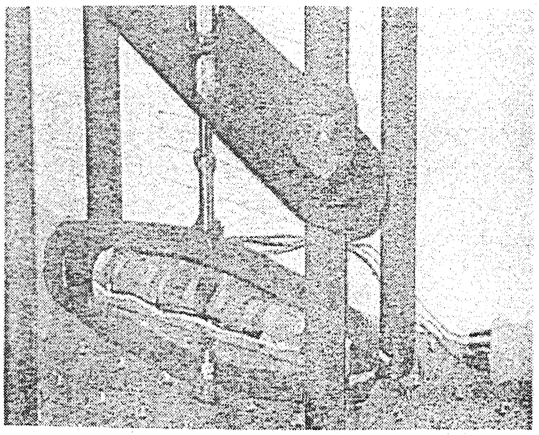

Fig. 1 – APA 500L stack actuator under blocked-free condition.

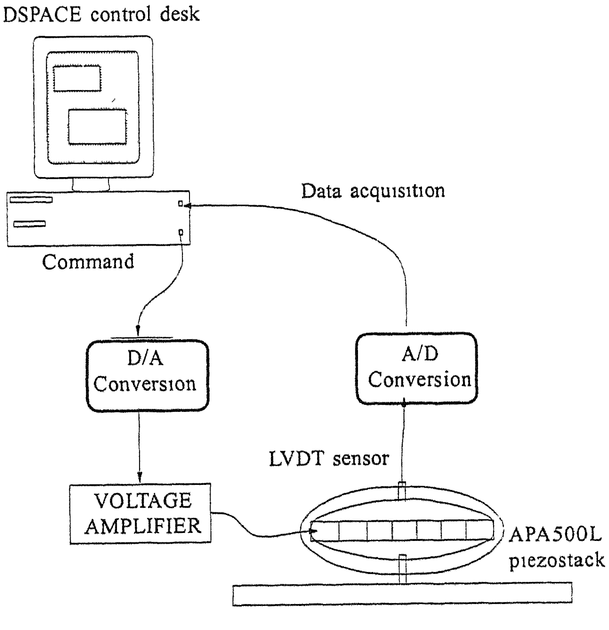

Fig. 2 – Experimental setup

It was found that hysteresis effects caused considerable change in the controller performance and a hysteresis compensation algorithm was proposed[12]. Fig. 3 shows the quasi static response of the stack actuator. It can be seen that even for this simple static hysteresis case, there is considerable deviation from the idealized linear behavior. Fig. 4 shows the actuator hysteresis at different frequencies some of which are close to the operational frequencies for helicopter vibration control. Clearly, there is significant hysteresis present which is also frequency dependant. Fig. 4 clearly shows the enormous importance of dynamic hysteresis in piezostack actuators.

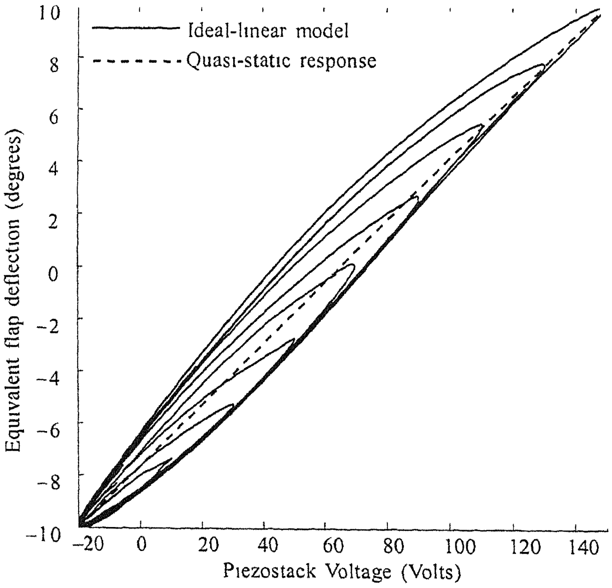

Fig 3 – Quasi static response of the stack actuator normalized to yield equivalent flap deflection

