Reliability Based Seismic Stability of Soil Slopes

Introduction

Earthquake induced slope failures occur in seismically active zones and lead to loss of lives and economic losses. The slope design in these situations needs to address the issues of uncertainty, safety and consequence costs in a rational manner. Conventional slope design based on the factor of safety cannot explicitly address uncertainty (Alonso, 1976). Geotechnical engineers have recognized the role of uncertainties in slope stability quite a few years back (Wu and Kraft, 1970; Alonso, 1976; Vanmarcke, 1977; Chowdhury et al., 1987; Li and Lumb, 1987; Chowdhury, 1996; Tang et al., 1999) but have been slow on implementing them in analysis and design and to assess the probability of success (satisfactory performance) or failure (unsatisfactory performance) of a structure. Christian et al. (1992) suggest that the effective applications of probability and reliability principles lie in identifying the relative probabilities of failure or in which the effects of uncertainties on design are clearly brought out. The impact of uncertainty on the reliability of slope design and performance assessment is often significant. Inherent variability of soil properties, scarcity of representative data, changing environmental conditions, unexpected failure mechanisms, simplifications and approximations adopted in geotechnical models, and human mistakes in design and construction are the factors contributing to uncertainty in geotechnical system modeling (Ramly et al., 2002). The evaluation of the role of uncertainty necessitates the implementation of probability concepts and reliability based design methods. Recognizing the aspects of safety, uncertainty and consequence costs, efforts are being made to formulate guidelines and codes.

Guidelines and Codes

Tolerable Risk Criteria

In a simple form, quantitative risk analysis of slope stability problems involves identification of hazards, which have potential for failure and damages leading to undesirable consequences. It is recognized that in many cases, the idea of annual probability of failure, depending on f-N relationships (frequency of fatalities (f), and number of fatalities (N)) is a useful basis (Fell and Hartford, 1997; Christian and Urzua, 1998) on which assessment of existing stability in terms of reliability and stabilization of slopes can be taken up. Some guidelines on tolerable risk criteria are formulated by a number of researchers and engineers involved in risk assessment (Morgenstern, 1997; Fell and Hartford, 1997). They indicate that the incremental risk from a slope instability hazard should not be significant compared to other risks and that the risks should be reduced to "As Low As Reasonably Practicable" (ALARP). In UK, risk criteria for land use planning made based on f-N curves (frequency - Number of fatalities) on annual basis suggest lower and upper limits of 10^{-4} and 10^{-6} per annum for probability of failure or risk. Risk assessment in the case of dams is reasonably well developed and practiced in many countries such as USA, Canada and Hong Kong. Very recently, US Army Corps of Engineers (1997) have made specific recommendations (Table 1) on targeted probabilities of failure and the corresponding reliability indices in geotechnical, water resources and infrastructure projects. The guidelines present the recommendations in terms of probability of failure p_f, or reliability index (β). Christian and Urzua (1998) proposed that it is necessary to study the extent of risk posed by earthquake as additional hazard in slope stability problems and presented a simple approach to estimate the probability of failure in seismic conditions. The annual probability of failure corresponds to an expected factor of safety E(F), which is variable and the variability is expressed in terms of standard deviation of factor of safety σ_F. If factor of safety is assumed to be normally distributed, reliability index (β) is expressed by

$$\beta = \frac{(E(F) - 1.0)}{\sigma_F} \qquad (1).$$

The proabablity of failure and the relaibility index are related by

$$p_f = 1.0 - \Phi(\beta) \qquad (2)$$

where, Φ(β) is the cumulative probability of standard normal variate.

TABLE 1. Relationship between probability of failure (p_f) & expected performance level (US Army Corps of Engineers, 1997)

Reliability Index, β	Probability of failure, p_f	Expected performance level
1.0	0.16	Hazardous
1.5	0.07	Unsatisfactory
2.0	0.023	Poor
2.5	0.006	Below average
3.0	0.001	Above average
4.0	0.00003	Good
5.0	0.0000003	High

The role of consequence costs is realised in recent times and has been receiving considerable attention in the geotechnical profession. Recently, Joint Committee on Structural Safety (JCSS, 2000) presented relationships between reliability index (β), importance of structure and consequences of failure. The committee divided consequences into 3 classes based on risk to life and/or economic loss, and they are presented in Tables 2 and 3 respectively. From the Tables 2 & 3, it can be inferred that if the failure of a structure is of minor consequence (i.e., $C^* \leq 2$), then a lower reliability index may be chosen. On the other hand, if the consequence costs are higher (i.e., C^* = 5 to 10) and if the relative cost of safety measures is small, higher reliability index values can be chosen. It can also be noted from the tables that reliability index in the range of 3 to 5 can be considered as acceptable in design practice.

