Delay Performance of Some Scheduling Strategies in an Input Queuing ATM Switch with Multiclass Bursty Traffic

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Abstract—We consider an $N \times N$ nonblocking, space division, input queuing asynchronous transfer mode (ATM) cell switch, and a class of Markovian models for cell arrivals on each of its inputs. The traffic at each input comprises geometrically distributed bursts of cells, each burst destined for a particular output. The inputs differ in the burstiness of the offered traffic, with burstiness being characterized in terms of the average burst length. We analyze *burst delays* in the situation in which some inputs receive traffic with low burstiness and others receive traffic with higher burstiness. Three policies for head-of-the-line contention resolution are studied: two static priority policies [viz., shorter-expected-burst-length-first (SEBF), longer-expected-burst-length-first (LEBF)] and random selection (RS).

Direct queuing analysis is used to obtain approximations for asymptotic (as $N \rightarrow \infty$) high and low priority mean burst delays with the priority policies. Simulation is used for obtaining mean burst delays for finite N and for the random selection policy. Numerical results show that, as the traffic burstiness increases, the asymptotic analysis can serve as a good approximation only for large switch sizes. Qualitative performance comparisons based on the asymptotic analysis are, however, found to continue to hold for finite switch sizes. It is found that the SEBF policy yields the best delay performance over a wide range of loads, while RS lies in between. SEBF drastically reduces the delay of the less bursty traffic (e.g., distributed computing traffic) while only slightly increasing the delay of the more bursty traffic, e.g., variable bit rate (VBR) video. LEBF causes severe degradation in the delay of less bursty traffic, while only marginally improving the delays of the more bursty traffic. RS can be an adequate compromise if there is no prior knowledge of input traffic burstiness.

I. INTRODUCTION

IN this paper, we consider an $N \times N$ nonblocking, space division, input queuing asynchronous transfer mode (ATM) cell switch, schematically shown in Fig. 1. Performance analysis of such a switch has been done with various traffic assumptions. The studies in [5], [6], and [10] are based on a Bernoulli model for cell arrivals, with each cell independently

Fig. 1. $N \times N$ switch with input queuing.

requesting each output with equal probability. This is referred to in the literature as an independent and uniform traffic model. The switch performance under independent but nonuniform traffic (i.e., the routing probabilities of cells to outputs are unequal) was studied in [12]. Performance analysis with a certain model for correlated input traffic has been reported in [13].

While the study in [13] is concerned with correlated input traffic, it assumes that the traffic on each input link has the same statistical behavior. However, a "local access" ATM switch will receive traffic from metropolitan area networks (MAN's) and also directly from broadband integrated services digital networks (B-ISDN) terminals [1]. It can be expected that since MAN's aggregate traffic from relatively slow sources, the cells on ATM links emanating from MAN's will display low serial correlation in their demands for the output links. The traffic from a B-ISDN terminal, e.g., a high definition television (HDTV) source, however, can be expected to display high serial correlation, even to the extent of delivering consecutive cells destined for the same switch output. Motivated by this situation, we have considered the following input traffic model.

At each input, the cell arrival process is characterized by an $N + 1$ state Markov chain $\{\xi_n, n \geq 1\}$ whose transition probabilities are depicted in Fig. 2. If $\xi_n = m$, $1 \leq m \leq N$, then a cell destined for output m arrives in the n th slot;

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whereas $\xi_n = \emptyset$ denotes that n th slot is empty. Clearly the number of consecutive slots with cells for the same output has a geometric distribution with mean $1/(1-p)$. We refer to this batch of consecutive cells destined for the same output as a “burst” of cells for that output. Upon the completion of a burst to an output, with probability q there is a geometrically distributed sequence of idle slots, and with probability $(1-q)$ a burst for a different output. The output for this new burst is chosen *uniformly* from among the other $(N-1)$ outputs. This cell arrival process model is similar to the one used in [3] and [14]. It follows that the cell arrival rate γ (in cells per slot) is given as $\gamma = (1-q)/(1-pq)$. As discussed above, in certain situations, some input processes display low serial correlation and others high serial correlation. We model this by asserting that a fraction α of the N inputs have traffic with mean burst length $1/(1-p_1)$ (called type 1 inputs) and the rest of the $(1-\alpha)N$ inputs have traffic with mean burst length $1/(1-p_2)$ (called type 2 inputs).

For input queuing with first-in first-out (FIFO) service discipline for each queue, a contention resolution scheme is needed in order to choose one head-of-the-line (HOL) cell when there are two or more input queues with their HOL cells destined for the same output. In most of the previous studies, it is assumed that among the k HOL cells contending for the same output, one cell is randomly (uniformly) selected for transmission across the switch fabric in one slot time. This is called the random selection (RS) policy. In [2] a priority selection policy is investigated. *Each input* receives two classes of traffic as independent Bernoulli streams. The priority introduced is mainly to support traffics with different delay requirements; e.g., real time traffic such as voice is assigned high priority to satisfy its strict delay requirement. We have shown [7], [8] that, when *different input links* have traffic with different statistical behavior, there can be a significant throughput advantage if, when the HOL cells from several input queues contend for the same output, the input with the smaller bursts (on the average) is given priority [referred to as shorter-expected-burst-length-first (SEBF)]. A remarkable finding is that, when there are two traffic classes, giving priority to less bursty traffic can give better throughput than if all the inputs were occupied by this less bursty traffic. Further, the results show that giving priority to the burstier traffic [referred to as longer-expected-burst-length-first (LEBF)] can result in a severe throughput degradation. We have done this by obtaining explicit expressions for the high and low priority saturation throughputs for the asymptotic case (the switch size $N \rightarrow \infty$).

In this paper, we study the impact of the HOL contention resolution policies on the mean burst delay. Note that for traffic such as packet voice or video, *burst delays are more meaningful* than individual cell delays. In particular, we compare the RS, SEBF, and LEBF scheduling strategies by comparing the mean burst delays with these schemes when we have the multiclass bursty traffic scenario described above. In Section II, we perform a detailed delay analysis for N (switch size) $\rightarrow \infty$ for the priority contention resolution scheme. Section II-A deals with the high priority mean burst delay, and Section II-B with the low priority mean burst delay.

Fig. 2. Markov chain transition diagram for the cell arrival process at each input. There are N states in which the input is carrying cells destined for a particular output; only two such states l and m are shown.

Section III deals with the numerical results of the analysis of Section II and comparisons with simulation results for finite switch size N . For the random selection scheme the analysis with multiclass bursty traffic is intractable, and hence this is studied by simulation. A comparison of the three schemes (i.e., RS, SEBF, and LEBF) based on the analytical and simulation results is presented in Section IV. We conclude in Section V.

A. Notational Note

We follow the convention that if X denotes a continuous random variable, $X(x)$ denotes its cumulative distribution function (cdf), $\tilde{X}(s)$ denotes the Laplace Stieltjes Transform of $X(x)$, and $\tilde{X}(s)$ denotes the Laplace Transform of $X(x)$. If X is discrete then $x(k)$ denotes its probability mass function (pmf), and $\tilde{x}(z)$ its probability generating function (pgf).

II. ASYMPTOTIC ($N \rightarrow \infty$) DELAY ANALYSIS FOR THE PRIORITY CONTENTION RESOLUTION SCHEME

A fraction α of the inputs carry type 1 traffic with burst length parameter p_1 and the remaining inputs carry type 2 traffic with parameter p_2 . HOL contention is always resolved in favor of type 1 inputs. Random selection is used within a type. Recall that if $p_1 < p_2$ we have the SEBF scheme, and if $p_1 > p_2$ we have LEBF scheme. We do the queue length analysis for infinite input buffers, and for the asymptotic case ($N \rightarrow \infty$).

Before any delay analysis can begin, we need to obtain the saturation throughput of the switch, and establish a stability condition for the queue length processes. We have performed a detailed saturation throughput analysis of the input queuing switch with multiclass traffic, as modeled above (see [7]). We have obtained closed form expressions for the asymptotic ($N \rightarrow \infty$) saturation throughput of high and low priority cells. Further, we have argued in [8] that saturation throughput yields a sufficient condition for the stability of the input queues; i.e., if the arrival rate at each input queue is less than the rate

at which cells are served from that input, if the queues were saturated, then the input queues will be stable. In the analysis presented in this paper, it is assumed that the stability condition is met, and in all the numerical results, the parameters are chosen so that the input queues are stable.

If we focus on all the HOL cells of both type 1 and type 2 input queues destined for a particular output, we can consider them as belonging to a logical queue. The queue length process of this logical queue is a 2-D stochastic process representing the number of type 1 and type 2 HOL cells contending for the particular output. From this logical queue, in each slot, either a type 1 HOL cell is randomly chosen for service or a type 2 HOL cell is chosen (if there are no type 1 HOL cells in this queue). Thus, associated with each *output* there will be a logical queue of HOL cells; this is called an *HOL contention process*.