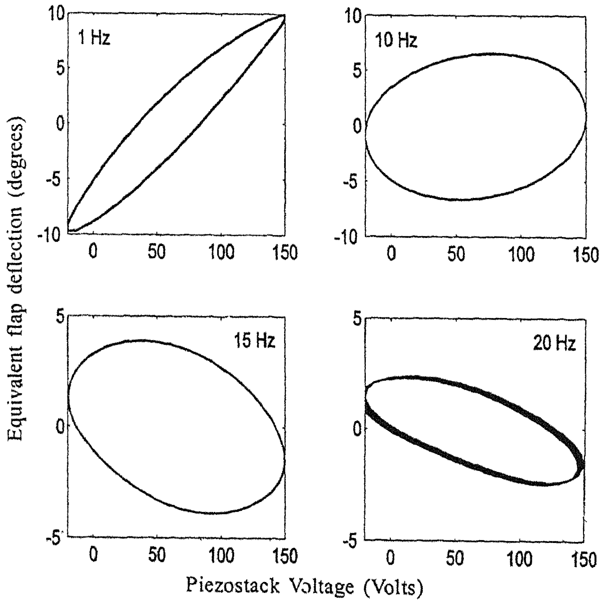

Fig. 4 – Actuator response at higher operational frequencies.

Global and local controllers were compared for their computational efficiency and for their robustness to measurement and model uncertainties[13]. It was found that the global controller was adequate for helicopter vibration problems, and the local controller only offered an incremental advantage which came at high computational expense. It was also found that the global optimal controller could be adjusted to ensure that each flap worked to its full authority[14]. This innovation in controller design allows for a greater amount of vibration reduction than the approaches proposed earlier.

Fig. 5 shows the schematic of the helicopter rotor blade with dual trailing edge flaps which was used for the numerical results. Earlier studies have shown that two small trailing edge flaps perform better than one large trailing edge flap. The piezoelectric hysteresis model is cascaded with the helicopter aeroelastic analysis and the voltage inputs needed for optimal vibration control is obtained. Here, the aeroelastic analysis involves the solution of periodic nonlinear partial differential equations using finite element in space and time along with the solution of vehicle equilibrium equation using the Newton-Raphson method.

Fig. 6 shows the voltage given to the flaps for optimal vibration control for a specific case of 3/rev actuation which is a simple approach for practical implementation on a 4-bladed rotor. The

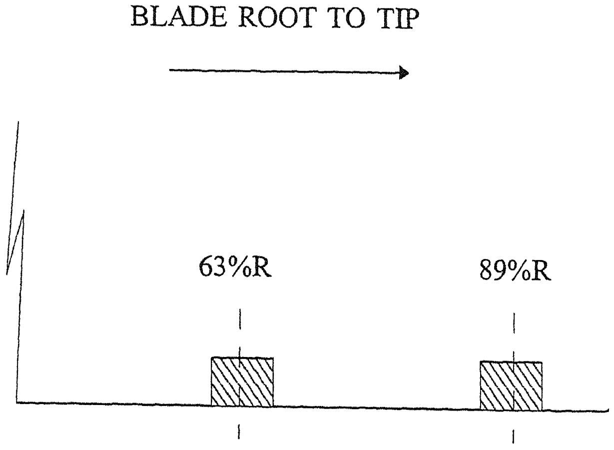

Fig. 5 – Schematic view of rotor blade with dual trailing edge flap.

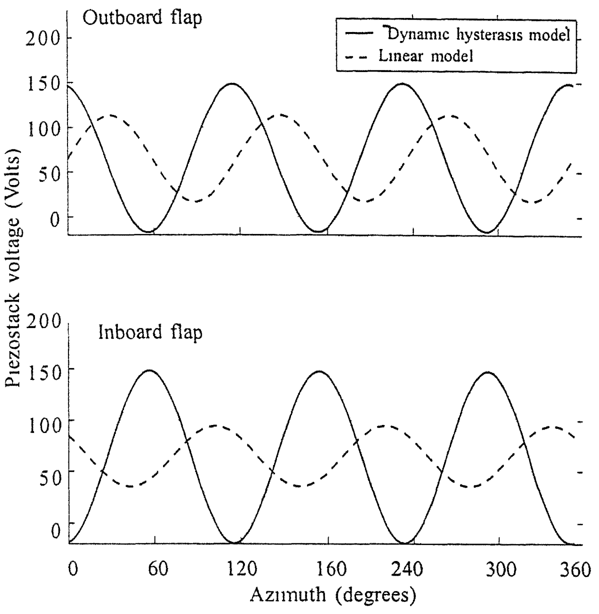

Fig 6 – Optimal 3/rev control input

results of using an idealized linear actuator model are compared with the results of using a nonlinear model with hysteresis. It is found that the linear model predictions are quite different from the nonlinear model predictions. Obviously, the use of an idealized linear piezoelectric stack actuator model would result in predictions for applied voltage which would result in poor performance for vibration control.

