Silo Music and Silo Quake: Granular Flow Induced Vibration

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Abstract

Acceleration and sound measurements during granular discharge from silos are used to show that silo music is a sound resonance produced by silo quake. The latter is produced by stick-slip friction between the wall and the granular material in tall narrow silos. For the discharge rates studied, the occurrence of flow pulsations is determined primarily by the surface properties of the granular material and the silo wall. The measurements show that the pulsating motion of the granular material drives the oscillatory motion of the silo.

1 Introduction

The discharge of granular materials from silos is often characterized by vibrations or pulsations of the silo, termed 'silo quake', and a loud noise, termed 'silo music' [8,9,10, 14,19,23, 27,30,31]. Both of these are undesirable as silo quake may cause structural failure and silo music is a source of noise pollution. Unfortunately, the numerous conflicting studies published in the literature [8,9,10, 14,19,23, 27,30,31] do not give the silo designer a simple model to understand the physical processes that cause the pulsations, and to guide silo design or modification that would prevent the pulsations or at least minimize their effect. The purpose of this study is to investigate the cause of the noise and the pulsations, and the interaction between the motion of the granular material and the motion of the structure.

Several studies of the discharge of granular material from silos have noted fluctuations in discharge rate and the production of noise and vibration [8,9,10, 14,19,23, 27,30,31]. The top of the granular material has been observed to move in discrete steps even though the discharge from the bottom of the silo was continuous [8,23]. For smooth-walled, tall, narrow silos, pulsations occurred during both mass and mixed flow. The pulsations were observed to stop at a critical height of granular material in the silo [9,26,]. Methods suggested for preventing pulsations include roughening the walls in the transition zone between the bunker and the orifice [9,14,26] and placement of inserts along the silo walls [8].

In an early study, Phillips [23] observed the motion of sand in a tube which had a glass face, and was closed at the lower end by a flat bottom having a central orifice. When the orifice was opened, the sand in the upper part of the tube moved downward intermittently in jerks. Phillips noted, "when the flow begins, a curious rattling sound is heard which changes to a distinct musical note". He also did experiments in which the tube was first partly filled with mercury and then filled with sand. Once again, the free surface of the sand descended intermittently when the mercury was allowed to flow through the orifice. He observed that the length of the column of sand increased by about 2% during the 'stick' phase. Further, the motion of the granular material caused the wall of the tube to vibrate. Thus both silo music and silo quake occurred in his experiments, and he suggested that the stick-slip motion of the sand may be responsible for these phenomena.

Some recent studies have suggested that the pulsations are intensified by a resonant interaction between the granular material and the silo structure [10,19,26,28]. However, in one of these studies, Tejchman [26] also noted that the magnitude and presence of the flow pulsations was influenced by environmental factors such as temperature and electrostatic effects, which suggests that while resonant interaction can intensify the pulsations it is not the only requirement for pulsations to occur. Olvarez and Clément [20] have also observed that the humidity, an environmental factor, can have a strong influence on the sliding motion of a slowly pushed granular column. They find that particle and wall frictional properties are important in determining stick-slip motion in granular systems.

Hardow et al. [8] conducted experiments on a silo in which the natural frequencies of the silo were significantly greater than the pulsation frequency. Thus pulsations occurred even in the absence of resonance. Hardow et al. [8] have proposed that the motion of the silo is driven by the rapid acceleration and deceleration of the granular material in the bin section, caused by the stress fluctuations in the granular material in the hopper section. As the granular material in the hopper region deforms, there are periods where the mass of granular material in the bin is not supported and collapses in a downward step creating a large impulse, which shakes the silo structure. This study observed pulsations during core flow in a silo that was 6 m high, 0.6 m deep and 1.2 m wide, and hence the flow kinematics were considerably different from those in tall narrow silos.

Wensrich [30,31] has suggested that these pulsations are due to compression and dilation waves in the granular material, which are created by stick-slip motion between the granular material and the silo walls. However, pulsations have also been observed in funnel flow bunkers, where the granular material at the walls does not slip during discharge [8]. Wensrich [30,31] has suggested that the pulsation creation mechanism is entirely different in funnel flow, but does not give evidence to support his conjecture.

Finally, Moriyama and Jimbo's [14] findings suggest that the magnitude of the pulsations is determined by how the granular material changes from a compressed state in the bunker to a dilated state in the hopper. Moriyama and Jimbo [14] also found that the likelihood of a silo discharging with pulsations was dependent on the method used to fill the silo. They did not propose a physical mechanism to explain their observations.

The aim of the present study, which is largely experimental, is to obtain a mechanistic understanding of silo music and flow pulsations. Through a combination of sound, bed height, and acceleration measurements, it is shown that silo music is driven by the stick-slip pulsating motion of the granular material during discharge and is associated with a sound resonance in the air column above the bed. Different wall and granular materials have been used to probe their role on flow pulsations and silo music during silo discharge.

2 Related studies on stick-slip friction in granular materials

To explain the rationale in the choice of experimental measurements, it is worth reviewing the generally understood kinematics of the discharge of granular material from a bin or hopper and relating these to stick-slip friction in granular materials. Experiments show that in a tall, flat-bottomed cylindrical bin, with walls having a lower friction coefficient than the internal friction angle of the granular material, there is a region of plug flow at the top of the full silo. As the silo empties, the size of the plug flow region decreases and eventually all of the flowing material is in converging flow. The discharge rate from the bin is independent of the height of material in the bin, provided the height is greater than a few multiples of the diameter of the orifice [18] and scales as $g^{1/2}D^{5/2}$, where D is the orifice diameter and g is the acceleration due to gravity.

Radiographic studies of slow dense granular flow in model bunkers show that velocity discontinuities exist at the transition from the bin to the hopper [1]. Measurements in a discharging bunker indicate that there is a dynamic arch at the transition where the nature of the material flow changes from one without deformation (above the arch) to one where material deforms (below the arch) as it approaches the orifice [22]. Pressure measurements [8, 10,14] near the transition from the bin to the hopper indicate that there is also a stress discontinuity [22] and that there can be large pulsating stresses, which correspond to the cyclical formation and breakage of the dynamic arch. This pulsating behavior only occurs for dense assemblies [22] and is very similar to silo quake.

The experiments that identified the dynamic arch [22] were conducted in a bunker where the bin to hopper transition determined the location of the dynamic arch. In a flat-bottomed silo (such as the one used in this study) the stagnant material adjacent to the orifice creates a hopper-like region. So the discharge from a flat-bottomed silo may be expected to show many of the features observed in bunkers. If the density of the material in the plug flow region above the dynamic arch is high, it must dilate as it crosses the arch in order to deform in the hopper-like region.

Nasuno et al. [17] carefully studied stick-slip motion in granular materials using a simple shear device with 70 - 110 μm glass beads and 100 - 600 μm sand. They observed stick-slip motion at low average slip rates, which became continuous at very large average slip rates. They also observed that at very small driving velocities, the period of stick-slip fluctuations was inversely proportional to the driving velocity. For glass beads, the system fluctuated with near constant period, while for sand, the period varied stochastically. As the sliding velocity was increased, the period became independent of velocity, and finally at large sliding velocities, the motion became continuous. In the simple shear experiments done by Nasuno et al. [17], the spring constant connecting the driving piston to the sliding mass was varied. The spring constant influenced both the pulsation frequency and the critical driving velocity at which the pulsation frequency became independent of the driving velocity.

