Transport through Quasiballistic Quantum Wires: The Role of Contacts

Siddhartha Lal,[1] Sumathi Rao,[2] and Diptiman Sen[1]

[1]*Centre for Theoretical Studies, Indian Institute of Science, Bangalore 560012, India*
[2]*Harish-Chandra Research Institute, Chhatnag Road, Jhusi, Allahabad 211019, India*

We model one-dimensional transport through each open channel of a quantum wire by a Luttinger liquid with different interaction parameters for the leads, the contacts, and the wire, and with two barriers at the contacts. We show that this simple model explains several features of recent experiments, such as the flat conductance plateaus observed at finite temperatures and lengths, and universal conductance corrections in different channels. We discuss the possibility of seeing resonancelike features at very low temperatures.

Recent advances in the fabrication of quantum wires within GaAs-AlGaAs heterostructures have made it possible to study their electronic transport properties in detail [1–8]. These studies show some puzzling features for the conductance, specifically, the flat conductance plateaus lying *below* integer multiples of $g_0 \equiv 2e^2/h$ [1,3,4,7,8]. At the same time, the theory of Tomonaga-Luttinger liquids (TLL) has provided a way of studying the conductance of strongly correlated electron systems in one dimension [9–13]. In this Letter, we propose a model for a quantum wire (QW) based on TLL theory which qualitatively explains a large number of experimental results.

Our model is motivated both by the way these systems are fabricated and by the experimental observations. The electrons are first confined to a two-dimensional region which is the inversion layer of a GaAs heterostructure. Then a gate voltage V_G is applied in a small region which further confines the electrons to a narrow constriction called the QW [1,3,4]. Within the QW, the electrons feel a transverse confinement potential produced by V_G; this produces discrete subbands or channels. As argued in Ref. [14], the electrons from the two-dimensional electron gas (2DEG) can enter the wire only if they are in a zero angular momentum state with respect to the end of the wire. Since the radial coordinate is the only variable governing the wave function of such a state, the 2DEG electrons engaged in transport may be modeled as one-dimensional noninteracting Fermi liquid systems lying on the two sides of the QW [10,11]; these two systems are called the *leads*. Inside the QW, the electrons interact via the Coulomb force. If this repulsion is approximated by a short range interaction, each open channel (defined below) can be modeled by a separate TLL. In addition, the charge and spin degrees of freedom are governed by independent TLLs in the absence of a magnetic field.

The simplest model incorporating these features is a one-dimensional system in which the TLL parameters (an interaction parameter K and the quasiparticle velocity v) are functions of the coordinate x as follows. If the QW lies in the range $0 < x < l$, (K, v) take the values (K_L, v_L) for $x < 0$ and $x > l$ (the leads), and the values (K_W, v_W) for $0 < x < l$ [10,11]. (The parameters K_W and v_W also carry charge and spin labels ρ and σ, as indicated below.) Here v_L is equal to the Fermi velocity $v_F = \sqrt{2E_{F2D}/m}$ of the electrons in the 2DEG (thus v_L depends on the density of the electrons in the 2DEG but *not* on any parameters of the QW), while v_W is the velocity of the quasiparticle excitations inside the QW. Since the electrons are taken to be noninteracting in the leads, $K_L = 1$, in the QW, repulsive interactions make $K_W < 1$. However, this model does not explain the observed conductances. The difficulty is that v_W varies from channel to channel and depends on V_G. The lowest energies E_s in each channel are given by the discrete energy levels for the transverse confinement potential; they can be shifted by changing V_G [15]. In the sth channel, E_{F1D} is given by $E_{F2D} - E_s$. If this is positive, the channel is open and the electrons have a bare velocity $v_{We} = \sqrt{2E_{F1D}/m}$ which is related to the velocity in the leads by $v_{We} = \sqrt{v_L^2 - 2E_s/m}$. Hence, the quasiparticle velocity v_W depends on the channel index s since it is related to v_{We} and K_W. However, the observed conductances exhibit features which are both channel and gate voltage independent.