Joint Committee on Structural Safety (JCSS, 2000) made specific recommendations which are quite similar to that of US Army Corps of Engineers and are given in Table 2. The relative cost of safety measure and the consequences of failure of the structure are also considered and related to probability of failure (p_f) and reliability index (β) and are given in Table 2 and 3. From Tables 1, 2 and 3, the following aspect points are clear.

1. The targeted reliability indices vary from 3 to 5, depending on the expected level of performance.

2. Consequence costs can also be considered in the analysis. If the consequence costs are not significant compared to initial costs ($C^* \leq 2$) (for example slope design in a remote area), lower reliability index can be allowed, whereas higher reliability index is required where the consequence costs are high (for example slope in an urban locality).

TABLE 2. Relationship between reliability index (β), importance of structure and consequences (JCSS, 2000)

1	2	3	4
Relative cost of safety measure	Minor consequence of failure	Moderate consequence of failure	Large consequence of failure
Large	β = 3.1	β = 3.3	β = 3.7
Normal	β = 3.7	β = 4.2	β = 4.4
Small	β = 4.2	β = 4.4	β = 4.7

TABLE 3. Classification of consequences (JCSS, 2000)

Class	Consequences	C^*	Risk to life and/or economic consequences
1	Minor	≤ 2	Small to negligible and small to negligible
2	Moderate	$2 < C^* \leq 5$	Medium or considerable
3	Large	$5 < C^* \leq 10$	High or significant

where, C^* is the normalised consequence cost (normalised with respect to initial cost).

This paper examines seismic slope stability in terms of reliability and consequence costs proposed in the context of the above guidelines. The objectives of the paper are i) to show that the reliability index is a better measure of safety than the conventional factor of safety, and ii) to show that it is possible to balance costs considering consequence costs, soil parameters, their variations and correlation, considering horizontal seismic coefficient and slope geometry. The following sections describe the mechanistic model adopted, calculation procedures and the results obtained.

Slope Reliability Analysis

The method of analysis described in this paper is a simplified approach for predicting the optimum slope angle for a given slope height and the soil properties. The relationship between the variability of soil strength parameters, c, cohesion, and ϕ, angle of internal friction, $\rho_{c,\phi,}$ correlation coefficient between cohesion and friction angle and p_f, the probability of failure of slope is explored to provide a probabilistic assessment of stability of slopes. Statistical analysis of actual data by many researchers (Lumb, 1966; Alonso, 1976; Harr, 1987; Christian et al., 1992; Duncan, 2000) has revealed that cohesion and friction angle follow normal or log-normal distribution, and that there exists a negative correlation between the above mentioned strength parameters. Studies also show that the use of log-normal or normal distribution is not significant if the coefficient of variation of parameters is less than 30% (Ang and Tang, 1975; Whitman, 1984). The effect of an earthquake on the soil mass comprising a slope is introduced as an increase in the inertia of the mass and is expressed in terms of the maximum acceleration experienced at the site of the slope.

Mechanistic Model

In the present analysis, the stability of soil slopes is analysed by assuming a wedge type failure surface. The slope geometry along with the planar failure surface is shown in figure 1.

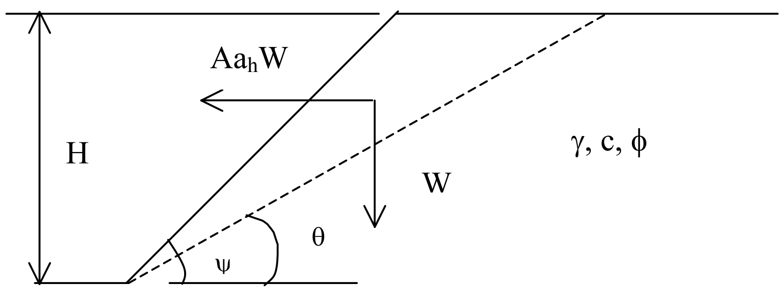

Figure 1. Slope Geometry along with planar failure surface

The static factor of safety corresponding to the assumed failure surface (Christian and Urzua, 1998) is

$$F = \frac{c + \frac{1}{2}\gamma H \frac{sin(\psi - \theta)}{sin\,\psi} cos\,\theta\, tan\,\phi}{\frac{1}{2}\gamma H \frac{sin(\psi - \theta)}{sin\,\psi} sin\,\theta} \tag{3}$$