Burst delay at an input queue is defined as the time from the first cell of the burst arriving at the input buffer, until the last cell of the burst is transmitted across the switch fabric to the output link. Burst delay has two components; the first is the time from when the first cell of the burst arrives at the input buffer, until this cell reaches the head of the input queue. This is the *burst waiting time*. When the leading cell of the burst reaches the head of the queue, it joins the HOL contention process associated with the output to which this burst is destined, and begins to contend for transmission to its designated output link. Once this cell is transmitted, the subsequent cells of the burst move to the head of the input queue to contend, in turn, for the same output until the last cell of the burst is transmitted. Thus, the second component of the burst delay is the *sojourn time* of the entire burst in an HOL contention process. This is called the *HOL contention delay*.

A. High Priority Burst Delay

We focus on a type 1 input queue (there are αN such queues), henceforth called the *tagged input queue*. The presence of low priority traffic on type 2 inputs is transparent to the high priority traffic. Let γ_1 be the cell arrival rate at each type 1 input.

Consider a *tagged burst* in the tagged input queue. Let S_H denote the HOL contention delay of this burst. We obtain the pgf of S_H . Focus on the *tagged output* to which the cells of this tagged burst are destined. We can view the HOL cells contending for this output as each representing an (HOL) *burst* at their respective inputs.

Let $k(\geq 0)$ index the slots. Consider the epoch of the boundary between two slots. Just prior to this epoch, one burst, among those queued for the tagged output, is selected at random and the first cell of the burst is transmitted. The same is done for the other outputs also. Now some bursts will have cells remaining in them and will continue to contend in the same contention processes, whereas for some bursts the last cell has just been transmitted. At the inputs corresponding to the latter category of bursts, new bursts may be waiting. These will result in *arrivals* to the HOL contention processes. Let H_k denote the number of HOL cells in the tagged output contention process *just after slot k* (the start of slot

$k + 1$) *but prior to new HOL burst arrivals*. If the HOL burst corresponding to a cell just served has not completed then this burst is included in H_k . Clearly

$$H_{k+1} = (H_k + A_{k+1} - Z_k)^+$$

where A_{k+1} is the number of new HOL burst arrivals (into the tagged output contention process) at the beginning of slot $k + 1$, and

$$Z_k = \begin{cases} 0, & \text{w.p. } p_1 \\ 1, & \text{w.p. } 1 - p_1. \end{cases}$$

In the asymptotic case, the process $\{A_k\}$ of arrivals of the first cells of bursts from other high priority inputs to the tagged HOL contention process is a sequence of independent Poisson random variables with mean $\alpha\lambda_1$, where λ_1 is the burst arrival rate at the high priority input queues ($\lambda_1 = \gamma_1(1 - p_1)$) [8]. The HOL contention process can thus be modeled as a *BD/D/1 queue with feedback*, where Batch Deterministic (BD) stands for the (deterministic) arrival of a (Poisson) batch of bursts at the beginning of every time slot, D stands for the one slot service time of a cell, and the feedback occurs with probability p_1 . Let H denote the stationary random variable for $\{H_k\}$. With standard techniques [11, Sec. 5.6], we obtain

$$\tilde{h}(z) = \frac{(1 - z)(1 - p_1 - \alpha\lambda_1)}{[1 - p_1(1 - z)]e^{-\alpha\lambda_1(1 - z)} - z}. \quad (1)$$

1) *PGF of High Priority HOL Contention Delay*: Since the process of arrivals of new HOL bursts to the tagged contention process is a sequence of independent and identically distributed (i.i.d.) (Poisson) random variables, the HOL contention time of the tagged high priority burst (S_H) is the time to absorption in a Markov chain. The state of this Markov chain is the number of HOL bursts (including the tagged burst) contending for the output. Recall that there is only one cell from the tagged burst in the tagged HOL contention process at any time. In each state of the Markov chain there is a probability that the cell from the tagged burst is selected for service, and that this is the last cell of the burst. This is the probability of absorption. Thus the absorbing state of the Markov chain corresponds to the successful transmission of the *last* cell of the tagged burst. This Markov chain has an infinite state space. We truncate the state space by assuming that there will not be more than m_1 bursts in the HOL contention process at any time and the error due to this truncation of state space is reduced to a negligibly small value by making m_1 sufficiently large. Thus we have an $(m_1 + 1)$ -state Markov chain with transition probability matrix

$$P = \begin{bmatrix} \mathbf{R} & \mathbf{R}_0 \\ \mathbf{0} & 1 \end{bmatrix}$$

where $\mathbf{R}$ is an $m_1 \times m_1$ substochastic matrix, such that $\mathbf{I} - \mathbf{R}$ is nonsingular ($\mathbf{I}$ is the identity matrix). The initial probability vector is (y, y_{m+1}) (given below). It now follows that S_H has discrete *phase type distribution (PH-distribution)* with representation $(y, \mathbf{R})$ [16, Sec. 2.2]. Letting $c(n)$ denote the pmf of the batch size of the new bursts arriving at the HOL

process at the beginning of each time slot, the elements of $\mathbf{R}$, i.e., r_{ij} , $1 \leq i, j \leq m_1$, are found to be

$$r_{ij} = \begin{cases} \frac{i-1}{i} (1-p_1)c(0), & 1 < i \leq m_1, \text{ if } j = i-1 \\ p_1 c(j-i) + \frac{i-1}{i} (1-p_1) \cdot c(j-i+1), & \text{if } j \geq i, 1 \leq i, j \leq m_1 \\ 0, & \text{otherwise} \end{cases},$$

where

$$c(n) = \frac{e^{-\alpha\lambda_1} (\alpha\lambda_1)^n}{n!}.$$

Further

$$\mathbf{y} = [y(n), 1 \leq n \leq m_1]$$

where

1) n HOL bursts, including itself

$\mathbf{y}$ is obtained from the stationary distribution of $\{H_k\}$, and the distribution of the size of the batch of bursts in which the tagged burst arrives at the output contention process.

Let, for $n \geq 1$, $\nu(n)$ be *batch biased probability* that tagged burst arrives in a batch of bursts of size n

Then [17, Sec. 2.5]

$$\begin{aligned} \nu(n) &= \frac{nc(n)}{\sum_{j=1}^{\infty} jc(j)} \\ &= \frac{e^{-\alpha\lambda_1} (\alpha\lambda_1)^{n-1}}{(n-1)!}. \end{aligned} \quad (2)$$

Since the bursts arrive in i.i.d. batches (that are Poisson distributed), the process indicating whether or not a batch is empty is a Bernoulli process. Hence we can make use of the discrete time version of Poisson Arrivals See Time Averages (PASTA), i.e., Geometric Arrivals See Time Averages (GASTA) [4], to assert that the stationary distribution of the number of HOL bursts found by a nonempty batch of bursts is equal to the stationary distribution of $\{H_k\}$. Hence, $y(n)$ can be computed as the convolution sum of $h(n)$ and $\nu(n)$

$$y(n) = \sum_{i=0}^{n-1} \nu(n-i)h(i), \quad n \geq 1.$$

Note that $h(n)$ is the pmf of H , and it is obtained from $\tilde{h}(z)$ (1) as

$$h(n) = \lim_{z \rightarrow 0} \frac{1}{n!} \frac{d^{(n)}}{dz} \tilde{h}(z).$$

We need only a finite number of terms of this distribution because of the truncation of the state space mentioned above. Finally, since S_H has discrete phase type distribution $(\mathbf{y}, \mathbf{R})$, we get the pgf of S_H as [16]

$$\tilde{s}_H(z) = z\mathbf{y}(\mathbf{I} - z\mathbf{R})^{-1}\mathbf{R}_0. \quad (3)$$

2) *Mean High Priority Burst Delay*: Observe that the HOL contention delay S_H of a burst can be viewed as its effective service time at the input queue. In the asymptotic case (i.e., $N \rightarrow \infty$), the bursts destined for the same output link are separated far apart (in time) on an input link so that the HOL contention delays of these bursts can be considered to be independent of each other. Also, as $N \rightarrow \infty$, the HOL contention processes for different outputs tend to be mutually independent [8]. Thus the HOL contention delays of successive bursts in an input buffer can be considered to be a sequence of i.i.d. random variables, which allows us to model the tagged input buffer as a $/GI/1$ queue. Li [12] has argued similarly that, in the limit ($N \rightarrow \infty$), each input queue can be considered as an independent queue with an i.i.d. service sequence.