We will therefore consider a different model which additionally has two *contact regions* of length d lying between the QW and the leads as shown in Fig. 1; thus, the total length of the QW system is $l + 2d$. The TLL parameters in the contact regions are denoted by (K_C, v_C); these will also carry spin and charge labels. The electrons may interact with each other in the contacts; the velocity v_C will be a function of v_F and the interaction parameter K_C. It is crucial that v_C should be independent of the gate voltage V_G. This is physically reasonable because V_G acts near the middle of the QW, whereas the contacts are at the ends. The point is that the electron density profile tends to adjust itself so as to shield the ends of the wire from the effects of the potential V_G. For instance, even if the applied gate voltage is abrupt similar to a square barrier potential, the actual voltage felt by the electrons is more smooth similar to an inverted harmonic oscillator potential [15], as a result of the shielding.
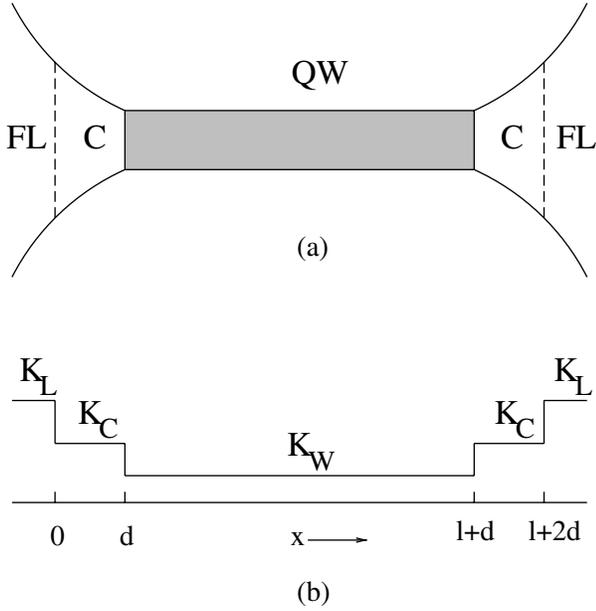
FIG. 1. Schematic diagram of the model showing the leads (marked FL for Fermi liquid), the contacts (C) of length d, and the quantum wire (QW) of length l. The interaction parameters in these regions are denoted by K_L, K_C, and K_W, respectively.

The contacts model the fact that the electrons do not directly go from the 2DEG to the QW; there is often a smooth transition region between the two. A recent experiment has explicitly studied the effect of a transition region between the 2DEG and the wire, and has shown that a region with a length of about 2–6 μm is required to cause backscattering [6]. This makes our modeling of the contact region as a Luttinger liquid of finite length d, rather than pointlike, quite plausible.

Note that the TLLs appearing in our model differ somewhat from a conventional TLL in which the electron velocity is related to the density. We are making the quasiballistic assumption that the electrons come in from the 2DEG, and shoot through the contact and wire regions where they interact with each other. Hence, the density of the electrons in the contacts and wire do not play a direct role in our model; the quasiparticle velocity v_W is determined primarily by v_F and the subband energies in the QW. The idea that the properties of the one-dimensional system are governed by E_{F2D} has been used earlier in Ref. [16] for a quantum point contact.

Given the Lagrangian density of a massless bosonic field in $1 + 1$ dimensions as [9]

$$\mathcal{L}(\phi; K, v) = (1/2Kv)(\partial\phi/\partial t)^2 - (v/2K)(\partial\phi/\partial x)^2,$$

the bosonized action for the model described above is

$$S_0 = \int dt \left[\int_{-\infty}^{0} dx\, \mathcal{L}_1 + \int_{l+2d}^{\infty} dx\, \mathcal{L}_1 + \int_{0}^{d} dx\, \mathcal{L}_2 \right.$$
$$\left. + \int_{l+d}^{l+2d} dx\, \mathcal{L}_2 + \int_{d}^{l+d} dx\, \mathcal{L}_3 \right], \quad (1)$$