Of the vertical and horizontal peak earthquake accelerations, the latter component is more often used in the current geotechnical practice, to approximately model the system response to earthquakes, and hence the same is used in the present analysis. If the ground acceleration is a_h and the amplification factor in the slope is A, the dynamic factor of safety (Christian and Urzua, 1998) becomes

$$F^* = \frac{c + \frac{1}{2}\gamma H \frac{sin(\psi - \theta)}{sin\,\psi}\left[cos\,\theta\, tan\,\phi - Aa_h\, sin\,\theta\, tan\,\phi\right]}{\frac{1}{2}\gamma H \frac{sin(\psi - \theta)}{sin\,\psi}\left[sin\,\theta + Aa_h\, cos\,\theta\right]} \tag{4}$$

Where, c is cohesion, γ is unit weight of soil, H is height of slope, ψ is slope angle, θ is slope of failure wedge in degrees, and ϕ is friction angle, A is amplification factor in the slope, and a_h is the peak horizontal acceleration.

The slope is assumed to be located in the seismically active region and the seismic loading is expressed in terms of the maximum horizontal ground acceleration, a_h, to be experienced by the slope during an earthquake. This is introduced into the analysis through a range of values (deterministic) equal to 10-20% of the acceleration due to gravity, g (i.e., a_h in the range of 0.10g to 0.20g) in which, g= 9.81 m/s^2. The assumed horizontal ground acceleration should have a lower probability of exceedence during the design life of the slope.

The results of reliability analysis are expressed in terms of reliability index (β), which is expressed in terms of Hasofer and Lind formulation (Madsen et al., 1986). It has been observed that the coefficients of variation of c and ϕ are in the range of 10-40% and 7-26% respectively (Harr, 1987; Becker, 1996; Duncan, 2000). The strength parameters, viz., cohesion, c, and angle of internal friction, ϕ, are taken as normally distributed random variables. The unit weight of soil is considered as deterministic parameter as its variation does not normally exceed 3-7% (Duncan, 2000). Madsen et al. (1986) and Becker (1996) explained the levels of reliability analysis, which can be performed in any design methodology depending on the importance of the structure. The term 'level' is characterized by the extent of information about the problem that is used and provided (Madsen et al., 1986). Level I reliability analysis uses only one value of each uncertain parameter (i.e., characteristic value). Load and Resistance Factor Design (LRFD) methods come under this category. Reliability methods, which employ two values of each uncertain parameter (i.e., mean and variance), supplemented with a measure of the correlation between parameters, are classified as level II methods. Reliability index methods are examples of level II methods. The reliability methods that employ the joint distribution of all uncertain parameters to evaluate the probability of failure are called level III methods. Level IV methods that are appropriate for structures that are of major economic importance, involve the principles of engineering economic analysis under uncertainty, considering costs and benefits, of construction, maintenance, repair, consequences of failure, and interest on capital, etc. Foundations for sensitive projects like nuclear power projects, transmission towers and highway bridges, etc. are suitable objects of level IV design. Level II method is performed in this study, as it is very difficult and uneconomical at least for the project concerned to get the actual variations of involved parameters and their distributions to be used with Level III analysis for precise evaluation of reliability of the system. The values of the parameters used in the analysis are shown in Table 4.