The actual cell arrival process on the tagged input, described in Section I, results in a cell arrival rate (in cells per slot) $\gamma_1 = (1 - q_1)/(1 - p_1 q_1)$, or alternatively, a burst arrival rate $\lambda_1 = \gamma_1(1 - p_1)$. For analytical simplicity we approximate the *burst* arrival process (i.e., the point process of the arrival epochs of the leading cells of the bursts) at the tagged input queue by a Bernoulli process with rate λ_1 , and assumes that a batch of cells constituting a burst arrive in a single slot. Note that the actual burst interarrival time is the sum of a geometric burst duration (at least one slot) and a geometric idle period (possibly zero slot). We expect that the assumption of geometric interarrival time will lead to an *over estimation* of the burst delay. Note that if the interarrival times are modeled exactly the batch arrival assumption in itself does not have any effect on the burst delay since the time it takes to transmit a cell across the switch is the same as the time it takes for a cell to arrive on the input link; so the next cell in the burst is already in the input buffer when the previous cell is fully transmitted.

Thus the tagged input is modeled as a Geom/GI/1 queue. Observe that the service time in this model, i.e., the HOL contention delay of the burst, is insensitive to the assumptions made above since it takes at least one slot for each HOL cell to get transmitted onto the output link.

Using the known results for the Geom/GI/1 queue [15] we obtain the mean waiting time for the tagged burst at the input queue. Let W_H denote the time from the arrival of the tagged burst, until it reaches the head of the queue. Then

$$EW_H = \frac{\lambda_1 E[S_H(S_H - 1)]}{2(1 - \lambda_1 ES_H)}$$

where ES_H and $E[S_H(S_H - 1)]$ are obtained from (3) as

$$\begin{aligned} ES_H &= \mathbf{y}(\mathbf{I} - \mathbf{R})^{-1}\mathbf{e} \\ E[S_H(S_H - 1)] &= 2\mathbf{y}\mathbf{R}(\mathbf{I} - \mathbf{R})^{-2}\mathbf{e} \end{aligned}$$

where $\mathbf{e}$ is the unit vector.

The mean burst delay at a high priority input queue is given by $EW_H + ES_H$. The mean burst delay that we compute with the Geom/GI/1 model should be an upper bound to the actual value, as the squared coefficient of variation of the actual burst interarrival time is less than that of the assumed geometric interarrival time. This observation is borne out by the simulation results presented in Section III.

Fig. 3. Schematic representation of the low priority burst sojourn time in an HOL contention queue.

B. Low Priority Burst Delay

Now we focus on a type 2 input queue, to which arrives low priority traffic with a different burst length parameter, p_2 . Let the cell arrival rate be γ_2 and the corresponding burst arrival rate at the input be λ_2 . Recall that there are $(1 - \alpha)N$ type 2 input queues.

First we compute the pgf of the HOL contention delay S_L of a low priority burst at the tagged input queue. The tagged low priority burst moves to the head of the queue and joins an HOL contention process. Consider the HOL contention process that this burst joins. In the asymptotic case ($N \rightarrow \infty$), independent Poisson batches of high priority and low priority HOL bursts with mean $\alpha\lambda_1$ and $(1 - \alpha)\lambda_2$, respectively, arrive at the tagged HOL contention process at the beginning of each time slot. A low priority cell in the HOL contention process is selected for transmission, only if there are no high priority cells in that process. It appears at first that in order to analyze S_L we will need to analyze the 2-D process of low and high priority bursts in the contention process. The mean of S_L (i.e., ES_L) can be obtained, however, by a direct method, as we show in the Appendix.

1) *PGF of Low Priority HOL Contention Delay:* Consider the tagged low priority burst. When it arrives in its HOL contention process, it observes an ongoing high priority busy period (see Fig. 3). The residual busy period B_0 is determined by the number of high priority bursts already there at the time of arrival of the tagged low priority burst, and the batch size of the HOL high priority bursts arriving into the same contention process along with the tagged low priority burst. The number of low priority and high priority bursts in the HOL contention process are dependent. Therefore we need the joint probability distribution of the number of high priority and low priority bursts in the HOL contention process.

Let $\mathbf{X}_k = [X_k^{(1)}, X_k^{(2)}]$, where $X_k^{(1)}$ [respectively, $X_k^{(2)}$] is the number of high priority (respectively low priority) bursts in the HOL contention process just after slot k but prior to new HOL burst arrivals. If the HOL burst corresponding to the cell just served has not completed then this burst is included in $\mathbf{X}_k$. Clearly $\{\mathbf{X}_k, k \geq 0\}$ is a 2-D homogeneous Markov chain. Let the transition probabilities of this Markov chain be denoted by $p_{i_1 i_2, j_1 j_2}$, i.e.,

$$p_{i_1 i_2, j_1 j_2} = \Pr[X_{n+1}^{(1)} = j_1, X_{n+1}^{(2)} = j_2 / X_n^{(1)} = i_1, X_n^{(2)} = i_2].$$

Let $\mathbf{X} = [X^{(1)}, X^{(2)}]$ be the stationary random vector for $\{\mathbf{X}_k\}$ and $x(k_1, k_2)$ be the joint probability distribution of $\mathbf{X}$. With C_1 and C_2 as defined earlier, $\bar{p}_i$ denoting $1 - p_i$,

$i = 1, 2$, we have the following set of equations for the transition probabilities

$$\begin{aligned} p_{00,00} &= c_1(0)c_2(0) + c_1(1)c_2(0)\bar{p}_1 + c_1(0)c_2(1)\bar{p}_2 \\ p_{0n_2, 0(n_2-1)} &= c_1(0)c_2(0)\bar{p}_2; \quad n_2 > 0 \\ p_{0n_2, 0(n_2+m_2)} &= c_1(1)c_2(m_2)\bar{p}_1 + c_1(0)c_2(m_2)p_2 \\ &\quad + c_1(0)c_2(m_2+1)\bar{p}_2; \\ &\quad n_2 > 0, m_2 \geq 0 \quad \text{or} \quad n_2 \geq 0, m_2 > 0 \\ p_{n_1 n_2, (n_1-1)(n_2+m_2)} &= c_1(0)c_2(m_2)\bar{p}_1; \quad n_1 > 0, n_2, m_2 \geq 0 \\ p_{n_1 n_2, (n_1+m_1)(n_2+m_2)} &= c_1(m_1)c_2(m_2)p_1 + c_1(m_1+1)c_2(m_2)\bar{p}_1; \\ &\quad n_1, n_2, m_2 \geq 0, m_1 > 0 \\ &\quad \text{or} \quad n_2, m_1, m_2 \geq 0, n_1 > 0. \end{aligned}$$

The equilibrium joint distribution is given by

$$x(k_1, k_2) = \sum_{l_1=0}^{\infty} \sum_{l_2=0}^{\infty} x(l_1, l_2) p_{l_1 l_2, k_1 k_2} \quad k_1, k_2 \geq 0. \quad (4)$$

Now defining the 2-D generating function

$$\tilde{x}(z_1, z_2) = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} z_1^{k_1} z_2^{k_2} x(k_1, k_2)$$

and applying it to (4), we obtain after some manipulations

$$\begin{aligned} \tilde{x}(z_1, z_2) &= \{(\tilde{x}_0(z_2)c_1(0)\tilde{c}_2(z_2) \\ &\quad \cdot [z_1(\bar{p}_2 + p_2 z_2) - z_2(\bar{p}_1 + p_1 z_1)] \\ &\quad + x(0,0)z_1 c_1(0)c_2(0)\bar{p}_2(z_2 - 1)\} / \\ &\quad \{z_2[z_1 - \tilde{c}_1(z_1)\tilde{c}_2(z_2)(\bar{p}_1 + p_1 z_1)]\} \end{aligned} \quad (5)$$

where

$$\tilde{x}_0(z_2) = \sum_{k_2=0}^{\infty} x(0, k_2) z_2^{k_2}.$$

To solve for the unknown function $\tilde{x}_0(z_2)$, we equate $\lim_{z_1 \rightarrow 1} \tilde{x}(z_1, z_2)$ to the generating function of the marginal distribution of the number of low priority bursts. Noting that the marginal process $\{X_k^{(2)}\}$ is just $\{X_k\}$ defined in the Appendix, we have

$$\lim_{z_1 \rightarrow 1} \tilde{x}(z_1, z_2) = \tilde{x}(z_2)$$

where $\hat{x}(z_2)$ is given by (A.9) in the Appendix. This yields

$$\hat{x}_0(z_2) = \frac{EC_2(1-p_2-E\Gamma)z_2[1-\hat{\gamma}(z_2)] + x(0,0)c_1(0)c_2(0)\bar{p}_2}{E\Gamma[\hat{\gamma}(z_2)(\bar{p}_2+p_2z_2)-z_2]} \cdot \frac{c_1(0)(1-p_2)\hat{c}_2(z_2)}{c_1(0)(1-p_2)\hat{c}_2(z_2)}.$$

The unknown quantity $x(0,0)$ is obtained by applying the normalizing condition, i.e.,

$$\lim_{z_1 \rightarrow 1} \lim_{z_2 \rightarrow 1} \hat{x}(z_1, z_2) = 1.$$

Thus, we get

$$c_1(0)c_2(0)x(0,0) = 1 - \frac{EC_1}{1-p_1} - \frac{EC_2}{1-p_2}.$$

Observe that the left hand side of the above formula is the fraction of idle slots at the output corresponding to the tagged HOL process. Since EC_1 and EC_2 are high and low priority burst arrival rates into this HOL process, and $1/(1-p_1)$ and $1/(1-p_2)$ are the respective mean burst lengths, this formula is *intuitively correct*.