where

$$\mathcal{L}_1 = \mathcal{L}(\phi_\rho; K_L, v_L) + \mathcal{L}(\phi_\sigma; K_L, v_L),$$
$$\mathcal{L}_2 = \mathcal{L}(\phi_\rho; K_{C\rho}, v_{C\rho}) + \mathcal{L}(\phi_\sigma; K_{C\sigma}, v_{C\sigma}), \quad (2)$$
$$\mathcal{L}_3 = \mathcal{L}(\phi_\rho; K_{W\rho}, v_{W\rho}) + \mathcal{L}(\phi_\sigma; K_{W\sigma}, v_{W\sigma}).$$

The charge and spin fields ϕ_ρ and ϕ_σ are continuous at the points $x = 0, d, l + d$ and $l + 2d$. They are related to the bosonic fields of the spin-up and spin-down electrons as $\phi_\rho = (\phi_\uparrow + \phi_\downarrow)/\sqrt{2}$ and $\phi_\sigma = (\phi_\uparrow - \phi_\downarrow)/\sqrt{2}$.

In addition to the five regions, our model includes two barriers which are motivated as follows. Since the geometry does not always change adiabatically from the 2DEG to the QW, one expects some scattering from the transition regions between the two [17]. Further, we have assumed the strength of the electronic interactions to change from zero in the 2DEG to a finite value in the contact regions; this can produce some barrierlike scattering [18]. Although the scattering produced by the changes in geometry and interaction can occur from anywhere in the contact regions, it is easier to study if we model it by spin-independent δ-function potentials placed at the junctions of the leads and contacts, i.e., at the points $x = 0$ and $x = l + 2d$. Following Ref. [19], we can show that the conclusions stated below do not change if we consider extended barriers, as long as they lie entirely within the contact regions which are insensitive to V_G.

In our model, the contact regions including the barriers are identical for all the subbands since the TLL parameters in the contacts depend only on E_{F2D}. It is only inside the quantum wire that the TLL parameters are different for different subbands. Although it is difficult to analytically derive our model, or even a more realistic version of it, from a microscopic Hamiltonian, we have presented above a justification for each aspect of it. Let us now consider what our model yields for the conductance. We first ignore the two δ-function barriers and the gate voltage, and consider the action in Eq. (1). The conductance can be obtained from the frequency-dependent Green's function $G_\omega(x, x')$ which can be computed exactly [11]. In the zero-frequency (dc) limit, we find that the conductance in each channel is given by $G = K_L g_0$ independent of all the TLL parameters in the contact and QW regions. For $K_L = 1$, this is the result expected for electrons in the absence of any scattering. This disagrees with the experimental observations which do show plateaus in the conductance, but at values which are renormalized down by a certain factor from the above values (see Fig. 2 of Ref. [3]). Further, the renormalization factor seems to be independent of the number of channels, although it is sample dependent [3,4,8].

The effect of the two barriers is best studied using the effective action technique [9]. We first integrate out the bosonic fields at all points except the junctions at $x = 0, d, l + d$, and $l + 2d$; we denote the fields at these four points by $\phi_{1a}, \phi_{2a}, \phi_{3a}$, and ϕ_{4a}, respectively, where $a = \rho$ or σ. In the high-frequency limit $\omega \gg v_C/d$ and

v_W/l, the action is given by [18]

$$S_{\text{eff}} = \sum_{a=\rho,\sigma} \int \frac{d\omega}{2\pi} |\omega| \left[\frac{K_L + K_{Ca}}{2K_L K_{Ca}} (\tilde{\phi}_{1a}^2 + \tilde{\phi}_{4a}^2) + \frac{K_{Ca} + K_{Wa}}{2K_{Ca}K_{Wa}} (\tilde{\phi}_{2a}^2 + \tilde{\phi}_{3a}^2) \right], \tag{3}$$