Nasuno et al. [17] also observed that a lengthy period of slow vertical dilation preceded rapid slip events in the horizontal direction. This dilation was carefully measured by Geminard et al. [7]. Their experiments suggest that in the shear zone particles climb slowly over each other. Once the particle is approximately 5% of a particle diameter over the particle below it, slip occurs and the top layer jumps forward before slowing down again and settling into another zone of particles [7]. In the experiments of Geminard et al. [7], the particle volume fraction was indistinguishable from the random close packed volume fraction of 63%.

These studies show that granular materials can undergo stick-slip motion, and that this can couple with the mechanical system (for example, a mass-spring system) in a complicated fashion, which depends on the system parameters. Together these studies suggest that stick-slip motion can occur in tall flat-bottomed silos during the discharge of granular materials. Excellent reviews on stick-slip friction can be found in Bowden and Tabor [5], Krim [11] and Berman et al. [3] who discuss the various postulated stick-slip friction mechanisms. The mechanism of most relevance to the present study is adhesive stick-slip friction, which occurs when there are slowly weakening, time-dependent forces between sliding surfaces. The hypothesis in this study is that adhesive stick-slip friction is the determining factor in cyclical dynamic arch formation and breakage, which creates impulses that drive the silo structure.

3 Experimental method

Aluminum, plain steel, and acrylic tubes, open at the top and covered at the bottom with a flat acrylic plate having a concentric orifice (see below for details of this plate), were used as silos. A number of experiments were conducted using silos resting on supporting springs (see figure 1), which in turn were attached to a steel frame that was rigidly connected to the laboratory walls. The silo was also equipped with rollers and sliders, which were attached to the steel frame. These allowed vertical oscillation of the silos and restricted lateral motion. The supporting springs had spring constants ranging from 4 to 2265 N/mm. Experiments were also done using an aluminum block in place of the spring, or simply bolting the silo directly to the supporting steel frame – these configurations afforded the two largest natural frequencies for vertical silo oscillation reported in this study. Properties of the tubes and granular materials are listed in tables 1, 2 and 3 respectively. Photographs of the granular materials, obtained using a microscope, are shown in figures 2a, 2b and 2c. The granular materials did not exhibit squeaking or booming when sheared. The temperature and humidity were recorded in each experiment. The temperature varied between 20˚C and 25˚C (from one day to another), and the relative humidity between 18 and 40%. During experiments with each tube and granular material combination (which lasted a few hours) the humidity variation was within 5% and the temperature variation was within 2˚C.

The angle of internal friction of each granular material was estimated by measuring the angle of repose of the granular material between two plane walls 1.9 cm apart. These values are in table 2.

The angle of wall friction for each granular material and tube combination was estimated by measuring the angle of inclination of the tube above the horizontal at which the granular material began to slide; these values are in table 3. This procedure did not allow an estimate of the angle of wall friction for the plain steel tube because the granular material began to slide over itself before it slid at the wall. Extensive measurements of the angle of wall friction for each tube and granular material combination were not made because studies of stick-slip friction [17] show that the frictional properties vary with stress level and with time. Given the time constraints for performing the study, it was not possible to do experiments similar to those of Nasuno et al. [17] to characterize the frictional properties of each granular material and tube wall combination. The experiments were therefore designed to test whether stick-slip friction was present based on the experimentally determined characteristics.

To determine if resonance (that is when the pulsation frequency f_p is equal to a natural frequency of the silo) was important, the dominant natural frequency of vertical silo oscillations, f_v, was changed by using different springs between the silo and the steel frame. The lowest natural frequencies of the empty silo tube were estimated using the method of Naeem and Sharma [16] was used with clamped-free boundary conditions. The lowest natural frequency estimates for each tube are given in table 1.

Acrylic plates with centrally located orifices (with diameters between 1.3 and 2.5 cm) were bolted to a 1.21 kg aluminum flange, which was screwed on the bottom of the tubes. To fill the silo, the orifice at the bottom of the tube was first sealed with a piece of duct tape. The granular material was poured into the silo through a funnel placed at the top of the tube. Stripping away the duct tape seal over the orifice initiated discharge. The mean discharge rate was measured using a stopwatch. For the acrylic tube, the height of material in the silo could also be measured during discharge to confirm that the discharge rate was constant with time. To ensure that the tubes had reached a steady state of wear, the granular material was discharged several times through the same tube before final measurements were taken. Steady state wear was reached when repeatable granular material acceleration measurements could be taken.

Accelerations were measured both in the granular material and on the silo structure. Vertical accelerations inside the granular material were measured using a unidirectional Kistler 8774A50 low-impedance ceramic shear accelerometer with an output sensitivity that deviated less than 1.5 % for frequencies between 10 Hz and 10 KHz. The accelerometer was embedded approximately 5 cm below the top surface of the granular material. This depth ensured that during discharge the accelerometer was held upright by the granular material and was still shallow enough that the acceleration could be measured for the bulk of the discharge. As the granular material discharged, the accelerometer cable was carefully fed into the silo to ensure that the cable did not affect the motion of the accelerometer. This accelerometer had a range of $\pm$ 500 m/s^2 and was accurate to within $\pm$ 5 m/s^2. It had a diameter of 0.8 cm, a length of 2.6 cm, and a mass of 4 g, and so was considerably larger than a sand grain. However, the wide frequency response allowed better time resolution of the bulk granular material acceleration than would be possible with smaller accelerometers of comparable cost.

Silo structural vibrations were measured using a Kistler 8784A5 low-impedance ceramic shear accelerometer, which had a greater sensitivity but a smaller range than the accelerometer used to measure granular material accelerations. To measure vertical accelerations, this accelerometer was wax mounted on the flange at the bottom of the silo. This accelerometer had a sensitivity that varied by less than 0.5 % for frequencies between 10 Hz and 6 KHz. It had a range of $\pm$ 50 m/s^2, an accuracy of $\pm$ 0.5 m/s^2, and a mass of 21 g.

The accelerometer output was sent through a Kistler 5118B2 signal conditioner to a Measurement Computing PCI-DAS1002 data acquisition card on a 400 MHz Pentium II computer. The sampling rate on the data acquisition card was 20 KHz. For both accelerometers, the manufacturer supplied calibration was used to convert the accelerometer voltage output to acceleration. The accelerometers could not be used simultaneously because only one data acquisition system was available.

The bulk of the sound measurements were taken in an apparatus made from an acrylic tube, for which the resonant frequency for vertical silo oscillations was not well controlled [24]. However, several measurements were then repeated in the experimental setup used for the acceleration measurements to check that the same results were obtained. In these experiments an omnidirectional Optimus 33-3026 lapel microphone with a constant amplitude response for a frequency range between 30 Hz and 15 KHz was used to collect the sound data through a sound card on a personal computer. During discharge, the sound was recorded and a discrete Fourier transform of one second of sound data was used to determine the dominant frequency as a function of time during discharge. In the acrylic tube, the time at which the top of the granular material crossed a marked height in the tube during discharge was also recorded using a stopwatch. From these measurements the height of the granular material as a function of time since discharge started was found.

4 Results

Sound measurements are shown for sand discharging from the acrylic tube, and acceleration measurements are shown for crushed glass and glass beads discharging from the aluminum tube.

Additional sound and acceleration measurements are reported in Quinn [24] and Muite [15], respectively.