An application of GASTA again yields that the joint probability distribution $x(k_1, k_2)$ is also the joint probability distribution of the number of high priority and low priority HOL bursts as found by a nonempty batch of low priority HOL bursts on its arrival.

Let $r(j_1, j_2)$ be the probability that the tagged low priority HOL burst starts contention with j_1 high priority HOL bursts and j_2 low priority HOL bursts. It is clear that (see Fig. 3)

$$r(j_1, j_2) = \sum_{k_2=0}^{j_2-1} \sum_{k_1=0}^{j_1} x(k_1, k_2)c_1(j_1-k_1)\hat{c}_2(j_2-k_2) \quad (6)$$

where $\hat{c}_2(n) = \Pr(\text{batch size of low priority burst bringing the tagged one into the HOL contention process is } n)$; this will be a *batch size biased probability*, i.e., $\hat{c}_2(n) = [nc_2(n)/EC_2]$.

In Fig. 3, B_0 is the residual high priority busy period as observed by the tagged low priority burst on its arrival into the HOL contention process, and U is the remaining sojourn time of the tagged burst in the HOL process. Thus

$$S_L = B_0 + U.$$

We now analyze the random variable U . Let $\{M_k, k \geq 0\}$ denote the number of low priority bursts (in the tagged HOL process) at the beginning of the $(k+1)$ st low priority cell service during the period U . Thus M_0 is the number of low priority bursts at the beginning of U (see Fig. 3). Let $\{T_k, k \geq 0\}$ denote the number of slots between successive low priority cell services during the period U . Observe that $\{(M_k, T_k), k \geq 0\}$ is a Markov Renewal process, and U is the time to "absorption" in this process, where absorption corresponds to the successful transmission of the last cell of the tagged low priority burst. Each time a cell is served, there is a probability that the cell from the tagged low priority burst is selected for service, and that this is the last cell of the tagged burst. This is the probability of absorption. We truncate the state space by assuming that there will not be more than m_2

low priority bursts in the tagged HOL contention process at any time.

Consider the discrete time Markov Renewal Process $\{(M_k, T_k), k \geq 0\}$ with transition probability matrix for the Markov chain $\{M_k, k \geq 0\}$ of the form

$$\mathbf{P} = \begin{bmatrix} \mathbf{Q} & \mathbf{Q}_0 \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$$

where $\mathbf{Q}$ is an $m_2 \times m_2$ matrix and $\mathbf{Q}_0$ is an $m_2 \times 1$ vector. Let α denote the absorbing state.

Using the random variable Γ from the Appendix, clearly, the elements of $\mathbf{Q}$, i.e., q_{ij} , $1 \leq i, j \leq m_2$, are given by

$$q_{ij} = \begin{cases} \frac{(i-1)}{i}(1-p_2)\gamma(0), & \text{if } j = i-1 \\ p_2\gamma(j-i) + \frac{i-1}{i} \cdot (1-p_2)\gamma(j-i+1), & \text{if } j \geq i \\ 0, & \text{otherwise} \end{cases}$$

and the elements of the vector $\mathbf{Q}_0$ by

$$q_{i\alpha} = \frac{1}{i}(1-p_2), \quad 1 \leq i \leq m_2.$$

Observe that

$$\Pr(T_k = n \mid M_{k+1} = j, M_k = i) = \frac{\Pr(T_k = n \mid M_k = i) \Pr(M_{k+1} = j \mid T_k = n, M_k = i)}{\Pr(M_{k+1} = j \mid M_k = i)}.$$

Let L_{ij} be a random variable with this pmf, i.e.,

$$l_{ij}(n) := \Pr(T_k = n \mid M_{k+1} = j, M_k = i)$$

and denote by $\tilde{l}_{ij}(z)$ the corresponding pgf. Also, denote by $U^{(i)}$ the time to absorption with the initial state i , i.e., $M_0 = i$. Then

$$\tilde{u}^{(i)}(z) = q_{i\alpha}\tilde{l}_{i\alpha}(z) + \sum_{j \neq \alpha} q_{ij}\tilde{l}_{ij}(z)\tilde{u}^{(j)}(z).$$

Since the sojourn time in the last state before absorption is 1 slot with probability 1 (see Fig. 3), $\tilde{l}_{i\alpha}(z) = z$ and we have

$$\tilde{u}^{(i)}(z) = zq_{i\alpha} + \sum_{j \neq \alpha} q_{ij}\tilde{l}_{ij}(z)\tilde{u}^{(j)}(z). \quad (7)$$

Setting

$$\tilde{\phi}_{i\alpha}(z) = zq_{i\alpha}$$

and

$$\tilde{\phi}_{ij}(z) = q_{ij}\tilde{l}_{ij}(z)$$

and using the vector notation, (7) can be written as

$$\begin{aligned} \tilde{\mathbf{u}}(z) &= \tilde{\Phi}_0(z) + \tilde{\Phi}(z)\tilde{\mathbf{u}}(z) \\ &= [\mathbf{I} - \tilde{\Phi}(z)]^{-1}\tilde{\Phi}_0(z) \end{aligned} \quad (8)$$

where the $m_2 \times 1$ vector $\tilde{\Phi}_0(z) = [\tilde{\phi}_{i\alpha}(z)]$ and the $m_2 \times m_2$ matrix $\tilde{\Phi}(z) = [\tilde{\phi}_{ij}(z)]$. $(\mathbf{I} - \mathbf{Q})$ is nonsingular, and hence the inverse in the above formula exists.

Now we are ready to obtain the generating function of the HOL contention delay S_L of the tagged low priority burst. With reference to Fig. 3, we can write

$$\tilde{s}_L(z) = \sum_{n=1}^{m_2} \sum_{l=0}^{\infty} p(l, n) z^l \tilde{u}^{(n)}(z) \quad (9)$$

where $\tilde{u}^{(n)}(z)$ is given by the n th component of the vector $\tilde{\mathbf{u}}(z)$ (8) and $p(l, n) = \Pr(B_0 = l, M_0 = n)$. Recall that B_0 is the residual high priority busy period seen by the arriving tagged low priority burst and M_0 is the initial state of the $(m_2 + 1)$ state Markov chain $\{M_k, k \geq 0\}$ defined above.

Define

- 1) $B_0^{(n)} :=$ busy period initiated by n high priority bursts,
- 2) $\Gamma^{(l)} :=$ total number of low priority bursts arriving at the tagged HOL process during l time slots.

Clearly

$$p(l, n) = \sum_{j_2=1}^n \sum_{j_1=0}^{m_1} r(j_1, j_2) b_0^{(j_1)}(l) \gamma^{(l)}(n - j_2) \quad (10)$$

where $r(j_1, j_2)$ is given by (6). As mentioned earlier we assume that not more than m_1 high priority bursts will be present in an HOL contention process at any time, which amounts to truncating the second summation in the above equation to m_1 . It is obvious that the pgf of $\Gamma^{(l)}$ is

$$\tilde{\gamma}^{(l)}(z) = e^{-l(1-\alpha)\lambda_2(1-z)}.$$

To obtain the pgf of $B_0^{(n)}$, note first that

$$\tilde{b}_0^{(j_1)}(z) = [\tilde{b}_0^{(1)}(z)]^{j_1}$$

and the pgf of the service time of high priority cells constituting a high priority burst is $z(1-p_1)/(1-p_1z) =: \tilde{\psi}(z)$. Moreover independent Poisson batches of high priority bursts are arriving in every time slot. It is easily seen that

$$\tilde{b}_0^{(1)}(z) = \tilde{\psi}\{z\tilde{c}_1(\tilde{b}_0^{(1)}(z))\}.$$

Finally, substituting $p(l, n)$ (10) and $\tilde{u}^{(n)}(z)$ (7) in (9), we obtain the pgf of the HOL contention delay of the tagged low priority burst, $\tilde{s}_L(z)$. To examine the effect of the truncation, later we will compare the mean value ES_L obtained from $\tilde{s}_L(z)$ with that found in the Appendix by (A.10).