Table 4. Values of parameters used in the analysis

Parameter	Mean value
Cohesion (c)	10 kPa
Angle of internal friction (ϕ)	30°
Unit weight of soil (γ)	19 kN/m^3
Height of slope (H)	6m
Slope angle (ψ)	44° - 60°
Failure angle (θ)	40°
Amplification factor (A)	1
Peak horizontal acceleration (a_h)	0 - 0.2
Correlation coefficient ($\rho_{c,\phi}$)	-0.75 - 0

The normalised cost of the slope is calculated for different sets of data and the optimum slope angle is obtained. The optimum design is the design, which minimizes the expected cost without compromising on the expected performance of the system. The cost of failure, C, reflects the damage caused by the failure plus loss of utility as a result of failure. Hence, the expected cost (E), initial cost (I), cost of failure (C), and the probability of failure (p_f) of any system can be expressed as

$$E = I + C \times p_f \quad (3)$$

Wu and Kraft (1970) demonstrated the advantage of arriving at the relative cost rather than actual cost of the system in getting the optimum section for a slope, by considering the cost of construction of 1:1 slope (I_o) as basis. Hence, (3) can be written as

$$E^* = I^* + C^* \times p_f \quad (4)$$

Where $E^* = E/I_o$, $I^* = I/ I_o$, $C^* = C/ I_o$, and $I = 0.5 * H^2 * \cot(\psi)$. Hence, initial cost of construction is proportional to the volume of earthwork involved. So, height of slope being constant, steeper is the slope, less is the earthwork involved and hence less is the initial cost. However, as the slope angle increases, probability of failure and therefore the total consequence cost increases.

The following sections discuss the application of the above methodology to arrive at the balanced section considering uncertainties in parameters, safety in terms of reliability index and economy.

Results and Discussion

Reliability Index (β) Versus Expected Factor of Safety (E(FS))

Figure 2 shows the variations of reliability index and expected factor of safety, for various possible combinations of horizontal earthquake coefficients (Aa_h) and correlation coefficients ($\rho_{c,\phi}$). Analyses are done for various slope angles in the range of 44^o-60^o using Aa_h of 0, 0.1 and 0.2 with coefficients of variation of cohesion and friction angle being 10%. It can be noted that the variation of factor of safety with horizontal earthquake coefficient (Aa_h) is very less when compared to that of reliability index. For $\rho_{c,\phi}$ of -0.75 between c and ϕ, β varies from 6.84 at Aa_h equals to 0 (i.e., static case) to 0.52 at Aa_h equals to 0.2, whereas the expected factors of safety for the above data are 1.38 & 1.02 respectively. As expected, the reliability index decreases with increase in earthquake coefficient. Lower the $\rho_{c,\phi}$, higher is the reliability index. The horizontal earthquake coefficient, Aa_h, being a destabilising parameter shows an adverse effect on the performance of structure. At higher values it even undermines the effect of $\rho_{c,\phi}$ on the stability. The effect of $\rho_{c,\phi}$ on β is well pronounced at low values of Aa_h than at higher values. In conventional analysis, the slope is considered unsafe (as the values are less than the recommended value of 1.5), whereas the slope can be considered as safe in the case of Aa_h equals to zero with any $\rho_{c,\phi}$ and also in case of Aa_h

equals to 0.1 and $\rho_{c,\phi}$ equals to -0.75, in terms of the reliability index values reported in Table 1 and 2. The results clearly show that factor of safety alone cannot adequately indicate status of safety in seismic conditions. The results also show that if one has the data with regard to coefficients of variation of cohesion and friction and $\rho_{c,\phi}$, one can arrive at the reliability index (β) for a given slope geometry and seismic coefficient.