The variation of the pulsation frequency (f_p) with the dominant natural frequency for vertical oscillations (f_v) is examined for all tube and granular material combinations, except for the plain steel tube, as pulsations did not occur in this tube. Silo pulsations also did not occur when sand was discharged from the aluminum tube, but did occur when sand was discharged from the acrylic tube. A few experiments with a smooth walled galvanized steel tube [15] showed that silo pulsations occurred when sand was discharge from this tube. This suggests that in tall and narrow silos, pulsations occur for particular granular material wall combinations. This is in agreement with studies which show that stick-slip friction depends on the composition of the sliding surface [3,5,11].

4.1 Sound measurements

Figure 3 shows the sound amplitude level as a function of time for the discharge of sand from the acrylic tube. No units are given because, although the amplitude is a direct voltage reading from the microphone that is linearly related to the sound decibel level, by moving the microphone, different absolute decibel levels can be recorded for the same sound signal. The discharge lasted for 51 s and as shown in the figure, silo music occurred for approximately half of this time. Figure 4 shows a typical power spectrum for the sound measurements during discharge, determined by analyzing data obtained over a 1 s time interval. There are three types of prominent peaks. The first peak is at a frequency of approximately 40 Hz and it will be shown later that this is the pulsation frequency for this particular granular material and silo combination. The second peak corresponds to the resonant frequency of the air column above the tube. This resonance is well documented and a good account can be found in Rayleigh [25]. At the time the data shown in the figure was collected, this frequency was 200 Hz. The fact that this peak represents a resonance frequency is demonstrated in Figure 5, which shows the quarter wavelength corresponding to this frequency as a function of time since the beginning of discharge. The wavelength, λ_a, is found from the relationship $\lambda_a = c/f_\mathrm{a}$ where c is the speed of sound in air and f_a is the frequency of the air column. Also shown is the height of the air column above the sand in the tube. This figure shows that the dominant quarter wavelength and the height of the air column are the same confirming the resonant behavior. It is clear from the *quarter* wavelength, that this resonance corresponds to a standing wave mode with a node at the granular material surface, and an anti-node at the open end of the tube (as the open end of the tube cannot be a node). Figure 6 also shows a number of other peaks at higher frequencies, which are simply the odd harmonics of the fundamental (lowest) resonance frequency of the air column.

4.2 Determination of the natural frequency for vertical oscillations of the silo

To determine the dominant natural frequency for vertical oscillations, f_v, the silo was filled with granular material and the orifice closed. The base of the filled silo was then struck with a soft mallet and the resulting acceleration during free oscillations recorded. The value of f_v was found either by using the largest peak in the power spectrum of the acceleration, or by counting the number of free oscillations during a specified time directly from the acceleration measurements. The two measurements gave essentially the same results; however, when f_v < 30 Hz, counting the number of oscillations in a specified time gave a more accurate measurement of the natural frequency than locating the center of the broad peak obtained from the power spectrum. Similarly, when the f_v > 30 Hz the power spectrum was a better indicator of the natural frequency because an unambiguous sharp peak could be located, while the acceleration vs. time trace showed rapidly decaying oscillations which were not easy to count. For spring constants, $k < 1000$ N/mm, $f_v \approx f_n = (1/2\pi)(k/m)^{1/2}$, where m is the oscillating mass and f_n is the theoretical natural frequency for a spring mass system. For $k > 1000$ N/mm, f_v was significantly less than f_n, possibly because of flange and tube deformations, which reduced the effective stiffness of the system. This effect was important for $f_v > 25$ Hz.

4.3 Acceleration measurements during discharge

Figure 6 shows measurements of the vertical acceleration of the silo when crushed glass was discharged through a 1.9 cm orifice. The accelerometer was mounted on the base of the silo. Once the flow started, there was a period of pulsations during which the silo experienced large negative accelerations towards the earth. Half way during the pulsations, the magnitude of the negative pulsations suddenly doubled. After the granular material fell below a critical level, the pulsations stopped, and the silo structure experienced only small accelerations until the flow ended. While the pulsations occurred regularly, this doubling of the pulsation magnitude was not always repeatable. It is not clear what changes in the flow resulted in these changes in the magnitude of the pulsations because the basic setup was unchanged from run to run.

The close-up of the acceleration measured during pulsations, figure 7, reveals that the periods of large negative accelerations were short compared to the gradual rebound after each pulsation. On this time scale, the pulsations had a very reproducible and steady frequency, but the absolute magnitude of the maximum acceleration varied from pulse to pulse.

Figure 8a shows measurements obtained with the accelerometer buried in the granular material. The flow conditions were the same as in figure 7, except that the orifice diameter was 1.3 cm instead of 1.9 cm. Also as in figure 7, negative accelerations are towards the earth. Figure 8a shows that large positive accelerations occurred in the granular material during pulsations, while figure 6 shows that the silo experienced large negative accelerations. The two figures show that during each pulsation, the granular material fell a short distance and impacted the tube wall and flange bottom.

The close-up of the pulsations in figure 9a shows that despite the difference in flow rate, $f_p = 30$ Hz for discharge of granular material through a 1.3 cm orifice (figure 7) and for discharge through a 1.9 cm orifice (figure 9a). Figures 7 and 9a also show that the pulsation frequency in the granular material is the same as the frequency with which the silo moves, suggesting that the motion of the granular material drives the motion of the silo. Figure 9a also shows that each pulsation was followed by a negative acceleration within the material and then a second large positive acceleration, after which the acceleration of the granular material was close to zero until the next pulsation. Figure 10a shows a power spectrum for the acceleration measured during 1 second of pulsations in the crushed glass. It has a peak at $f_p = 30$ Hz followed by a flat band region between 200 and 1000 Hz after which the power spectrum decays.

Figures 8b, 9b, and 10b are similar to figures 8a, 9a and 10a, but are for glass beads discharging through a 1.9 cm orifice. In figure 8b there is an acceleration spike before flow starts because a tap of the tube caused the granular material to consolidate after the tube was filled, but before discharge had begun. The pulsations again stop at a critical height and the individual pulsations can be seen in figure

8b. The nature of each pulsation for glass beads (figure 9b) is a little different than for the crushed glass (figure 9a) and this is reflected in their power spectra, (figures 10a and 10b). Both spectra have the same high frequency decay for frequencies above 1000 Hz; however, for frequencies below 1000 Hz the glass beads have a larger number of distinct harmonics than the crushed glass. The crushed glass power spectrum is typical of white noise with a high frequency cutoff, while the glass bead power spectrum is typical of a signal produced by a well correlated periodic, but non-sinusoidal function [2].

Surprisingly, all the power spectra for acceleration measurements inside the granular material for all tube and granular material combinations that pulsated decayed for frequencies above 1000 Hz. The high frequency cutoff of 1000 Hz was neither due to any limitation of the accelerometer (which could measure frequencies up to 10 KHz) or the lowest natural frequency of the tube (which was varied in these experiments and did not affect the high frequency cutoff). It may be related to the tube diameter, or to the particle size and particle density, which were not varied in the experiments.

In figures 7, 9a, and 9b the maximum downward accelerations of the silo and particles are roughly comparable, whereas the maximum upward acceleration of the granular material is significantly greater than that of the silo. This suggests that during each pulsation, the granular material slips past the silo walls and is forced to rest over a very short time period. This impact creates a shock wave that travels through the granular material and is recorded as the large upward acceleration. The granular material and silo then move together so that the resulting accelerations are of similar magnitude.