2) *Mean Low Priority Burst Delay:* Following the discussion of Section II-A, the tagged low priority input queue is modeled as a Geom/GI/1 queue in order to compute the mean burst waiting time. Again, using similar notations as in Section II-A, mean delay of a low priority burst is given by

$$EW_L + ES_L = \frac{\lambda_2 E[S_L(S_L - 1)]}{2(1 - \lambda_2 ES_L)} + ES_L$$

where ES_L and $E[S_L(S_L - 1)]$ are obtained from (9) as

$$ES_L = \sum_{n=1}^{m_2} \sum_{l=0}^{\infty} p(l, n) [l \tilde{u}^{(n)}(1) + \tilde{u}^{(n)'}(1)] \quad (11)$$

$$E[S_L(S_L - 1)] = \sum_{n=1}^{m_2} \sum_{l=0}^{\infty} p(l, n) [l(l-1) \tilde{u}^{(n)}(1) + 2l \tilde{u}^{(n)'}(1) + \tilde{u}^{(n)''}(1)]. \quad (12)$$

TABLE I
MEAN HOL CONTENTION DELAY FOR LOW PRIORITY BURST COMPUTED FROM TWO METHODS; $ES_L^{(1)}$ IS FROM (A.10) AND $ES_L^{(2)}$ IS FROM THE GENERATING FUNCTION (11); $p_1 = 0.3$, $p_2 = 0.7$, $\gamma_2 = 0.2$

γ_1	SEBF		LEBF	
	$ES_L^{(1)}$	$ES_L^{(2)}$	$ES_L^{(1)}$	$ES_L^{(2)}$
.1	3.891	3.889	1.963	1.91
.2	4.171	4.164	2.054	1.999
.3	4.496	4.475	2.158	2.097
.4	4.879	4.823	2.276	2.2
.5	5.335	5.210	2.413	2.305

Note that $\tilde{u}^{(n)'}(1)$ and $\tilde{u}^{(n)''}(1)$ are the elements of the vectors $\tilde{\mathbf{u}}'(1)$ and $\tilde{\mathbf{u}}''(1)$, respectively. We obtain $\tilde{\mathbf{u}}'(z)$ and $\tilde{\mathbf{u}}''(z)$ by differentiating $\tilde{\mathbf{u}}(z)$ (8). Thus

$$\tilde{\mathbf{u}}'(z) = [\mathbf{I} - \tilde{\Phi}(z)]^{-1} \tilde{\Phi}'(z) [\mathbf{I} - \tilde{\Phi}(z)]^{-1} \tilde{\Phi}_0(z) + [\mathbf{I} - \tilde{\Phi}(z)]^{-1} \tilde{\Phi}_0'(z)$$

$$\tilde{\mathbf{u}}''(z) = 2\{[\mathbf{I} - \tilde{\Phi}(z)]^{-1} \tilde{\Phi}'(z)\}^2 [\mathbf{I} - \tilde{\Phi}(z)]^{-1} \tilde{\Phi}_0(z) + [\mathbf{I} - \tilde{\Phi}(z)]^{-1} \tilde{\Phi}''(z) [\mathbf{I} - \tilde{\Phi}(z)]^{-1} \tilde{\Phi}_0(z) + 2[\mathbf{I} - \tilde{\Phi}(z)]^{-1} \tilde{\Phi}'(z) [\mathbf{I} - \tilde{\Phi}(z)]^{-1} \tilde{\Phi}_0'(z) + [\mathbf{I} - \tilde{\Phi}(z)]^{-1} \tilde{\Phi}_0''(z).$$

III. NUMERICAL RESULTS: ANALYSIS AND SIMULATION

In getting the numerical results from the analysis, we have made some simplifications. As mentioned earlier we assume that not more than m_1 high priority bursts and m_2 low priority bursts will be present in an HOL contention process at any time. Further, we assume that the initial busy period as observed by the arriving tagged low priority burst will not be more than m_3 slots. This amounts to truncating the second summation in (9) to m_3 . The numerical error due to these truncations is reduced to a negligibly small value by choosing sufficiently large values for the parameters m_1 , m_2 , and m_3 .

We have used Mathematica [18] for numerical computation. We obtain the 2-D distributions $x(k_1, k_2)$, for $1 \leq k_1 \leq m_1$ and $1 \leq k_2 \leq m_2$, from the 2-D generating function $\tilde{x}(z_1, z_2)$ using the following fact:

$$x(k_1, k_2) = \lim_{z_1 \rightarrow 0} \lim_{z_2 \rightarrow 0} \frac{1}{k_1! k_2!} \frac{d^{(k_1)}}{dz_1} \frac{d^{(k_2)}}{dz_2} \tilde{x}(z_1, z_2).$$

To quantify the numerical error due to the above mentioned truncations, we compared ES_L computed from (11) with that obtained from the closed form expression in (A.10). Tables I and II illustrate this comparison. Recall that in the SEBF policy the low priority traffic is type 2 with mean burst length $(1 - p_2)^{-1}$, and in the LEBF policy the low priority traffic is type 1 with mean burst length $(1 - p_1)^{-1}$. In the typical numerical illustrations of Tables I and II we have used $p_1 = 0.3$, $p_2 = 0.7$, $m_1 = m_2 = 6$, and $m_3 = 8$. Table I shows the comparison with type 2 cell arrival rate γ_2 fixed at 0.2, and Table II shows the results with type 1 cell arrival rate γ_1 fixed at 0.45. The agreement is quite good and we accept these values of m_1 , m_2 , and m_3 as adequate.

Next we compare the analytical results with those from simulations. We recall that the analytical results are for switch

TABLE II
MEAN HOL CONTENTION DELAY FOR LOW PRIORITY BURST COMPUTED FROM TWO METHODS; $ES_L^{(1)}$ IS FROM (A.10) AND $ES_L^{(2)}$ IS FROM THE GENERATING FUNCTION (11); $p_1 = 0.3$, $p_2 = 0.7$, $\gamma_1 = 0.45$

γ_2	SEBF		LEBF	
	$ES_L^{(1)}$	$ES_L^{(2)}$	$ES_L^{(1)}$	$ES_L^{(2)}$
.05	4.637	4.627	1.835	1.824
.15	4.933	4.922	2.155	2.101
.25	5.274	5.261	2.551	2.412
.35	5.672	5.646	3.049	2.731

Fig. 4. Comparison of mean burst delay (normalized w.r.t. the mean burst length) for single class traffic; $p = 0.1$.

Fig. 5. Comparison of mean burst delay (normalized w.r.t. the mean burst length) for single class traffic; $p = 0.6$.

size $N \rightarrow \infty$. We have used SIMSCRIPT II.5 for simulation. Figs. 4–6 illustrate the comparison of normalized mean burst delay (normalized with respect to the mean burst length) obtained from the *asymptotic* analysis and from the simulation with different values for the switch size parameter N . These results are for single class traffic with burst length parameter of 0.1, 0.6, and 0.9, respectively, i.e., mean burst lengths of $\frac{1}{0.9}$, 2.5, and 10, respectively.

The curves labeled *hybrid approach* are obtained in the following way. The pgf of the effective service time of a burst in an input buffer (HOL contention time for the burst) is obtained by the asymptotic analysis of Section II-A. The distribution of the effective burst service time is then obtained by numerical inversion of the pgf [9]. We have simulated a single server FIFO queue with the service time distribution obtained in this

Fig. 6. Comparison of mean burst delay (normalized w.r.t. the mean burst length) for single class traffic; $p = 0.9$.

manner, and with the actual burst arrival process as described in the Introduction. Recall that in the Geom/GI/1 analysis we have used the geometric approximation for the burst arrival process. The hybrid approach reduces simulation time, and also captures the cell arrival process more accurately than the geometric approximation.

We observe from these figures that mean burst delay increases as N increases, and the asymptotic analysis gives an upper bound. For Bernoulli traffic (i.e., $p = 0$ in our model), it was observed in [10] that the mean *cell* delay for $N \geq 16$ is closely approximated by the asymptotic analysis ($N \rightarrow \infty$). Note that in our work we have studied *burst* delays, which we believe are more appropriate for packetized isochronous traffic. Our results in Figs. 4–6 show, however, that progressively larger values of switch size N are needed to come close to the asymptotic analysis as the burst length parameter p increases. For $p = 0.1$, the asymptotic analysis is a good approximation for $N = 64$. For larger values of p , even for $N = 64$ or $N = 128$ the asymptotic analysis yields only an upper bound. The Geom/GI/1 analysis is close to the more accurate hybrid results for $p = 0.1$, but becomes a loose upper bound for $p = 0.6$ and $p = 0.9$. The fact that Geom/GI/1 yields an upper bound (even for the asymptotic mean delay) is as expected in our discussion accompanying the analysis (end of Section II-A). We note from Fig. 6 that the *hybrid approach* is a tighter upper bound for $N = 128$, suggesting that for $p = 0.9$ one needs to go upto $N = 128$ to get close to the asymptotic results.