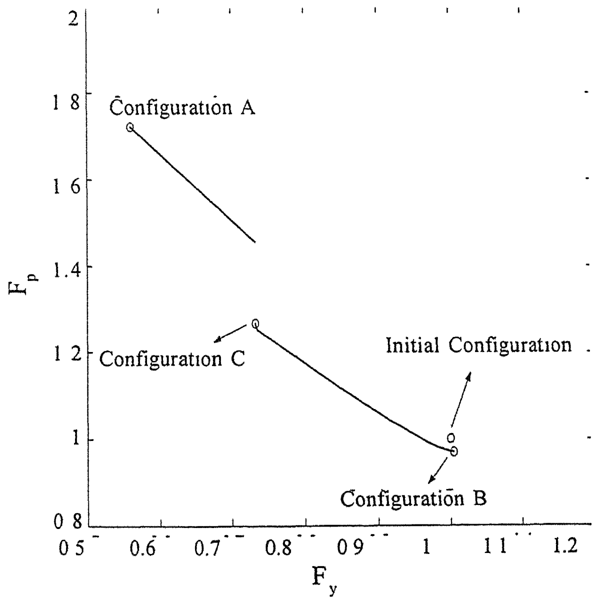

Fig. 7 – Optimal points for vibration (Fy)and flap power (Fp) minimization.

The optimal placement of two flaps along the rotor blade was studied and it was found that trying to reduce both vibration and flap power resulted in a disjoint Pareto optimal design space[15-16], as shown in Fig. 7. However, practical design points can be obtained for the optimal flap locations which also use less power. In Fig. 7, A represents the lowest vibration point, B the point corresponding to least flap power required and C the best compromise point where substantial reduction in vibration occurs while using less flap power.

The trailing edge flap motion using the APA500L actuator was demonstrated experimentally using a simple amplification mechanism. Efforts are now underway to demonstrate this system in the wind tunnel and in flight. Controllers based on neural networks and fuzzy logic provide robust approaches to handle nonlinear systems and can be addressed in future research. Uncertainty in piezoelectric material properties should also be analyzed for robust design. The potential of magnetostrictive materials for actuation should also be investigated.

Active twist control of vibration

Besides trailing edge flaps, active twist is also a possible approach for vibration reduction in helicopters. However, this approach is classified as more long term in nature due to the need to design the rotor blade using integrated smart material actuators which would also require accounting for strength and other issues. Moreover, it is more difficult to move the complete blade in a higher harmonic motion compared to moving one or two trailing edge flaps. However, the promise of an actively adapting rotor blade is great both from vibration and noise reduction perspectives. Thakkar and Ganguli[17,18] studied the possibility of using shear mode of the piezoelectric materials for twisting a rotor blade modeled as a box-beam. Early efforts on actively twisting the rotor blade used actuators at 45 degrees angles placed on the top and bottom surfaces of the rotor beam blade. In these works, the direct piezoelectric coefficient or d_{31} is used. The shear strain coefficient of piezoelectrics is much higher than the direct strain coefficient and therefore can lead to higher actuation authority.

The effect of high electric field on shear actuation of smart structures was studied by Thakkar and Ganguli[19] and it was found that the nonlinear relation of the strain coefficient with electric field could be used to further amplify the control authority of the actuators. The relationship between the piezoelectric shear coefficient d_{15} and the applied electric field E_a was found from experimental results for PZT-5H to be

$$d_{15} = 737.3411 + 159.699E_a + 102.653E_a^2$$
$$-39.006E_a^3 + 5.495E_a^4$$

The nonlinearity here occurs at a threshold value of E_a = 110 V/mm. The limiting value of the amplitude of the applied voltage is 490 V/mm for PZT-5H which was the material used in this research. Fig. 8 shows the reduction in six vibratory hub loads acting at the rotor hub using the active twist concept. These results are at a typical moderate non-dimensional speed of μ = 0.2 and show that a considerable level of vibration reduction can be obtained by a closed loop controller which exploits the piezoelectric nonlinearity at high voltages.

Thakkar and Ganguli also studied the use of new single crystal smart materials which have very high shear strain coefficients for twisting the blade. It was also found that actuation using single crystal piezoceramics could be used for dynamic stall suppression as the angle of attack across the rotor disk was actively reduced[18].