4.4 Dependence of the pulsation frequency of the granular material on the natural frequency of vertical silo oscillations

Figures 11 and 12 show the variation of the pulsation frequency (f_p) for different granular material and silo wall combinations, as a function of f_v. To determine f_p, peaks per unit time above a certain threshold in the acceleration time data were counted. The threshold was determined by looking at the acceleration time trace and picking a threshold value approximately equal to a half of the maximum acceleration. The threshold was adjusted depending on the type of acceleration time graph to ensure the correct periodicity was obtained. In particular, by comparing figures 9a and 9b one observes that if the threshold is too low, a higher periodicity would be measured in some experiments because the "aftershock" would also be included. Similarly, if the threshold is set too high, some quakes could be missed because as shown in figures 8a and 8b, the peak amplitude could vary during quaking. The pulsation frequency was determined for each second of flow pulsations and an average pulsation frequency during pulsating discharge obtained. The standard deviation in the average frequency measured during a single discharge was typically less than 10%.

When f_v < 25 Hz, f_p had no dependence on f_v as shown in figures 11 and 12. Figure 12 also shows that doubling the orifice diameter and hence increasing the discharge rate by a factor of 6 had a negligible effect on the pulsation frequency. (Doubling the orifice diameter gives a six-fold increase in discharge rate in a silo of constant cross sectional area, because the discharge rate is proportional to the orifice diameter to the power 2.5 [18]). When f_v > 25 Hz, f_p had a positive correlation with f_v for all granular material tube wall combinations that pulsated, except for the acrylic and crushed glass combination. The figures show that glass beads have similar frequency behavior in the acrylic and aluminum silos. Crushed glass has a lower pulsation frequency in the aluminum silo as compared to the acrylic silo. As sand did not pulsate during discharge from the aluminum silo, no data points are shown.

4.5 Critical height

The critical height (H_c) is taken as the height of the granular material above the base of the silo at which pulsations stop. The time at which this occurred was recorded from the acceleration measurements, and as the discharge rate was independent of time, the critical height could be calculated. This method gave critical heights that were in agreement with direct measurements made for the transparent acrylic silo. For $f_v < 25$ Hz, H_c did not vary by more than 0.1 m when the orifice diameter and spring constant were changed. For $f_v > 25$ Hz, the variation of H_c with silo and granular material properties was not closely examined.

For all the granular materials, the values of H_c for the acrylic silo were smaller than those for the aluminum silo (table 4). In the aluminum silo, crushed glass had a significantly smaller value of H_c than glass beads. In the acrylic silo, all three granular materials had similar values of H_c.

As frictional properties can depend on stress level [17], an experiment was performed in the aluminum silo where the top of the granular material was loaded with a known weight after the silo had been filled. As explained in Vanel et al. [29] and Ovarlez et al. [21], such a test may yield different stress transmission characteristics for different experimental procedures. However it can also increase the stress level inside a granular material. The weights were placed on top of the center of the granular material in the filled silo and did not touch the walls of the silo. The values of H_c and f_p were calculated using time and acceleration measurements during discharge. The experiments showed that f_p was independent of the overload. For glass beads, H_c did not vary with overload (figure 13). However, for crushed glass, H_c decreased linearly with as the overload increased. Experiments in the acrylic silo gave similar results.

5 Discussion

In this section, a mechanism for the production of pulsations is suggested. The results are then compared with those obtained in previous work on pulsating granular materials and some suggestions for further work are made.

5.1 A mechanism for producing silo-quake

Using the background on stick-slip friction in granular materials, one can compare the experimental observations in this study with those in previous studies to qualitatively explain the physical mechanism for stick-slip motion. The dynamic arch is a force chain – that is, a fragile network through which stresses are transmitted [6]. The arch is fragile, and consequently when the material below it has discharged enough so that the arch is unsupported from below, the slow creep typically observed in adhesive stick-slip begins. After a long enough time, adhesive friction forces are sufficiently weak that the arch will break. Once the arch breaks, complete slip occurs, a quake is observed and a new arch is created. This quake can setup structural vibrations of decaying amplitude that then collapse the new arch; thereby creating a system with self-sustaining pulsations. This is the pulsation process observed in the present study, where the discharge rate is *fast* enough (between 1 – 8 cm/s) that it does not affect the f_p unlike in Wensrich's study [30, 31].

In Wensrich's experiments [30, 31], the entire bottom of a cylindrical model silo was slowly lowered. There is no region of converging flow, but there is a region near the bottom where the granular material dilates as the piston descends. Here the arch may be regarded as the boundary between the dense and dilated material.

For the *slow* discharge rates examined by Wensrich [30, 31], creep and external perturbations did not determine f_p. Instead the arch collapsed whenever the particles below the arch had dropped enough to lose contact with the arch. Consequently, it is entirely reasonable that f_p was inversely proportional to the discharge velocity. Wensrich estimated the distance that particles at the base of the silo moved between pulsations was 3% of a particle diameter, which is comparable to the 5% of a particle diameter dilation distance required for slip to occur in stick-slip simple shear experiments with granular materials [7]; however, further studies are required to determine if the proportionality constant (i.e. critical distance) in this regime is always a small fraction of the mean particle diameter.

5.2 Comparison with previous work

Wensrich [30, 31] observed that the acceleration produced by each quaking impulse grew with distance traveled by the wave carrying the information of the impulse from the dynamic arch to the top of the silo. In contrast, the granular material accelerations recorded in these experiments with the accelerometer at a fixed depth below the free surface did not change appreciably as the bed height decreased during discharge.

The granular material accelerations measured by Wensrich [30, 31] were less than 15 m/s^2, while those measured in this study were typically more than 100 m/s^2. Tejchman [26] also observed silo wall acceleration levels greater than 100 m/s^2. Nonlinear effects may be responsible for the height independence of the acceleration at the large accelerations seen in this study.

It is interesting to contrast the acceleration power spectra obtained with crushed glass and glass beads to see the effect of particle shape on granular dynamics. The power spectra for acceleration measurements in the glass beads (figure 10b) showed many harmonics of f_p before the high frequency decay region was approached. The power spectra for crushed glass (figure 10a) showed only a few harmonics, followed by a band limited white noise region and then a high frequency decay. This suggests that the glass beads showed a highly correlated distributed response to slip which originated at the arch. Crushed glass had a significantly less correlated response, quite possibly because of the heterogeneity in particle shape and particle contacts between them. This is consistent with the suggestions by Mair et al. [12] that smooth round particles have force chains that are stable over a narrow range of orientations whereas, rough particles such as the crushed glass have force chains that are stable over a wider range of orientations. Consequently, the force chains in the glass beads break in a highly correlated manner during a pulsation, whereas those in the crushed glass break in a less correlated manner.

The hypothesized difference in stress chain behavior between smooth and rough particles suggested by Mair et al. [12] may also explain the difference in the value of the critical height when an overload is imposed on the granular material. For rough particles the critical height decreased linearly with imposed overload, whereas for smooth particles, the critical height was independent of the imposed overload. As the force chains in a granular material composed of smooth spheres will have narrow directionality, the effects of the imposed overload will be transmitted to the side walls of the silo rapidly, and will not affect stress levels between the silo wall and the granular material a significant distance away from the overload. These force chains form a bridge so that the bulk of the overload is transmitted to the silo walls. For rough particles, the force chains are less concentrated because friction and asperity interlocking allows rough particles to transmit forces in a variety of directions without failure. Because bridging in the granular material is less effective, the imposed overload is not screened and its effects on the stress field can be transmitted further in to the granular material. Consequently, the critical height decreases because stresses at the arch are large and allow pulsations to occur for a longer time during discharge, in agreement with studies that slip-stick friction is dependent on the local stress level [17].