There is one important reason why asymptotic analysis can only be expected to yield upper bounds: in the asymptotic case, the input queues saturate at a lower arrival rate than for finite N (see [8]). In [7], we have stated the following formula for the asymptotic saturation throughput $T(p)$ for the single class case, with each input carrying traffic of burstiness parameter p

$$T(p) = \frac{2 - \sqrt{4 - 2(1 - p)}}{(1 - p)}.$$

Calculation using this formula yields $T(0.1) = 0.574$, $T(0.6) = 0.527$, and $T(0.9) = 0.506$. We observe from Figs. 4–6 that, as expected, the switches with a finite number of inputs saturate at cell arrival rates higher than the asymptotic saturation throughput.

Fig. 7. Normalized mean burst delay for type 1 and type 2 traffic with the two priority selection contention resolution policies; analytical results ($N \rightarrow \infty$) with $p_1 = 0.1$, $p_2 = 0.9$, $\gamma_1 = 0.6$, $\alpha = 0.5$.

Fig. 8. Normalized mean burst delay for type 1 and type 2 traffic with the two priority selection contention resolution policies; simulation results with $N = 16$, $p_1 = 0.1$, $p_2 = 0.9$, $\gamma_1 = 0.6$, $\alpha = 0.5$.

We note that exact analysis for any reasonably large finite input queuing switch is intractable; asymptotic analysis becomes tractable because it is assumed that as $N \rightarrow \infty$ the arrival process into the output contention queues converges to Poisson (see Section II). Our results show that this more tractable analysis yields upper bounds on mean burst delay, and an accurate approximation for large switch sizes. We observe in the next section, however, that asymptotic analysis yields qualitative comparisons that continue to hold for finite switch sizes.

IV. COMPARATIVE PERFORMANCE OF THE SCHEDULING STRATEGIES

In this section we compare the effect of SEBF, LEBF, and RS scheduling strategies on the mean burst delay, for a local access ATM switch with two classes of bursty traffic. Since the analysis of the switch with the RS scheme is intractable we have only simulation results (for finite N) for it, whereas for the SEBF and LEBF schemes we have both analytical (for $N \rightarrow \infty$) and simulation (for finite N) results. First we compare the two priority contention resolution policies, and seeing that the LEBF scheme is not desirable, we compare the SEBF with RS scheme. In the analytical as well as simulation results that we present here, we have fixed α (the fraction of switch inputs carrying type 1 traffic which is less bursty) to be 0.5.

Figs. 7–12 illustrate the comparison of the normalized (with respect to mean burst length) mean burst delay with the two priority contention resolution policies, namely, SEBF and LEBF policies.

Figs. 7, 8, 11, and 12 show the results with the cell arrival rate at type 1 inputs ($p_1 = 0.1$ or $p_1 = 0.9$) kept at a constant value equal to 0.6 (which corresponds to type 1 throughput per output link of 0.3), and the cell arrival rate at type 2 inputs ($p_2 = 0.9$ or $p_2 = 0.99$) varying. Figs. 7 and 11 give the asymptotic analysis results, whereas Figs. 8 and 12 give the simulation results for a finite switch with $N = 16$.

Figs. 9 and 10 illustrate similar results with type 1 offered load varying and type 2 offered load fixed at a constant value, and with a different set of values for the burstlength parameters

Fig. 9. Normalized mean burst delay for type 1 and type 2 traffic with the two priority selection contention resolution policies; analytical results ($N \rightarrow \infty$) with $p_1 = 0.3$, $p_2 = 0.7$, $\gamma_2 = 0.2$, $\alpha = 0.5$.

Fig. 10. Normalized mean burst delay for type 1 and type 2 traffic with the two priority selection contention resolution policies; simulation results with $N = 16$, $p_1 = 0.3$, $p_2 = 0.7$, $\gamma_2 = 0.2$, $\alpha = 0.5$.

($p_1 = 0.3$ and $p_2 = 0.7$). In all the simulation studies, the cell arrival process described in the Introduction was used.

Figs. 7–12 illustrate some interesting results. There are combinations of cell arrival rates for the two bursty classes such that while both type 1 and type 2 input queues are stable with SEBF policy, type 1 (shorter mean burst length) queues are unstable with reversed priority; further, the degradation

TABLE III
ASYMPTOTIC ($N \rightarrow \infty$) STABILITY CONDITIONS ARE AT THE TYPE 1 AND TYPE 2 INPUT QUEUES FOR SEBF AND LEBF HOL CONTENTION RESOLUTION STRATEGIES, FOR THE PARAMETERS IN THE EXAMPLES IN SECTION IV. γ_1 AND γ_2 ARE TYPE 1 AND TYPE 2 CELL ARRIVAL RATES; THE DASHES IN THE γ_1 AND γ_2 COLUMNS MEAN THAT CELL ARRIVAL RATES ARE ARBITRARY; $\bar{\alpha} := (1 - \alpha)$

p_1	p_2	α	γ_1	γ_2	asymptotic stability condition			
					SEBF		LEBF	
					(priority to type 1)		(priority to type 2)	
					type 1 queues $\alpha^{-1}T_H(p_1, \alpha)$	type 2 queues $\bar{\alpha}^{-1}T_L(p_1, p_2, \alpha, \alpha\gamma_1)$	type 1 queues $\alpha^{-1}T_L(p_2, p_1, \bar{\alpha}, \bar{\alpha}\gamma_2)$	type 2 queues $\bar{\alpha}^{-1}T_H(p_2, \bar{\alpha})$
0.1	0.9	0.5	0.6	—	$\gamma_1 < 0.7514$	$\gamma_2 < 0.4668$	$\gamma_2 < 0.0601$	$\gamma_2 < 0.6742$
0.9	0.99	0.5	0.6	—	$\gamma_1 < 0.6742$	$\gamma_2 < 0.4552$	$\gamma_2 < 0.0310$	$\gamma_2 < 0.6674$
0.3	0.7	0.5	—	0.2	$\gamma_1 < 0.7286$	$\gamma_2 < 0.4036$	$\gamma_1 < 0.5674$	$\gamma_2 < 0.6906$

Fig. 11. Normalized mean burst delay for type 1 and type 2 traffic with the two priority selection contention resolution policies; analytical results ($N \rightarrow \infty$) with $p_1 = 0.9$, $p_2 = 0.99$, $\gamma_1 = 0.6$, $\alpha = 0.5$.

Fig. 13. Normalized mean burst delay for type 1 and type 2 traffic with the random selection and the priority selection contention resolution policies; simulation results with $N = 16$, $p_1 = 0.3$, $p_2 = 0.7$, $\gamma_2 = 0.2$, $\alpha = 0.5$.

Fig. 12. Normalized mean burst delay for type 1 and type 2 traffic with the two priority selection contention resolution policies; simulation results with $N = 16$, $p_1 = 0.9$, $p_2 = 0.99$, $\gamma_1 = 0.6$, $\alpha = 0.5$.

of type 2 (longer mean burst length) mean burst delay with SEBF as compared to LEBF is not significant. Thus without significantly affecting the delay performance of type 2 traffic, that of type 1 traffic is improved drastically.

In Fig. 13, we compare the normalized mean burst delays with the RS and SEBF policies, for type 1 and type 2 input traffic. We see the clear advantage of SEBF over RS for type 1 traffic. Also we observe that the type 2 traffic delay performance is almost the same under SEBF and RS for the typical load range shown.

The broad conclusions made above could have been anticipated from the asymptotic saturation throughput formulas derived in [7]. We consider two traffic types with burst parameters p_1 and p_2 . We define $T_H(p_1, \alpha) =$ asymptotic

(as $N \rightarrow \infty$) high priority saturation throughput (at each switch output port) when type 1 traffic occupies a fraction α of the switch inputs, and is given priority on output contention. $T_L(p_1, p_2, \alpha, T_H) =$ asymptotic low priority saturation throughput (at each switch output port) when type 1 traffic occupies a fraction α of the switch inputs, type 1 traffic is given priority, and the high priority throughput per switch output is T_H (of course, $T_H < T_H(p_1, \alpha)$).