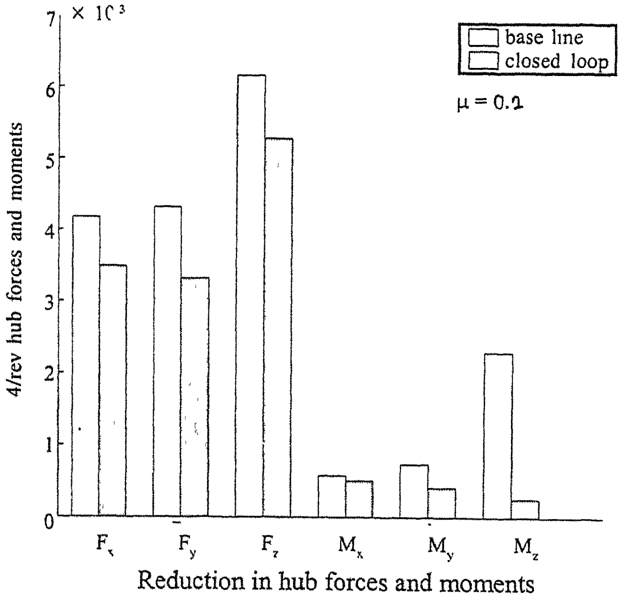

Fig. 8 – Reduction of vibratory hub loads using active twist.

For dynamic stall suppression, an objective function based on angle of attack variation around the rotor disk was proposed and the feedback controller gains were optimized to minimize this function.

A review paper on the active twist rotor concept was written[20] to provide a good background to anyone planning to study this area. Pawar and Jung[21] have recently extended the approach to dissimilar rotor blades.

As single crystal piezoceramics have become more practical materials and are now commercially available, their use in the development of smart composite rotors should be addressed. Strength design plays an important role in the active twist rotor concept and needs to be carried out.

Uncertainty quantification in nonlinear aeroelasticity

A significant contribution of research in recent years is in uncertainty quantification in rotorcraft aeroelasticity. It is known that all system modeling suffers from aleatory or random uncertainty and epistemic or model uncertainty[22]. Almost all research in helicopter dynamics focuses on improving the structural and aerodynamic modeling which is an attempt to alleviate epistemic uncertainty. However, the random uncertainties cannot be reduced by improved modeling and need to be addressed by uncertainty quantification methods such as Monte Carlo simulations.

Murugan *et. al*[23,24] made a beginning in research in this area by studying the effect of uncertainty in composite material properties on the rotor blade frequencies, blade response, vibration and aeroelastic stability. A cross section analysis was used to model the composite rotor blade and obtain the 1D elastic properties for implementing in a beam analysis. In helicopter aeroelasticity, beams undergoing flap and lag bending, torsion and axial motions are used for structural modeling as they provide accurate results in a computationally efficient manner.

Composite materials have a high level of scatter with a coefficient of variance ranging from 5-12 percent for the Young's modulus, Poisson ratios and shear modulus due to manufacturing issues. Thus, there is variation between different composite structures which needs to be accounted for in the design process. Monte Carlo simulations were used for quantifying the effects of propagating composite material uncertainty first through the cross section analysis and then through the nonlinear rotor aeroelastic analysis.

Monte Carlo simulations (MCS) are the most direct and non-intrusive approach for stochastic analysis. MCS involves the generation of a large sample of random points where the deterministic simulation code is run. Typically, the probability distribution functions of the input variables are needed and the probabilities of the output variables provide the required uncertainty behavior of the system. A key aspect in MCS is the number of sampling points at which the simulation needs to be run. Monitoring the convergence of standard deviation of a key output variable is one way for estimating the sample size. Fig. 9 shows the standard deviation of the blade flap response with the number of sampling points. It is clear that a sample size of 5000 provides a good estimate of the stochastic effects.