where the tildes denote Fourier transforms in time. We see that the four fields decouple at high frequencies or high temperatures; in that limit, ω is related to T by $\hbar\omega \sim 2\pi k_B T$. In the low-frequency limit $\omega \ll v_C/d$ and v_W/l, the action is given by

$$S_{\text{eff}} = \int \frac{d\omega}{2\pi} \frac{|\omega|}{2K_L} (\tilde{\phi}_{1\rho}^2 + \tilde{\phi}_{4\rho}^2 + \tilde{\phi}_{1\sigma}^2 + \tilde{\phi}_{4\sigma}^2)$$
$$+ \sum_{a=\rho,\sigma} \int dt \left[\frac{v_{Ca}}{K_{Ca}d} \{(\phi_{1a} - \phi_{2a})^2 + (\phi_{3a} - \phi_{4a})^2\} + \frac{v_{Wa}}{K_{Wa}l} (\phi_{2a} - \phi_{3a})^2 \right]. \tag{4}$$

Now we introduce the δ-function barriers and the gate voltage in the QW region; they are given by $V[\delta(x) + \delta(x - l - 2d)]$ and $(eV_G/\sqrt{\pi}) \int_d^{l+d} \partial\phi_\rho/\partial x$, respectively, where e is the electron charge. This part of the action takes the form

$$S_{\text{gate}} + S_{\text{barrier}} = \frac{eV_G}{\sqrt{\pi}} \int dt \,[\phi_{3\rho} - \phi_{2\rho}] + \frac{V}{2\pi\alpha} \Sigma_i \int dt \,[\cos(2\sqrt{\pi}\,\phi_{1i}) + \cos(2\sqrt{\pi}\,\phi_{4i} + 2\eta)], \tag{5}$$

where i is summed over $\uparrow,\downarrow$, α is a short distance cutoff, and η is given in terms of the wave numbers in the contact regions and the QW as $\eta = 2k_C d + k_W l$. After adding this action to (4) and integrating out ϕ_{2a} and ϕ_{3a}, we obtain the following low-frequency effective action in terms of $\tilde{\phi}_\rho = (\tilde{\phi}_{1\rho} + \tilde{\phi}_{4\rho})/\sqrt{2}$, $\tilde{n}_\rho = \sqrt{2/\pi}\,(\tilde{\phi}_{4\rho} - \tilde{\phi}_{1\rho})$, and $\tilde{\phi}_\sigma$, $\tilde{n}_\sigma$ (defined similarly to their charge counterparts),

$$S_{\text{eff}} = \int \frac{d\omega}{2\pi} \frac{|\omega|}{K_L} \left[\left(\tilde{\phi}_\rho - \frac{\eta}{2\sqrt{\pi}} \right)^2 + \frac{\pi}{4} \left(\tilde{n}_\rho - \frac{\eta}{\pi} \right)^2 + \tilde{\phi}_\sigma^2 + \frac{\pi}{4} \tilde{n}_\sigma^2 \right]$$
$$+ \int dt \left[\frac{U_\rho}{2} (n_\rho - n_{0\rho})^2 + \frac{U_\sigma}{2} n_\sigma^2 + \frac{V}{2\pi\alpha} \left\{ \cos(\sqrt{\pi}\,\phi_\rho)\cos(\sqrt{\pi}\,\phi_\sigma)\cos\left(\frac{\pi n_\rho}{2}\right)\cos\left(\frac{\pi n_\sigma}{2}\right) \right. \right.$$
$$\left. \left. + \sin(\sqrt{\pi}\,\phi_\rho)\sin(\sqrt{\pi}\,\phi_\sigma)\sin\left(\frac{\pi n_\rho}{2}\right)\sin\left(\frac{\pi n_\sigma}{2}\right) \right\} \right]. \tag{6}$$