Influence of coefficients of variation of basic variables on Normalised costs

Figure 3(a) through 3(d) show the variation of normalised expected cost for various combinations of $\rho_{c,\phi}$, ψ, cv_c and cv_ϕ, for a typical case with Aa_h equals to 0.2. The normalised total cost is plotted on ordinate and the slope angle on abscissa. The normalised total cost corresponding to a slope angle is obtained by dividing the total cost (i.e., sum of initial cost and total consequence cost) for that particular slope with initial cost of a 45^o slope. All the variables and their variations are considered in arriving at the probability of failure, which is one of the two multiplicands in the calculation of total consequence cost. Normalised total cost of unity for any given slope means that the total cost of that slope (i.e., Initial cost + probability of failure × consequence cost) is equal to initial cost of 1:1 slope. From the above figures it is evident that as uncertainty of basic parameters in terms of coefficients of variation increases, normalised total cost increases and optimum slope angle decreases. This is because as uncertainty in strength parameters increases the probability of failure or unsatisfactory performance of the system increases. This in turn increases the total consequence cost and so the normalised expected cost. For any particular data set, if $\rho_{c,\phi}$ increases, there will be a substantial decrease in normalised cost and a corresponding increase in optimum slope angle. For example considering figure 3(a), upto a slope angle of 56^o, $\rho_{c,\phi}$ does not have any appreciable effect on normalised total cost. It means that if one chooses the slope angle within 56^o, it implicitly accounts for any value of $\rho_{c,\phi}$ under study (between 0 and -0.75). The reliability index values corresponding to these points for various combinations of $\rho_{c,\phi}$, cv_c & cv_ϕ are presented in Table 5. Even if one does not have any clear idea about $\rho_{c,\phi}$ one can safely provide this slope angle. If $\rho_{c,\phi}$ is known, higher reliability index can be assigned. It can also be noted that as cv_c and cv_ϕ increase, the optimum slope angle decreases. The variation of E^* is more pronounced at higher values of $\rho_{c,\phi}$ and also at steeper slope angles.

Table 5.Typical values of reliability index for $Aa_h = 0.2$

Coefficient of variation of c and ϕ (cv_c & cv_ϕ)	Slope angle ($\psi°$)	Reliability index values for different values of ($\rho_{c,\phi}$)			
		0	-0.25	-0.50	-0.75
5%	56	2.889	3.328	4.057	5.658
10%	52	2.962	3.378	4.039	5.349
15%	48	3.271	3.643	4.180	5.055
20%	46	3.029	3.309	3.685	4.227

Influence of Seismic Coefficient (Aa_h) on Normalised costs

Figure 4 shows the variation of normalised costs (E^*) as function of slope angle (ψ°) for different values of Aa_h. From the figure it can be noted that normalised cost decreases with increase of slope angle and reaches a minimum value close to a particular slope angle called optimum slope angle, beyond which it starts increasing. For Aa_h equals to 0.1 and 0.2, (with $\rho_{c,\phi} = 0$, and $C^* = 5$), the optimum angles obtained from the analyses are 58° and 53° respectively. For Aa_h equals to zero, i.e., for the static condition, the optimum angle is not found within the domain of study [44°-60°] and beyond 60°, the slopes are no more admissible for the given set of data. The figure clearly shows that as Aa_h increases, the optimum slope angle reduces. There is no variation in E^* with respect to Aa_h for slope angles upto 52°.

Role of Normalised Consequence Cost (C^*)

Figure 5 shows a typical result of effect of consequence costs due to failure of slope on expected cost for Aa_h and $\rho_{c,\phi}$ equal to 0.2 and -0.25 respectively. It can be noted that the normalised cost increases with the increase in consequence cost. The optimum slope angle also changes with the consequence cost. Lower consequence cost results in lower overall cost of the slope. For a given consequence cost, the increase in normalised cost is rather steep for slope angles higher than the optimum angle. It can also be noted that for slope angles lesser than 54º there are no variations and corresponding slope gives a balanced design independent of consequence costs.

Concluding Remarks

This paper demonstrates that slope stability evaluation using reliability considerations is a rational way complementary to the conventional factor of safety approach, and that it is possible to arrive at the optimum angle for a given geometry of slope and the soil properties taking into account risk, seismic effects using a horizontal seismic coefficient, variability of soil in a probabilistic frame work. A pseudo static probabilistic stability analysis of soil slopes is carried out taking into account the uncertainties associated with the soil parameters, correlation between cohesion and friction angle, initial cost and consequence costs. Relationships between reliability index and horizontal earthquake coefficients for chosen slope geometry and property variations are studied. While results are valid for the conditions used in the problem, the same methodology can be applied to any problem of geotechnical interest.