The Janssen solution for the effect of an overload on the stress field in a granular medium [18] in a silo predicts that effect of the overload on the stress field inside the granular material decays faster as the angle of wall friction is increased. Crushed glass has a larger angle of wall friction than the glass beads and so the finding that an overload has more effect on the crushed glass than the glass beads does not agree with the predictions from the Janssen solution. Nedderman [18] has suggested that the Janssen solution is not a good method for predicting stress levels inside a granular material when an overload is imposed. Further work examining the shear and wall normal stresses in silos for different shapes and distributions of particle sizes with varying overloads would help in obtaining appropriate constitutive relations to describe granular material stress fields macroscopically, an area which is the subject of current debate [29,21].

Mair et al. [12] found that particle shape influences granular material sliding characteristics. The present study confirms these findings, because granular materials made of the same glass with similar sizes but different shapes had different pulsation frequencies in the aluminum tube. In particular, surfaces that are rough are less likely to have stick-slip friction because the asperities can lock and prevent slip occurring at all contact points. This explains why no pulsations occurred in the fully rough plain steel tube. These results are in accord with findings by Tejchman [26], Moriyama and Jimbo [14], and Jahagirdar [9] that to prevent silo quake, mass flow silos should have rough walls.

Hardow et al. [8], whose study did not examine a variety of granular materials, suggested that wall friction was not the cause of silo quake, a finding that this study has shown is not always correct. As their study was for a core flow silo, they did not consider the possibility that stick-slip friction can occur at sliding surfaces inside the granular material, as shown in the study by Nasuno et al. [17] and not just between the granular material and the silo wall.

Finally, Hardow et al. [8], suggested that resonance is not always required for silo quake while other studies have suggested it is an important factor in amplifying the amplitude of the pulsations [10, 19,26, 28]. This study does not give a conclusive answer to this question, because careful measurements of the filled silo tube radial vibration natural frequencies are required. The results of the calculations for the natural frequencies of the empty tubes given in table 1 suggest that for the acrylic tube, tube resonance frequencies could be important. However, acrylic is also a material with well established stick-slip properties [4], it is difficult to decompose material surface elastic properties from structural elastic properties. For the aluminum tube, resonance occurs for $f_v > 25$ Hz. For $f_v < 25$ Hz, it is difficult to state whether resonance occurs, as only the dominant natural frequency of vertical oscillations of the silo has been determined. If a system is designed such that its lowest natural frequency for structural oscillations is much larger than the pulsation frequency, it would show that resonance is not essential for silo quake to occur.

6 Conclusion

This study has shown that stick-slip motion generates silo music and silo quake. Silo music is driven by the stick-slip pulsating motion of the granular material during discharge and is associated with a resonance in the air column above the bed. When the pulsating motion disappears, so does the silo music. Over the range of discharge rates studied here (equivalent to average velocities of descent through the tube of 1 – 8 cm/s), the pulsation frequency was independent of discharge velocity. Both silo music and flow pulsations stopped abruptly when the bed height fell below a critical value. The critical height could be changed by placing an overload in the case of crushed glass, but not in the case of the smooth glass beads. This may be rationalized, although only speculatively at this point, by differences in stress chain behavior.

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8 References

1. Baxter G.W., Behringer R.P., Fagert T., Johnson G.A., *Pattern Formation in Flowing Sand*, Phys. Rev. Lett., **62(24)**, 2825 (1989) .
2. Bergé P., Pomeau Y., Vidal C., Order within Chaos, Wiley (1984).
3. Berman A.D., Ducker W.A., Israelachivili J.N., *Experimental and theoretical investigation of stick-slip friction mechanisms* in Persson B.N.J., Tosatti E.,ed. Physics of Sliding Friction, Kulwer Academic Publishers (1996).
4. Bouissou S., Petit J.P., Barquins M., *Normal load, slip rate and roughness influence on the polymethylmethacrylate dynamics of sliding 1. Stable sliding to stick-slip transition*, Wear **214**, 156 (1998).
5. Bowden F.P., Tabor D., The friction and lubrication of solids, Oxford University Press (1950).
6. Cates M.E., Wittmer J.P., Bouchaud J.P., Claudin P., *Jamming, force chains, and fragile matter*, Phys. Rev. Lett. **81**(9), 1841 (1998).
7. Géminard J.C., Losert W., Gollub J.P., *Frictional mechanics of wet granular material*, Phys. Rev. E **59**(5), 5881 (1999).
8. Hardow B., Schulze D., Schwedes J., *An Experimental Analysis of the 'Silo Quaking' phenomenon*, Proc. Of the 3rd World Congress on Particle Technology, Brighton, England (1998).

9. Jahagirdar S., An Experimental Study of Sound Emission during Granular Flow. Department of Chemical Engineering, Indian Institute of Science, Bangalore, India (1999).
10. Kmita J., *Silo as a system of self-induced vibration*, ASCE J. of Struct. Eng. **111,** 190 (1985).
11. Krim J., *Friction at macroscopic and microscopic length scales*, Am. J. Phys. **70(9)**, 890 (2002).
12. Mair K., Frye K.M., Marone C., *Influence of grain characteristics on the friction of granular shear zones*, J. of Geo. Res. **107(B10),** 2219 (2002).
13. McCabe R.P., *Flow patterns in granular materials in silos*, Geotechnique **24**, 45 (1974).
14. Moriyama R., Jimbo G., *Reduction of pulsating wall pressure near the transition point in a bin*, Bulk Solids Handling **8,** 421 (1988).
15. Muite B.K.,The effects of boundaries on granular and fluid mechanics, M.S.E. Thesis, Princeton University, Princeton, New Jersey, USA (2003).
16. Naeem M.N., Sharma C.B., *Prediction of natural frequencies for thin circular cylindrical shells,* Proc. Instn. Mech. Engrs., **C214**, 1313, (2000).
17. Nasuno S., Kudrolli A., Bak A., Gollub J.P., *Time-resolved studies of stick-slip friction in sheared granular layers*, Phys. Rev. E **58**(2), 2161 (1998).
18. Nedderman R.M., Statics and Kinematics of Granular Materials, Cambridge University Press (1992).
19. Niedostatkiewicz M., Tejchman J., *Experimental and theoretical studies on resonance dynamic effects during silo flow,* Powder Handling and Processing **15(1)**, 36 (2003).
20. Ovarlez G., Clément E., *Slow dynamics and aging in a confined granular flow*, Phys. Rev. E **68**, 031302 (2003).
21. Ovarlez G., Fond C., Clément E., *A giant overshoot effect in the Janssen granular column,* Phys. Rev. E **67**, 060302 (2003).
22. Perry M.G., Handley M.F., *The Dynamic arch in free flowing granular material discharging from a model hopper*, Trans. Instn. Chem. Engrs. **45**, T367 (1967).
23. Phillips C.E.S., *Electrical and other properties of sand*, Proc. Roy. Instn. Gt. Britain **19,** 742 (1910).
24. Quinn S.F., Silo Music, B.S. Thesis, Princeton University, Princeton, New Jersey, USA (2002).
25. Rayleigh J.W.S., The Theory of Sound vol. II, Dover Publications (1945).
26. Tejchman J., *Technical concept to prevent the silo honking*, Powder Technol. **106**, 7 (1999).
27. Tejchman J., Gudehus G.,*Silo-music and silo-quake, experiments and a numerical cosserat approach,* Powder Technol. **76**, 201 (1993).
28. Tejchman J., Niedostatkiewicz M., *Resonance effects during granular flows in silos*, Proc. International Congress for Particle Technology, Nuremburg, Germany (2001).
29. Vanel L., Claudin Ph., Bouchaud J.-Ph., Cates M.E., Clément E., Wittmer J.P., *Stresses in Silos: Comparison between theoretical models and New Experiments*, Phys. Rev. Lett. **84(7)**, 1439 (2000)
30. Wensrich C.M. *Experimental behaviour of quaking in tall silos*, Powder Technol. **127**, 87 (2002).
31. Wensrich C.M., Analytical and Numerical Modeling of Quaking in Tall Silos, PhD Thesis, University of Newcastle, Australia (2002).