Here we have shifted the fields $\tilde{\phi}_\rho$ and $\tilde{n}_\rho$ by factors proportional to η. $U_\rho = \pi\Lambda_{C\rho}\Lambda_{W\rho}/(\Lambda_{C\rho} + 2\Lambda_{W\rho})$ is the charging energy of the charges confined between the two barriers with $\Lambda_{C\rho} = v_{C\rho}/(dK_{C\rho})$, $\Lambda_{W\rho} = v_{W\rho}/(lK_{W\rho})$, and U_σ, $\Lambda_{C\sigma}$, and $\Lambda_{W\sigma}$, are defined similarly with $\rho \rightarrow \sigma$. $n_{0\rho} = \eta/\pi - eV_G/(\pi^{3/2}\Lambda_{W\rho})$ is the average number of charges between the two barriers. This effective action has the symmetry that if $n_{0\rho}$ is tuned to be an odd integer (using the gate voltage V_G), then there are two degenerate ground states [9]. Tunneling between these two ground states in the weak barriers limit corresponds to a resonance in the transport of electrons through the system. For weak barriers $V \ll U_\rho$ and U_σ, we can expand the terms involving U_ρ, U_σ, and V in (6) around $n_\rho = n_{0\rho}$ and $n_\sigma = 0$; this gives an effective barrier which vanishes if $n_{0\rho}$ is an odd integer. We thus require η to be a constant plus an odd integer times π for resonance.

We now do a renormalization group analysis to see how the barrier strengths scale with the length and temperature [9,12,13] and compute the conductance. We work in the weak barrier regime as we believe that the two junction barrier strengths are weak; any renormalization of their strengths will also be small since the total length of the contacts and QW is small. We define $T_d = \hbar v_{C\rho}/(2\pi k_B d)$ and $T_l = \hbar v_{W\rho}/(2\pi k_B l)$. If we assume that $d \ll l$ for simplicity, then $T_d \gg T_l$. The conductance g to leading order in the barrier strengths is obtained in three regimes:

(i) $T_d \ll T$, (ii) $T_l \ll T \ll T_d$, and (iii) $T \ll T_l$. In the high temperature regime (i), the particles are incoherent, and we find

$$g = g_0 K_L [1 - c_1 T^{2(K_{\text{eff}}-1)}(|V(0)|^2 + |V(l + 2d)|^2)]. \tag{7}$$

Here c_1 is a dimensionful constant containing factors of the velocity $v_{C\rho}$ and the cutoff α (but it is independent of all factors dependent on V_G), while $K_{\text{eff}} = K_L K_{C\rho}/(K_L + K_{C\rho}) + K_L K_{C\sigma}/(K_L + K_{C\sigma})$. In the intermediate temperature regime (ii), the particles are coherent only over the contact regions, and we get

$$g = g_0 K_L \{ 1 - c_2 T_d^{2(K_{\text{eff}} - \tilde{K}_{\text{eff}})} T^{2(\tilde{K}_{\text{eff}}-1)}$$
$$\times [|V(0)|^2 + |V(l + 2d)|^2] \}. \tag{8}$$

Here c_2 is a constant similar in nature to c_1, but it can depend on v_W and, hence, on V_G. $\tilde{K}_{\text{eff}}$ is also dependent on interactions within the wire, and is given as $\tilde{K}_{\text{eff}} = K_L K_{W\rho}/(K_L + K_{W\rho}) + K_L K_{W\sigma}/(K_L + K_{W\sigma})$. In the low temperature regime (iii), the particles are phase coherent over the whole wire, giving

$$g = g_0 K_L [1 - c_3 T^{2(K_L-1)} T_d^{2(K_{\text{eff}}-\tilde{K}_{\text{eff}})} T_l^{2(\tilde{K}_{\text{eff}}-K_L)}$$
$$\times |V(0) + V(l + 2d)|^2], \tag{9}$$

where the two barriers are seen coherently. c_3 is a constant similar in nature to c_2. (Similar expressions