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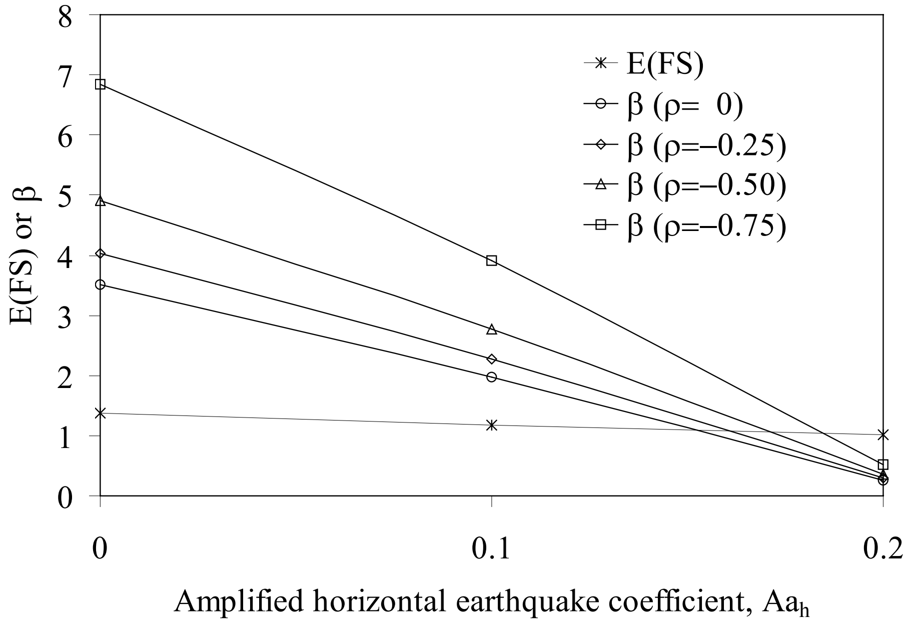

Figure 2. Variation of expected factor of safety (E(FS)) and Reliability index (β) as function of Aa_h for cv_c & cv_ϕ = 10%

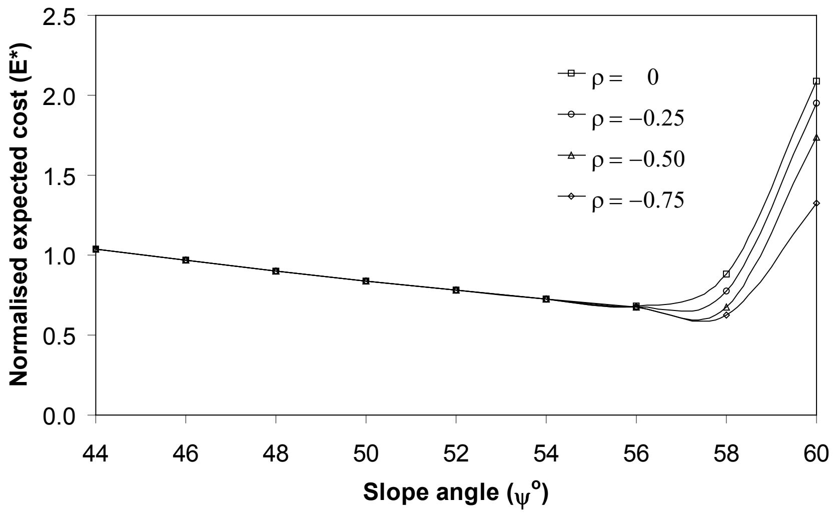

Figure 3(a). Normalised expected cost vs. slope angle for Aah = 0.2, C^*=5, cv_c & cv_ϕ = 5%

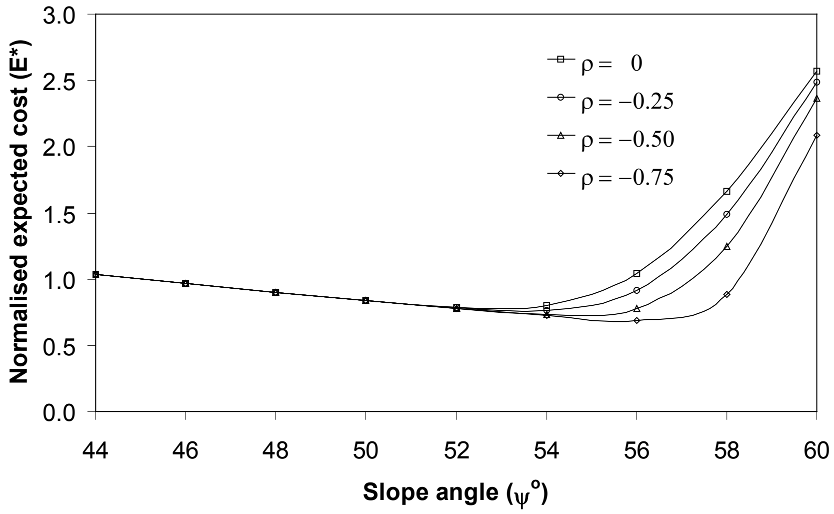

Figure 3(b). Normalised expected cost vs. slope angle for $Aa_h = 0.2$, $C^*=5$, cv_c & $cv_\phi = 10\%$