Table 1
Tube properties

Tube Material	Length (m)	ID (cm)	Wall Thickness (cm)	Surface Finish	Calculated lowest Natural Frequency (Hz)
6061-T6 Aluminum Alloy	1.8	6.37±0.01	0.51±0.1	Smooth	26
Plain steel	1.8	6.38±0.01	0.57±0.01	Rough	25
Cast Acrylic	1.5	6.35±0.01	0.64±0.01	Smooth	12

Table 2
Granular Material Properties

Material	Supplier	Particle Size (µm)	Particle Density (g/cm^3)	Angle of Repose (°)
Crushed Glass	Potters Industries	450±50	2.5±0.1	34±1
Ballotini Impact Beads	Potters Industries	480 ±60	2.5±0.1	26±1
Washed and Ignited Standard Ottawa Sand	EMD Science	400±100	2.7±0.1	33±1

Table 3

Angle of wall friction for the silo wall and granular material combinations

Silo wall material Granular material	Acrylic[b]	Aluminum[b]	Plain steel
Crushed glass	28°	33°	Fully rough
Glass beads	17°	17°	Fully rough
Sand	25°	30°	Fully rough

Note: [b]The angles are accurate to within ± 2°

Table 4

Variation of critical height with silo wall and granular material properties

Silo wall material	Granular material	Critical height[a] (m)
Aluminum	Crushed glass	0.9
Aluminum	Glass beads	1.3
Acrylic	Crushed glass	0.8
Acrylic	Glass beads	0.6
Acrylic	Sand	0.7

Note: [a] The accuracy of the critical height data is ± 0.1 m.

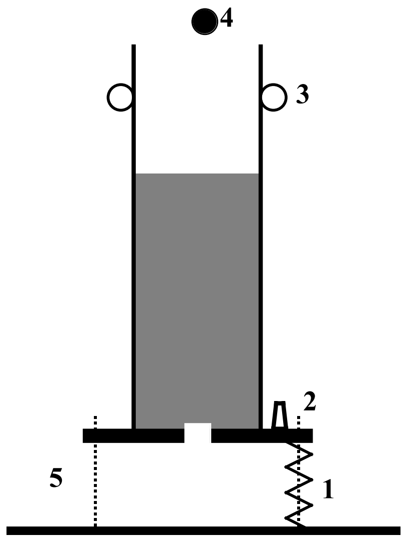
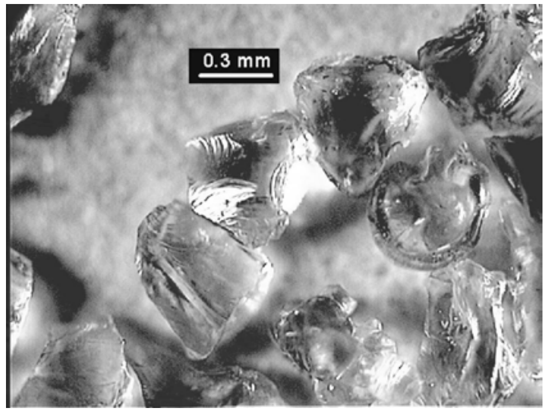
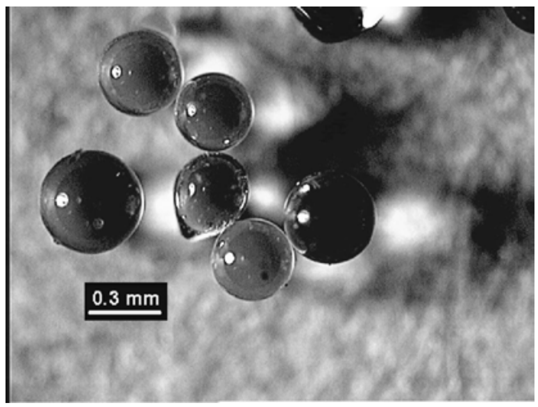


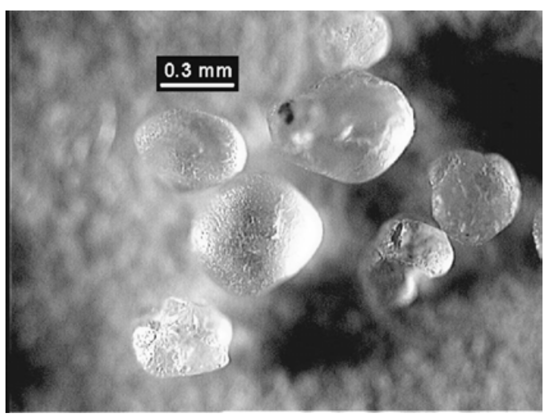
Fig. 1: Experimental setup for vertical acceleration and sound measurements. The numbers indicate: (1) spring on positioning slider, (2) accelerometer, (3) positioning roller, (4) microphone and, (5) positioning slider.



(a)



(b)



(c)

Figure 2: Photographs of the granular materials: a) crushed glass, b) glass beads, and c) sand.

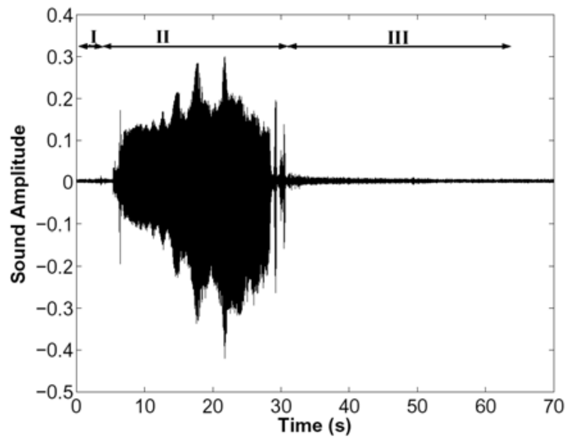


Fig. 3: Variation of sound amplitude with time during discharge of sand from an acrylic tube of 76 mm outer diameter, wall thickness 3 mm, and having an orifice of diameter 19 mm: region I – no flow, region II – flow with pulsations and, region III – flow after pulsations have ended.