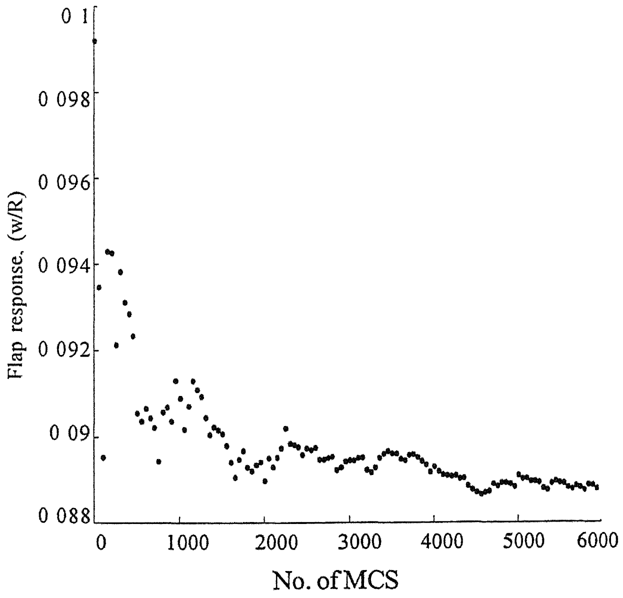

Fig. 9 – Convergence of Monte Carlo simulation results with increasing number of sampling points.

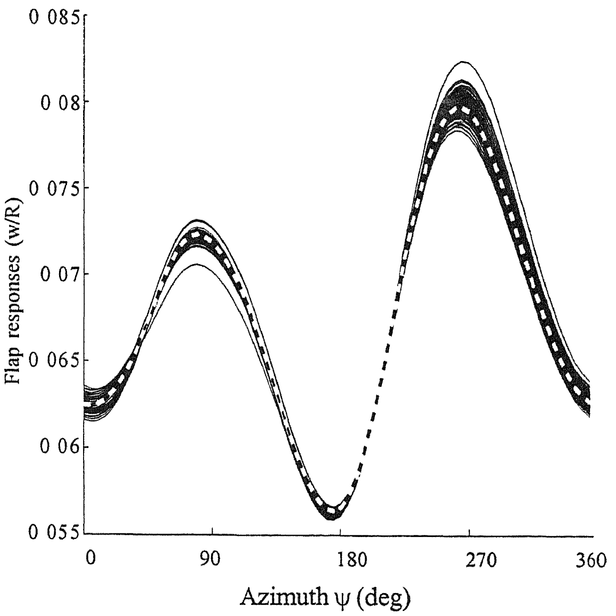

Fig. 10 – Monte Carlo simulations of flap bending response of rotor blade

Fig. 10 shows the dispersion in flap bending tip response around the rotor disk. There is considerable dispersion in the predictions from the baseline deterministic value. Fig. 11 shows the bounds on the torsion response obtained through MCS. It can be observed that there is considerable change in both the magnitude and phasing of the torsion response due to uncertainty in composite material properties. Torsion or elastic twist is very

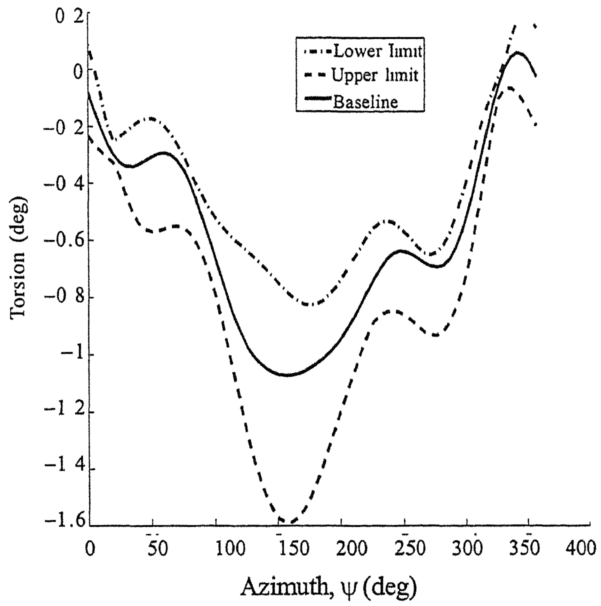

Fig 11 – Bounds on the torsion response from Monte Carlo simulations important in aeroelasticity, since any change in elastic twist affects the blade section angle of attack which can dramatically change the airloads. It should also be noted that helicopter rotor blades are very flexible structures and this leads to significant stochastic effects due to uncertainty propagation.

Fig. 12 shows the first harmonic of the torsion response of the blade. Fig. 13 and 14 show the probability density function of the sine and cosine components of the first harmonic torsion response, respectively. The appearance of non-Gaussian density function involving dual peaks is seen. Such behavior is typical of the propagation of uncertainty through nonlinear systems and clearly shows the importance of considering stochastic methods in rotary wing aeroelasticity.