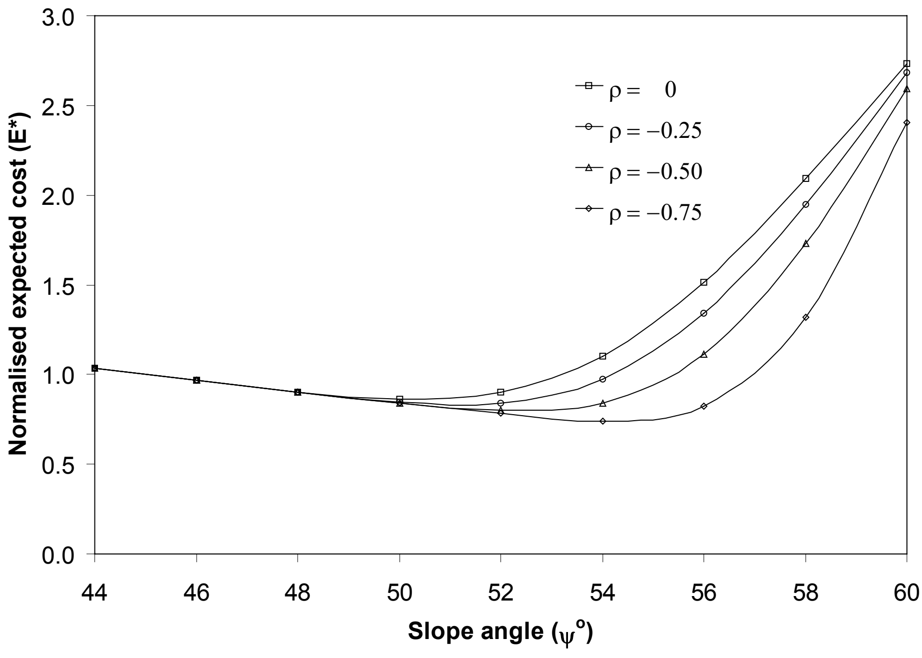

Figure 3(c). Normalised expected cost vs. slope angle for $Aah = 0.2$, $C^*=5$, cv_c & $cv_\phi = 15\%$

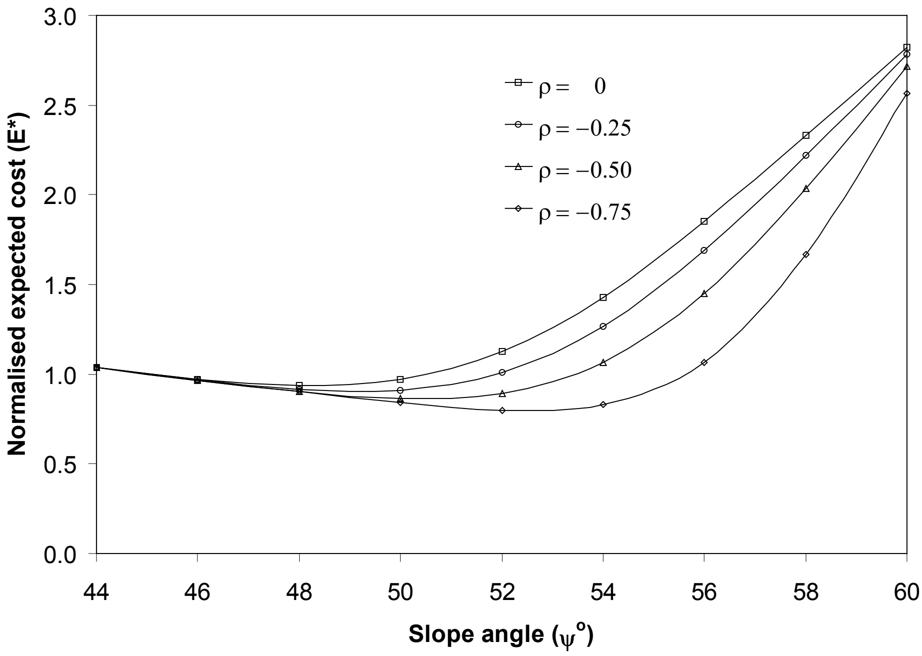

Figure 3(d). Normalised expected cost vs. slope angle for $Aa_h = 0.2$, $C^{*}=5$, cv_c & $cv_\phi = 20\%$