can be derived if $T_d \ll T_l$, but the conclusions stated below remain unchanged.) As observed in several experiments, the expressions (7)–(9) show that, as *either* the temperature T is raised *or* the length $l + 2d$ is decreased, the conductance corrections become smaller and the conductance approaches integer multiples of g_0 [1,3]. Furthermore, in the high temperature limit $T \gg T_d, T_l$, the conductance correction in (7) is seen to be independent of the QW parameters. Hence, it is independent of V_G and the channel index. Thus, we obtain renormalizations to the ideal values which are themselves plateaulike and uniform for all channels. Such corrections to the conductance explain some of the puzzling features observed in Ref. [3]. Their system has a fairly long contact region of $d \sim 2$–6 μm, which corresponds to $T_d \sim 0.2$–0.7 K; this is much less than the temperature range shown in Fig. 3 of Ref. [3]. Similar flat and uniform conductance corrections have been observed in Refs. [4,8]; this suggests that their systems also include contact regions and that $T \gg T_d$. Interestingly, the low temperature corrections do depend on the QW parameters and, hence, on V_G. Thus, a concrete prediction of our model is that one would fail to see flat plateaus in the conductance for $T \ll T_d$ indicating that the low temperature corrections are dependent on V_G. This has been observed in a recent experiment (see Fig. 3 in Ref. [8]), where the conductances at 1 K show flat and channel independent plateaus, but at 50 mK are neither flat nor channel independent.

To test our model, we have quantitatively fitted the high temperature conductance data presented in the inset of Fig. 3 of Ref. [3]. We find that the best fit for the conductance correction $\delta g \equiv g - 2e^2/h$ is given by

$$\delta g = -0.3512\, T^{-0.1058 - 0.0345\, T}, \tag{10}$$

with a value for the correlation coefficient $R^2 = 0.9955$. This does not match the simple form given in Eq. (7). The T dependent piece in the exponent implies that our model is only qualitatively correct, and a quantitative fit needs a more sophisticated treatment. We will consider elsewhere the effects of an extended transition region between the leads and the contacts (i.e., the parameters K and v varying smoothly as a function of x), as well as more extended barriers lying within the contacts [18]. We find that, although the temperature dependence of the conductance correction then becomes more complicated than that presented in Eq. (7), the flatness of the conductance plateaus and the uniform renormalization for all the subbands remain unaffected. We should emphasize here that a temperature dependence of the conductance correction of the form that we have obtained (decreasing at high temperatures) is a nontrivial effect of the electron interactions, and our simple model has already captured this qualitatively. A noninteracting theory does not have a temperature dependence of this kind.

Finally, we observe that the existence of two weak barriers at the contacts can lead to the occurrence of resonances in regime (iii), where there is phase coherence over the whole wire. Resonant transmission can occur only when $n_{0\rho}$ (defined earlier) is an odd integer, i.e., the phase $\eta = 2k_C d + k_W l = (2n + 1)\pi + eV_G/(\pi^{1/2}\Lambda_{W\rho})$. Experimentally, E_{F1D} and therefore k_W is tuned by V_G as one sweeps across the states of each open channel. Hence, we expect enhanced conductances at energies where k_W matches the condition given above. Such peaks in the conductance of an open channel may already have been seen at $T = 50$ mK at conductances close to multiples of g_0 in Ref. [8]. We expect these resonances caused by the contact barriers to survive when the channels are moved laterally, unlike resonances which may be caused by impurities present inside the wire. (We note that resonances can occur in a noninteracting system also, but the dependence of the resonance condition on the wire parameters and the temperature is different there [20]).

In summary, we have presented a simplified model which can be applied to a large class of quantum wires. Our model shows that a TLL can provide the correct approach towards understanding the observed conductance corrections, although a more accurate modeling is required to get a completely quantitative understanding of the data. We will present elsewhere [18] the details of our calculations as well as some extensions of experimental interest, such as the effects of impurities inside the QW, and a magnetic field where some additional features are observed.

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