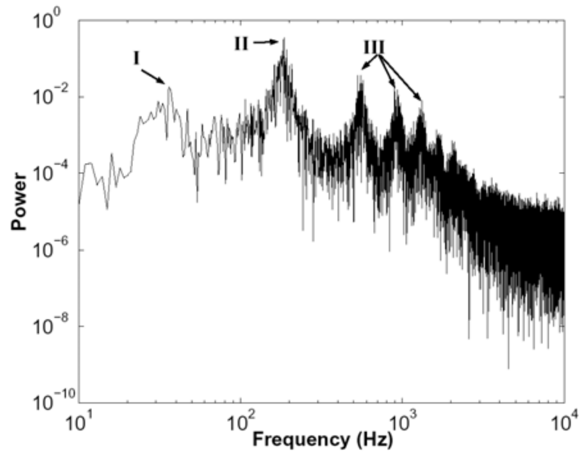


Fig. 4: **Typical power spectrum for one second of sound measurements during silo music when sand is discharged from an acrylic tube of 7.6 cm outer diameter, wall thickness 0.3 cm, and having an orifice of diameter 1.9 cm: (I) - the pulsation frequency, (II) - the dominant sound frequency and, (III) - the higher harmonics of the dominant sound frequency.**

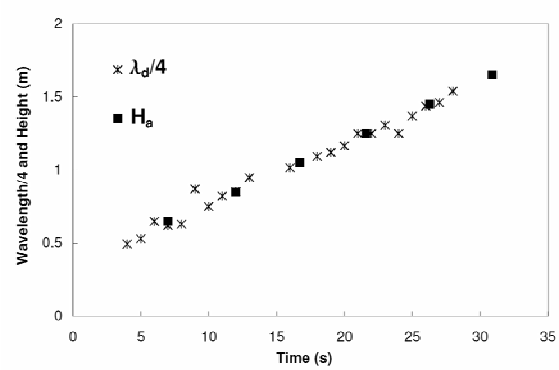


Fig. 5: Variation of the dominant quarter wavelength (*$\lambda_d/4$*) and the height of the air column (*H_a*) with time during discharge of sand from an acrylic tube of 7.6 cm outer diameter, wall thickness 0.3 cm, and with an orifice of diameter 1.9 cm.

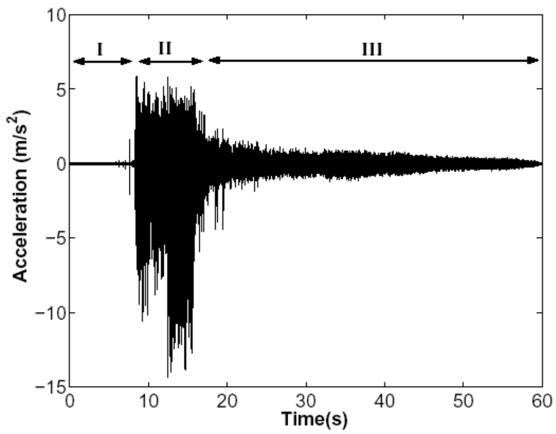


Fig. 6: Vertical acceleration measurements on the base of the aluminum silo during discharge of crushed glass through a 1.9 cm orifice. The silo had a dominant natural frequency of vertical oscillations of 8 Hz: region I – no flow, region II – flow with pulsations and, region III – flow after pulsations have ended.

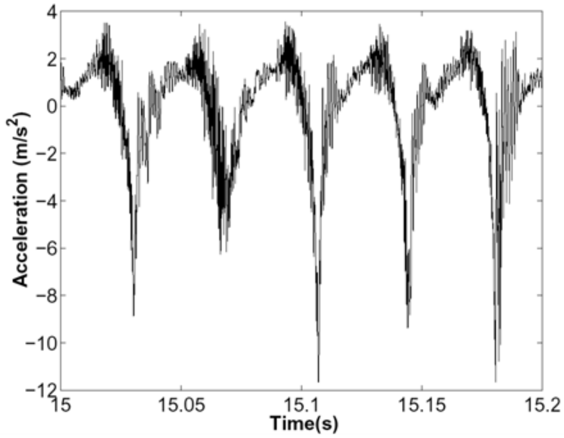
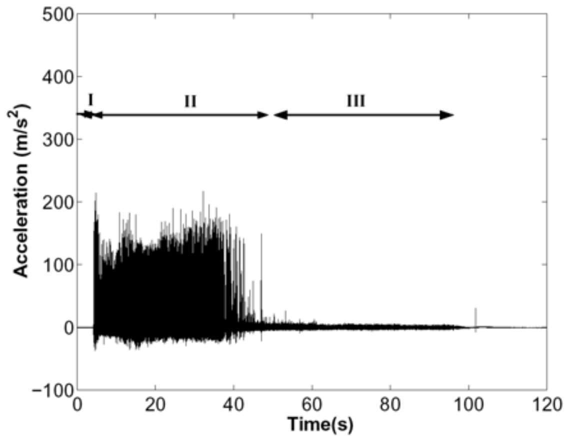
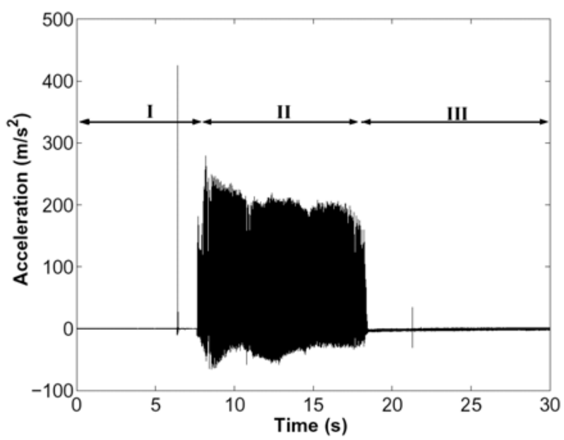


Fig. 7: Close-up showing individual pulsations measured by the accelerometer on the silo structure for the flow in Fig. 6.

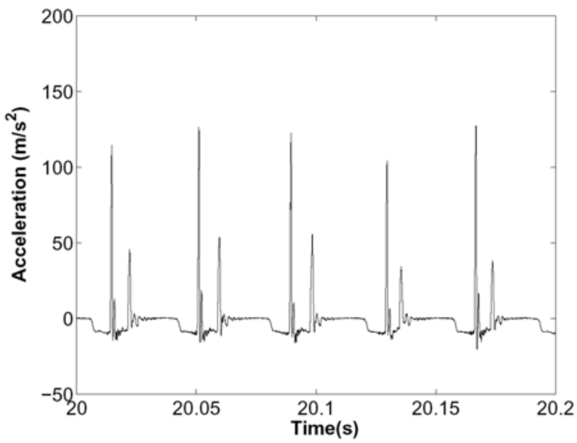


(a)

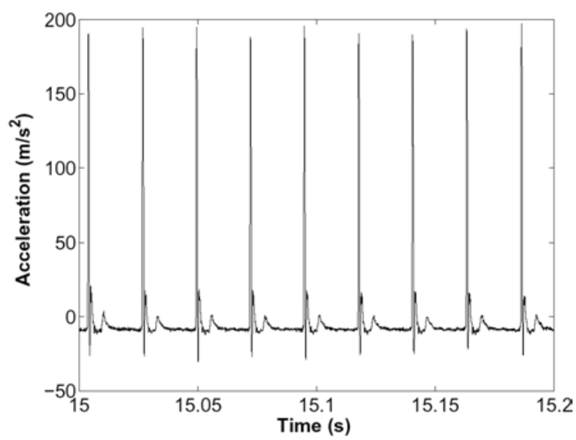


(b)

Fig. 8: Vertical acceleration measurements made when the accelerometer was embedded in the granular materials ((a) crushed glass, (b) glass beads) and allowed to translate with it during discharge from the aluminum silo: region I – no flow, region II – flow with pulsations and region III – flow after pulsations have ended. In a) the silo had a 1.3 cm orifice and a dominant natural frequency of vertical oscillations of 8 Hz and in b) the silo had a 1.9 cm orifice and a dominant natural frequency of vertical oscillations of 6 Hz.