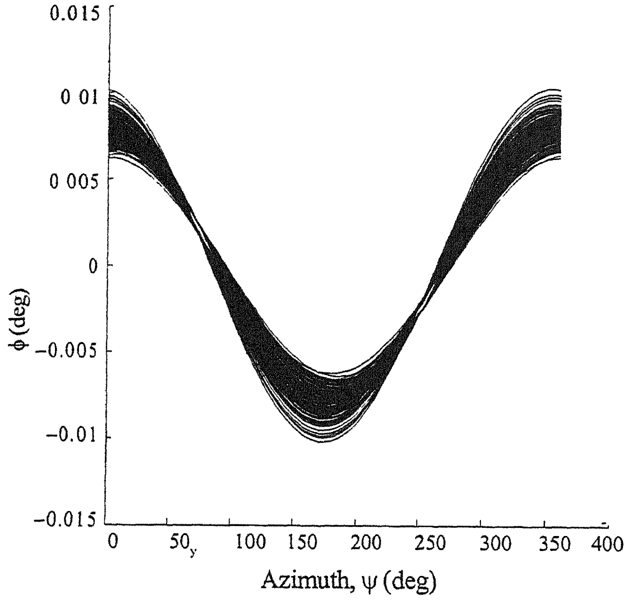

Fig. 12 – Monte Carlo simulation of the first harmonic of torsion response

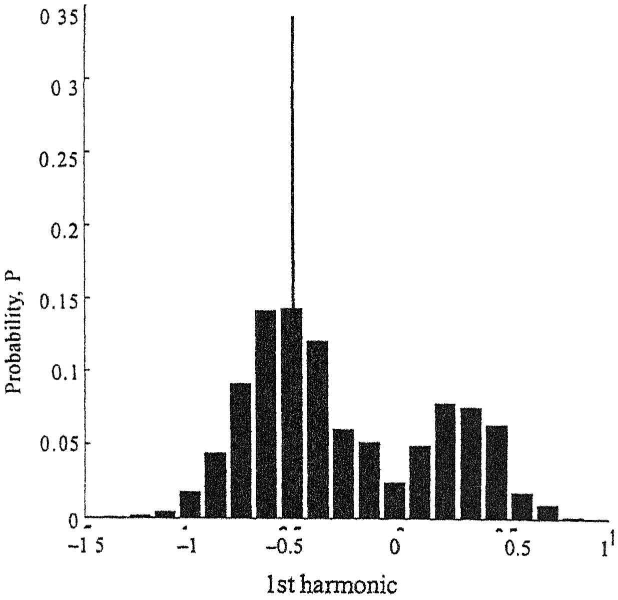

Fig. 13 – Probability density function of 1st sine harmonic of torsion response

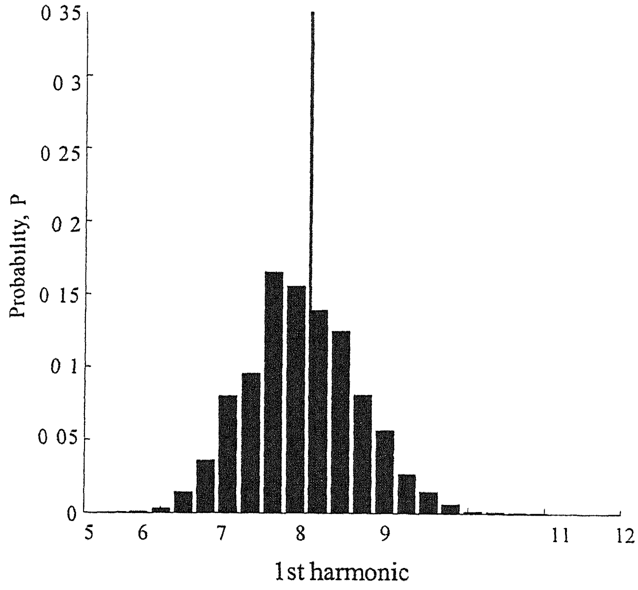

Fig 14 – Probability density function of 1st cosine harmonic of torsion response

It was also found that the effect of uncertainty is considerable, particularly on vibration and stability[24]. In fact, vibration predictions show a high level of sensitivity to uncertainty as they come from higher harmonics of the rotor response and blade loads, which are themselves very sensitive to uncertainty. The lack of good predictions of helicopter aeroelastic analysis with experimental data may not be only due to modeling deficiencies of an epistemic nature but could also come from uncertainty in the material, geometric, aerodynamic properties as well as in the boundary conditions. The influence of uncertainty sources in addition to material properties should be investigated in the future.