The following formulas have been derived in [7]:

$$\begin{aligned}
 T_H(p_1, \alpha) &= \frac{(1 + \alpha) - \sqrt{\{(1 + \alpha)^2 - 2\alpha(1 - p_1)\}}}{(1 - p_1)} \\
 T_L(p_1, p_2, \alpha, T_H) &= [b(p_1, p_2, \alpha, T_H) - \sqrt{b^2(p_1, p_2, \alpha, T_H) - 4a(p_1, p_2, T_H)c(p_1, \alpha, T_H)}] / [2a(p_1, p_2, T_H)]
 \end{aligned}$$

where

$$\begin{aligned}
 a(p_1, p_2, T_H) &= (1 - p_1)(1 - p_2)(1 - T_H), \\
 b(p_1, p_2, \alpha, T_H) &= 2(2 - \alpha)(1 - p_1)(1 - T_H) \\
 &\quad + (1 - p_2)[2T_H - T_H^2(1 - p_1)] \\
 &\quad - 2(1 - p_1)(1 - p_2)T_H(1 - T_H), \\
 c(p_1, \alpha, T_H) &= 2(1 - \alpha)(1 - p_1)(1 - T_H)^2.
 \end{aligned}$$

Using these formulas with the parameters in the figures in this section, we get the results shown in Table III. Recall that γ_i is the cell arrival rate at the type i input; and if the input

queues are stable with this (γ_1, γ_2) then the throughput at each output is going to be $(\alpha\gamma_1, (1 - \alpha)\gamma_2)$.

In Table III, the first row corresponds to Figs. 7 and 8, the second row corresponds to Figs. 11 and 12, and the third row corresponds to Figs. 9 and 10. All the arrival rates shown in Table III are input queue arrival rates. Observe from the first two rows that with SEBF and $\gamma_1 = 0.6$ the type 1 input queues are stable, since $0.6 < 0.7514$ and $0.6 < 0.6742$, while γ_2 can go up to 0.4668 or 0.4552, respectively. With LEBF, however, while the stable arrival rate for type 2 increases to 0.6742 or 0.6674 (last columns of the first two rows), the type 1 queues (which have a fixed arrival rate of $\gamma_1 = 0.6$) become unstable for γ_2 as small as 0.0601 or 0.0301, respectively. Similarly, in the third row of Table III, with SEBF the type 1 stability condition is $\gamma_1 < 0.7286$. The value in the 7th column of the third row is $0.5^{-1}T_L(0.3, 0.7, 0.5, 0.5 \times 0.7286) = 0.4036$, which is the maximum stable arrival rate for type 2 cells when the type 1 queues are saturated; since $\gamma_2 = 0.2 < 0.4036$, the type 2 queues will be stable since γ_1 must be less than 0.7286. With LEBF, however, while the type 2 stable arrival rate increases to 0.6906, even with γ_2 as small as 0.2 the type 1 stable arrival rate drops to 0.5674 (from 0.7286 for SEBF).

We can make the following general remarks, from the preceding figures and Table III:

- 1) Table III corroborates the delays curves in the figures in this section.
- 2) As observed before, asymptotic saturation throughput is an underestimate for finite-sized switches, but the qualitative comparative conclusions drawn from the asymptotic analysis also apply to switches of finite size.
- 3) While the saturation throughput results suggest the phenomenon that the delays curves show, the dramatic degradation in type 1 delay with LEBF, and the very slight degradation in type 2 delay with SEBF, can best be appreciated from the delay curves. Hence the delay analysis in this paper is a necessary complement to the saturation analysis in [7].

V. CONCLUSION

In this paper, we have studied the impact of cell scheduling strategies on the mean burst delay for a local access ATM switch with mixed bursty traffic. We have found that, unlike Bernoulli arrivals, with bursty cell arrivals, asymptotic (as $N \rightarrow \infty$) analysis yields good approximation only for large switch sizes. We have demonstrated a hybrid technique, for estimating input queuing delays, that utilizes an analytically derived effective service time distribution at an input queue, along with simulation of an input queue. We have found, in addition, that the qualitative nature of results predicted by the asymptotic analysis applies well to finite switch sizes.

We showed that the SEBF policy can yield significantly lower mean delay for the short burst length traffic without much effect on the longer burst length traffic. This means that if the less bursty traffic is delay sensitive, then SEBF policy has great advantage. On the other hand, if the more bursty traffic is delay sensitive then by giving it a low priority (as in SEBF), we may degrade the delay performance of this

traffic but not significantly. Recall from the Introduction that a motivating scenario for our model was that the less bursty traffic could be from metropolitan or campus area networks. Such traffic would carry interactive services, e.g., interactive simulations with an image on a graphics workstation in one location being constantly updated by a program running on a supercomputer in a different location. Such traffic would have short bursts but would be delay sensitive. We also find that the delay performance of RS lies in between that of the two static priority policies; hence RS can be an adequate compromise if there is no prior knowledge of input traffic burstiness.

APPENDIX

MEAN LOW PRIORITY HOL CONTENTION DELAY

At the end of a slot in which a low priority cell is served, a high priority busy period (possibly of zero slot duration) is initiated by a Poisson batch of high priority bursts. At the termination of the busy period again a low priority cell gets service. We call the high priority busy period followed by the one slot service time of a low priority cell, together as a low priority cell completion time.

Define the following random variables

- 1) C_1 (respectively C_2) := Poisson batch size of high priority (resp. low priority) HOL bursts that arrive at the beginning of each time slot,
- 2) B := high priority busy period initiated by a Poisson batch of high priority HOL bursts (this "busy period" could be of zero slot duration if the Poisson batch is empty),
- 3) G := low priority cell completion time ($= B + 1$),
- 4) Γ := total number of low priority bursts arriving at the HOL contention process during one low priority cell completion time,
- 5) F := total service time (in no. of slots) of a Poisson batch of high priority bursts.

Then with the usual notation for the pgfs and pmfs (see Section I-A), and recalling that the high priority burst length is geometric with parameter p_1 , we obtain the following expressions:

$$\begin{aligned} \tilde{f}(z) &= \sum_{k=0}^{\infty} \left[\frac{z(1-p_1)}{1-zp_1} \right]^k \frac{e^{-\alpha\lambda_1} (\alpha\lambda_1)^k}{k!} \\ &= e^{-\alpha\lambda_1(1-z)/(1-zp_1)}. \end{aligned}$$

Observe that F can be viewed as the total amount of work (in slots) brought in by an arriving batch of high priority HOL bursts. For a work-conserving server, the busy period distribution is invariant to the service discipline. Hence, to compute the busy period distribution, we consider the following discipline: serve one cell of the first arriving batch, and then serve the busy period of the Poisson batch of bursts that arrives after this slot; the total time for this will have pgf $z\tilde{b}(z)$. This is repeated k times with probability $f(k)$. Hence

$$\begin{aligned} \tilde{b}(z) &= \sum_{k=0}^{\infty} [z\tilde{b}(z)]^k f(k) \\ &= \tilde{f}(z\tilde{b}(z)). \end{aligned}$$

Further, since $G = B + 1$

$$\hat{g}(z) = z\hat{f}(z\hat{b}(z))$$

and, noting that $G \geq 1$

$$\begin{aligned}\hat{\gamma}(z) &= \sum_{n=1}^{\infty} [\hat{c}_2(z)]^n g(n) \\ &= \hat{g}[\hat{c}_2(z)]\end{aligned}\quad (\text{A.1})$$

where $\hat{c}_2(z)$ is the pgf of the low priority arriving batch.

Recall that we are focusing on the HOL contention process which a tagged low priority burst has joined. Consider the ends of those slots in which a low priority cell is transmitted. Let $\{X_k^-, k \geq 1\}$ denote the number of low priority HOL bursts at the k th such epoch, where X_k^- does not include the newly arriving low priority HOL bursts, nor does it include the burst corresponding to the cell just transmitted when this burst is not yet complete (i.e., this burst is viewed as a feedback arrival). Owing to the asymptotic analysis and the consequent i.i.d. arrivals of new HOL bursts of both priorities, $\{X_k^-, k \geq 1\}$ is a Markov chain.

$$X_{k+1}^- = X_k^- + A_{k+1} - 1$$

where $X_k^- + A_{k+1} \geq 1$ since we consider the ends of those slots in which a low priority cell is transmitted. In the above expression

$$A_{k+1} = A_{k+1}^- + Z_k,$$

where Z_k ($= 0$ if the burst just served is complete, and $= 1$ otherwise) is a sequence of i.i.d random variables with

$$Z_k = \begin{cases} 1 & \text{w.p. } p_2 \\ 0 & \text{w.p. } (1 - p_2) \triangleq \bar{p}_2 \end{cases}$$

and A_{k+1}^- is the number of low priority burst arrivals *after* the end of the k th low priority service completion epoch, but *before* the end of the $(k+1)$ st such epoch. It is easily seen that

$$P(A_{k+1}^- = i) = \gamma(i), \quad i \geq 0 \quad \text{if } X_k^- + Z_k \geq 1$$

and further, if $X_k^- + Z_k = 0$, then a low priority cell is there to be served at the end of a high priority busy period in which at least one low priority burst arrives; hence

$$P(A_{k+1}^- = i) = \frac{\gamma(i)}{1 - \gamma(0)}, \quad i \geq 1 \quad \text{if } X_k^- + Z_k = 0$$