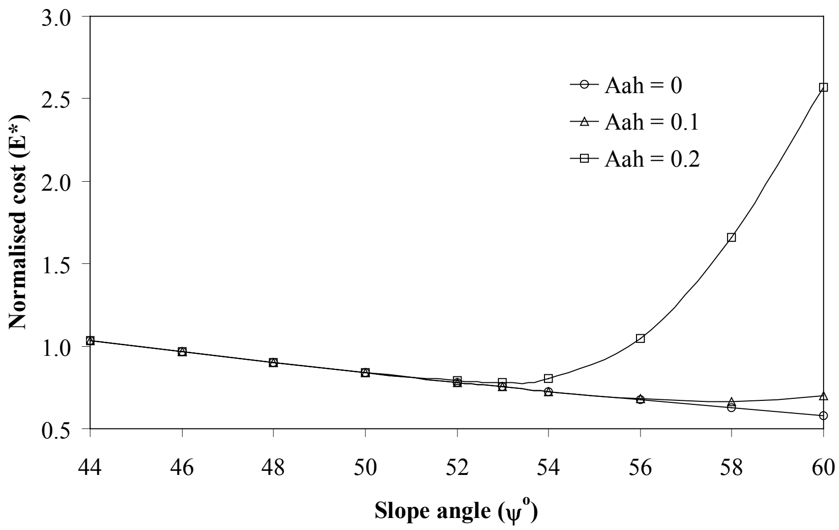

Figure 4. Normalised cost as function of slope angle for $\rho_{c,\phi}=0$ & $C^*=5$, cv_c & $cv_\phi = 10\%$

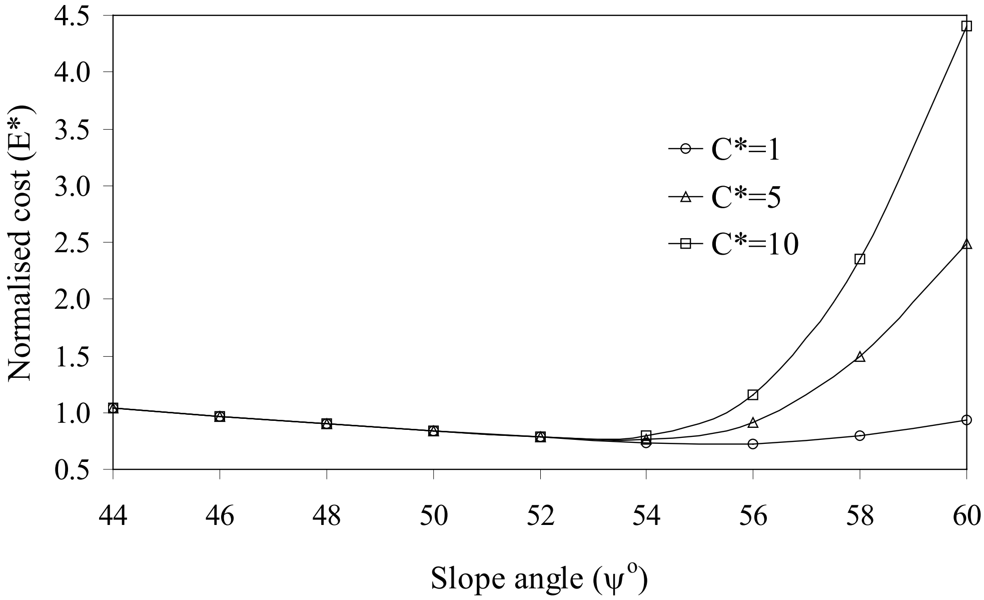

Figure 5. Normalised cost as function of slope angle for Aa_h=0.2, $\rho_{c,\phi}$= -0.25, cv_c & cv_ϕ = 10%