Monte Carlo simulations were found to be very computationally expensive for the helicopter aeroelastic analysis problem. Future work should address more efficient approaches such as the use of metamodels for the aeroelastic analysis and polynomial chaos expansions. Stochastic finite element analysis can also be used to reformulate the rotor aeroelastic analysis and lead to fast analysis which is important for reliability based design optimization.

Smart flapping wings

Recently, there has been a serious effort to design a class of very small flight vehicles called micro air vehicles (MAVs) as their applications range from the military, surveillance in restricted spaces, planetary exploration, search-and-rescue to many more[25]. Essential requirements of such vehicles are power efficiency, high maneuverability and low-speed flight. Nature provides flapping flyers such as birds and insects which represent a very successful design for intelligent MAVs with much better performance than conventional wings and rotors. Hence, birds and insects serve as a natural source of inspiration for the development of MAV. For a flapping-wing MAV, the wings are not only responsible for lift, but also for propulsion and maneuvers. Therefore, MAV flapping wing design represents one of the major challenges to efficient flight in the low Reynolds-number regime.

Biomimetic flapping wing mechanisms are used to achieve a deeper insight as well as qualitative and quantitative comprehension of flapping flight. However, these dynamically scaled flapping wing mechanisms may not be suitable for use in small or micro-scale flying vehicles as they are bulky and flap at very low frequency. Moreover, current flapping wing mechanisms rely on pneumatic and motor-driven flapping actuators which lead to high weight and system-complexity[26]. Natural flapping flyers generate lift and thrust using complex wingbeat kinematics which cannot be easily mimicked with these conventional actuators. Another plausible alternative may be to use actuators made of smart materials.

Smart materials, especially piezoelectric materials, are widely used in smart structures as sensors and actuators because they have several attractive features such as high bandwidth, high output force, compact size and high power density. However, for creating the large displacements needed for flapping, some kind of motion amplification mechanisms are necessary in order to obtain large deflection because the piezoelectric effect is intrinsically small and leads to a small deflection when extracted directly from the bending piezoelectric unimorph/bimorph. Piezoelectric fan (piezofan) is one of the simple motion amplifying mechanisms. Piezofan couples a piezoelectric unimorph to an attached flexible wing and is capable of producing large deflection, as high as 47^0, especially at resonance[27]. Piezoelectric fans are popular as a very compact, low power, noiseless air cooling technology for portable electronic devices such as cellular phones, DVD players, laptop computers etc. However, there accurate modeling for flapping wing applications is a challenge and needs nonlinear modeling.

Mukherjee and Ganguli[28] used an energy method to derive the non-linear governing equations of motion of a smart flapping wing. Flapping wing is actuated from the root by a PZT unimorph in the piezofan configuration. Dynamic characteristics of the wing, having the same size as dragonfly *Aeshna Multicolor*, are analyzed using numerical simulations. It is shown that flapping angle variations of the smart flapping wing are similar to the actual dragonfly wing for a specific feasible voltage. An unsteady aerodynamic model based on modified strip theory is used to obtain the aerodynamic forces. It is found that the smart wing generates sufficient lift to support its own weight and carry a small payload. It is therefore a potential candidate for flapping wing of micro air vehicles.

Fig. 15 shows the schematic representation of the piezoelectric actuated dragonfly like flapping wing. Here the wing moves like a fan at relatively high frequencies and is thus able to generate high levels of lift. Nonlinear modeling is required as the deflections can become quite large as shown in

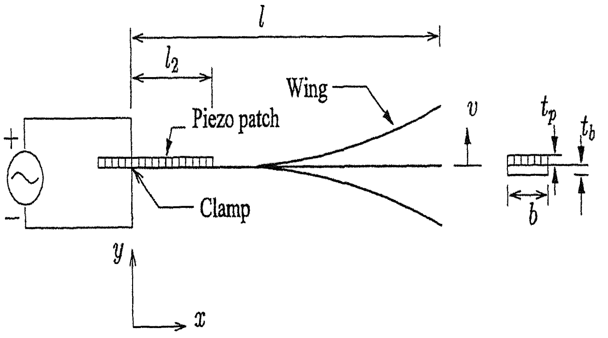

Fig. 15 – Schematic of piezoelectrically actuated dragonfly inspired flapping wing

Fig. 16. Here, the flapping angle is of the order of 30 degrees which is too high for a linear analysis. Fig. 17 shows the flapping angles obtained with the piezofan and compares it with experimentally measured data for a dragonfly wing. It can be seen that the piezofan is capable of generating large levels of deformation and can achieve flapping angles similar to those of a dragonfly. The dragonfly results were experimentally obtained by Zeng *et. al.*[29]. Finally, Fig. 18 shows that there is sufficient net lift generated by two wings to support a small payload. Therefore, the dragonfly inspired piezofan based flapping wing is a viable candidate for micro air vehicles.