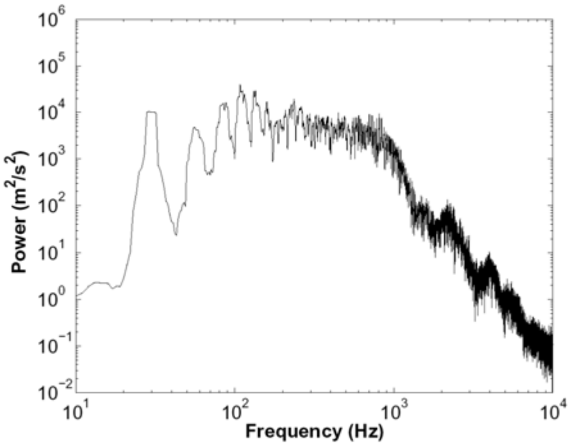


(a)

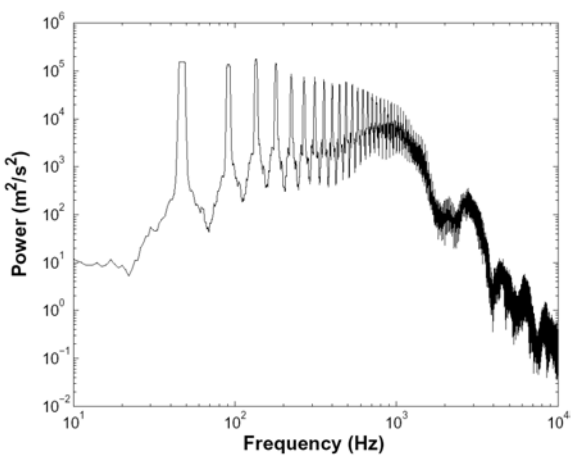


(b)

Fig. 9: Close-up showing individual pulsations measured by the accelerometer embedded in the granular material for the flow in Fig. 8: a) close-up from Fig. 8a and b) close-up from Fig.8b.



(a)



(b)

Fig 10: Power spectra for one second of the measurements in Fig. 8. The power spectra have been averaged over 4 points in frequency to make average trends clearer: a) 20th second of measurements of Fig. 8a and b) 15th second of measurements of Fig. 8b.

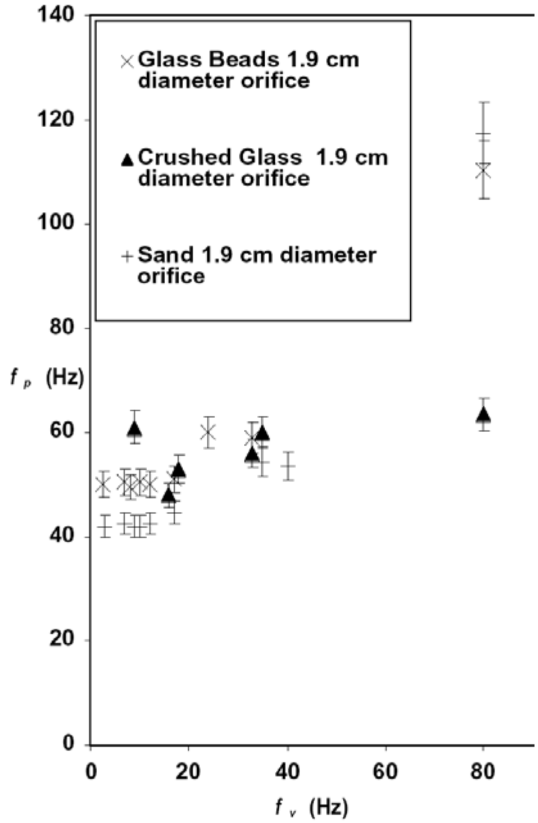


Fig. 11: Variation of the pulsation frequency (f_p) of the filled silo with the dominant natural frequency for vertical oscillations of the silo (f_v) for granular materials discharging from the acrylic tube.

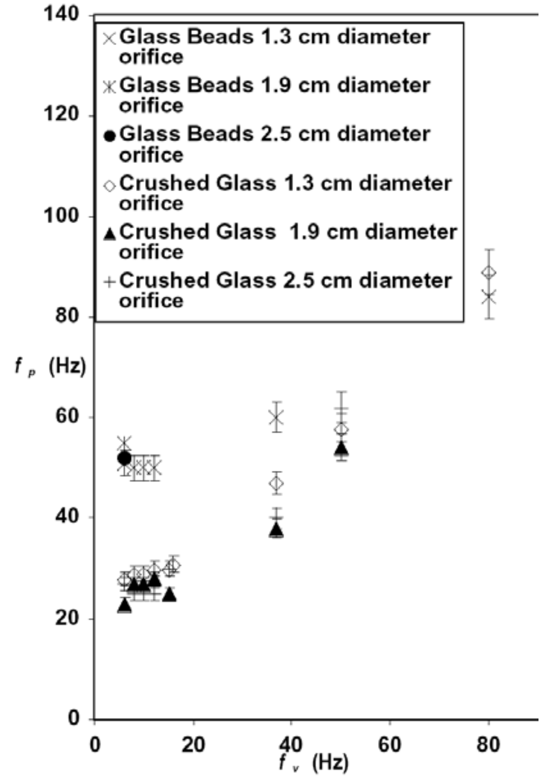


Fig. 12: Variation of the pulsation frequency (f_p) of the filled silo with the dominant natural frequency for vertical silo oscillations (f_v) for granular materials discharging from the aluminum tube.

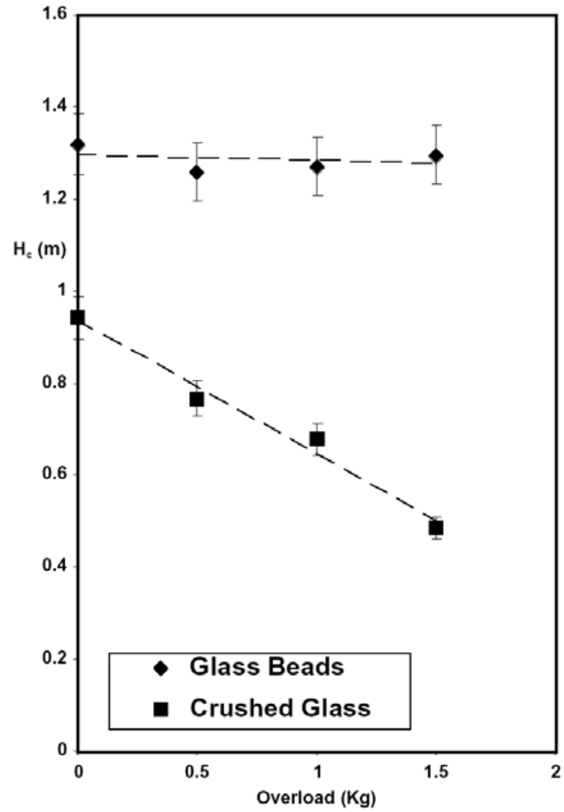


Fig. 13: Variation of critical height with overload for glass beads and crushed glass in the aluminum tube when the dominant natural frequency of vertical oscillations of the filled silo is 8 Hz and the silo has a 1.9cm orifice.