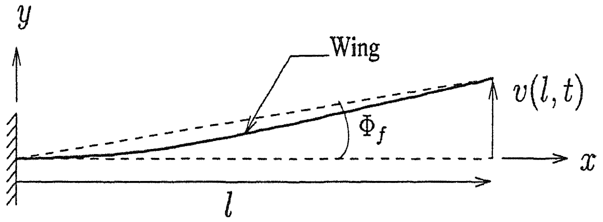

Fig 16 – Schematic diagram of calculating flapping angle.

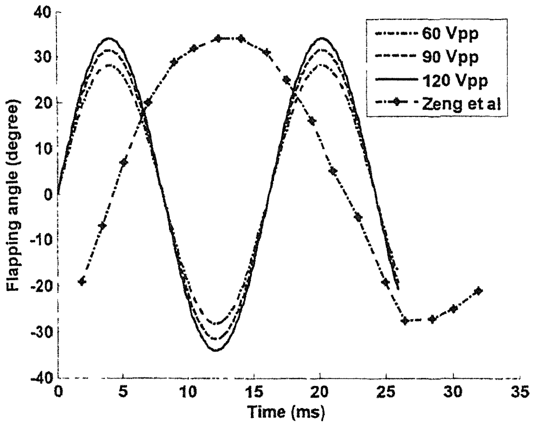

Fig.17 – Comparison of flapping angle variations with experimental data.

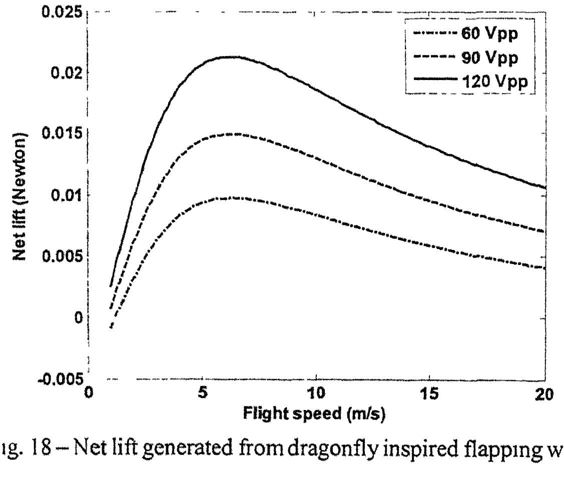

Fig. 18 – Net lift generated from dragonfly inspired flapping wing.

The flapping wing problem is a very interesting and difficult problem in aeroelasticity. Further work should include torsional motion and computational fluid dynamic modeling for better predictions. Stability analysis of the nonlinear system should be conducted to identify possible unstable modes. A prototype development is also needed to take the concepts into the real world.

Closing Remarks

A brief discussion of some of the new developments in nonlinear aeroelasticity is given in this paper. The following closing comments are made from this review.

1. It can be observed that smart materials are a key source of nonlinearity. In the active twist rotor, the nonlinear dependence of the piezoceramic shear coefficient d_{15} with voltage allows larger levels of actuation.

2. However, for the trailing edge flap device, hysteresis nonlinearity causes problems in controller performance and requires the use of hysteresis compensation algorithms or controllers developed using aeroelastic analysis models with embedded dynamic hysteresis models.

3. The propagation of random material properties through the nonlinear aeroelastic analysis was found to give considerable dispersion in response, vibration and stability.

4. Flapping wings need to be highly flexible structures which move at high frequencies. However, nonlinear modeling becomes necessary for such structures. There is a need to address nonlinear stability and control issues for flapping wing vehicles. Accurate modeling is the first step in that direction.

The research discussed in this paper showed considerable influence of nonlinear effects in aeroelasticity even though the problems considered steady state periodic motion. Transient response needs to be studied as it can show chaotic behavior.

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