$\{X_k^-, k \geq 1\}$ is an embedded Markov chain and its transition probability matrix has the form

	0	1	...	$i-1$	i	$i+1$	...
0	$w(0)$	$w(1)$			$w(i)$		
1	$t(0)$	$t(1)$			$t(i)$		
2	0	$t(0)$			$t(i-1)$		
...							
i	0			$t(0)$	$t(1)$	$t(2)$	$t(3)$
...							

where

$$\begin{aligned}\forall j \geq 0, \quad w(j) &= \gamma(j)p_2 + \frac{\gamma(j+1)\bar{p}_2}{1 - \gamma(0)} \\ t(0) &= \gamma(0)\bar{p}_2 \\ \forall j \geq 1, \quad t(j) &= \gamma(j-1)p_2 + \gamma(j)\bar{p}_2\end{aligned}$$

It follows that the equilibrium distribution $\{x^-(j), j \geq 0\}$ is given by

$$x^-(j) = x^-(0)w(j) + \sum_{k=1}^{j+1} x^-(k)t(j+1-k).$$

Taking transform and rearranging terms

$$\hat{x}^-(z) = \frac{x^-(0)[\hat{t}(z) - z\hat{w}(z)]}{[\hat{t}(z) - z]}\quad (\text{A.2})$$

where

$$\begin{aligned}\hat{w}(z) &= \sum_{j=0}^{\infty} z^j \left[p_2 \gamma(j) + \bar{p}_2 \frac{\gamma(j+1)}{1 - \gamma(0)} \right] \\ &= p_2 \hat{\gamma}(z) + \bar{p}_2 \frac{\hat{\gamma}(z) - \gamma(0)}{z[1 - \gamma(0)]}\end{aligned}\quad (\text{A.3})$$

$$\begin{aligned}\hat{t}(z) &= \gamma(0)\bar{p}_2 + \sum_{j=1}^{\infty} [p_2 \gamma(j-1) + \bar{p}_2 \gamma(j)]z^j \\ &= \hat{\gamma}(z)(\bar{p}_2 + zp_2).\end{aligned}\quad (\text{A.4})$$

Substituting (A.3) and (A.4) in (A.2) and simplifying, we obtain

$$\begin{aligned}\hat{x}^-(z) &= \frac{x^-(0)\bar{p}_2\gamma(0)[1 - \hat{\gamma}(z)]}{[1 - \gamma(0)][\hat{\gamma}(z)(\bar{p}_2 + zp_2) - z]} \\ \hat{x}^-(1) &= \frac{x^-(0)\gamma(0)E\Gamma\bar{p}_2}{[1 - \gamma(0)](1 - p_2 - E\Gamma)}\end{aligned}$$

also

$$\hat{x}^-(1) = 1.$$

It follows that

$$x^-(0) = \frac{(1 - p_2 - E\Gamma)[1 - \gamma(0)]}{\bar{p}_2\gamma(0)E\Gamma}$$

and

$$\hat{x}^-(z) = \frac{(1 - p_2 - E\Gamma)[1 - \hat{\gamma}(z)]}{E\Gamma[\hat{\gamma}(z)(1 - p_2 + zp_2) - z]}\quad (\text{A.5})$$

where $E\Gamma$ is the expectation of Γ , and is obtained from the pgf of Γ (A.1).

Continuing the analysis, let k index the successive slots. Let $\{X_k, k \geq 1\}$ denote the number of low priority bursts, in the tagged HOL contention process, at the end of the k th slot, after the possible low priority burst departure in that slot but before new HOL burst arrivals (see Fig. 14). Let $\{\sigma_i, i \geq 1\}$ denote the slot indices in which a low priority cell is served in the tagged HOL contention process. From the definition of the process $\{X_k^-\}$ above, it is then clear that, for $i \geq 1$

$$X_{\sigma_i} = X_i^- + Z_i$$

with Z_i as defined above. Let $\{\sigma_i, j \geq 1\}$ be the subsequence of $\{\sigma_i, i \geq 1\}$ such that $Z_{\sigma_j} = 0, j \geq 1$, i.e., $X_{\sigma_j} = X_{\sigma_j}^-$.

Fig. 14. Pictorial depiction of the processes $\{X_k\}$ and $\{X_k^-\}$. Here $X_1 = 2, X_2 = 3, X_3 = 4, X_4 = 3, X_5 = 3, X_6 = 4, X_7 = 4$. Cell services occur in slots 2, 4, 5, and 7. Hence, $X_1^- = 2, X_2^- = 3, X_3^- = 2, X_4^- = 4$. The solid lines show the transitions in $\{X_k\}$. The dashed lines show the "pseudo" transitions in which a serviced cell is immediately fed back.

$j \geq 1$. The epochs $\{\sigma_{ij}, j \geq 1\}$ are low priority burst departure epochs. Since $\{Z_i, i \geq 1\}$ is a Bernoulli process, it follows that the subsequence $\{X_{ij}^-, j \geq 1\}$ of the process $\{X_i^-, i \geq 1\}$ is obtained by Bernoulli sampling of $\{X_i^-, i \geq 1\}$.

For $m \geq 0$, define the following stationary probabilities for the process $\{X_k, k \geq 1\}$:

- 1) $x(m) : \Pr\{X_k = m\}$,
- 2) $d(m) : \Pr\{\text{a low priority burst departure leaves behind } m \text{ low priority bursts}\}$, and
- 3) $a(m) : \Pr\{\text{an arriving nonempty batch of low priority bursts finds } m \text{ low priority bursts already in the HOL process}\}$.

Note that since the low priority burst arrivals are i.i.d, the epochs of arrivals of nonempty bursts is a Bernoulli sequence. From the preceding observations it follows, using the discrete time version of PASTA, i.e., GASTA [4] that

$$d(m) = x^-(m) \quad m \geq 0 \quad (\text{A.6})$$

$$a(m) = x(m) \quad m \geq 0. \quad (\text{A.7})$$

We now use a level crossing argument on $\{X_k, k \geq 1\}$ to relate $d(m)$ and $a(m)$, and hence to relate $x(m)$ and $x^-(m)$. Let, for $m \geq 0, n \geq 1$,

- 1) $\text{up}_n(m) : \text{number of up-transitions in } \{X_k, 1 \leq k \leq n\} \text{ that cross level } m$, and
- 2) $\text{down}_n(m) : \text{number of down-transitions } (m+1) \downarrow m \text{ in } \{X_k, 1 \leq k \leq n\}$.

Clearly

$$|\text{up}_n(m) - \text{down}_n(m)| \leq 1.$$

It follows that

$$\lim_{n \rightarrow \infty} \frac{\text{up}_n(m)}{n} = \lim_{n \rightarrow \infty} \frac{\text{down}_n(m)}{n}.$$

Let

$$\delta_n = \text{no. of burst departures in } [0, n]$$

then

$$\lim_{n \rightarrow \infty} \frac{\text{down}_n(m)}{n} = \lim_{n \rightarrow \infty} \frac{\delta_n}{n} \frac{\text{down}_n(m)}{\delta_n}.$$

Now, for a stable system

$$\lim_{n \rightarrow \infty} \frac{\delta_n}{n} = EC_2 \quad \text{w.p. 1}$$

(where w.p. 1 denotes "with probability one") and

$$\lim_{n \rightarrow \infty} \frac{\text{down}_n(m)}{\delta_n} = d(m) \quad \text{w.p. 1.}$$

Further, defining $\bar{C}_2(j) = \sum_{i=j}^{\infty} c_2(i)$

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{\text{up}_n(m)}{n} &= [1 - c_2(0)] \\ &\cdot \left[\frac{a(0)\bar{C}_2(m+1)}{1 - c_2(0)} + \frac{a(1)\bar{C}_2(m)}{1 - c_2(0)} \right. \\ &\quad \left. + \cdots + \frac{a(m)\bar{C}_2(1)}{1 - c_2(0)} \right] \quad \text{w.p. 1} \end{aligned}$$

thus, w.p. 1, $\forall m \geq 0$

$$EC_2 d(m) = \sum_{i=0}^m a(i) \bar{C}_2(m+1-i).$$

Taking transforms

$$\begin{aligned} EC_2 \tilde{d}(z) &= \sum_{m=0}^{\infty} z^m \sum_{i=0}^m a(i) \bar{C}_2(m+1-i) \\ &= \sum_{i=0}^{\infty} a(i) \sum_{m=i}^{\infty} \bar{C}_2(m+1-i) z^{m+1-i} z^{i-1} \\ &= [\tilde{C}_2(z) - 1] z^{-1} \tilde{a}(z) \end{aligned}$$

or

$$\tilde{a}(z) = \frac{z EC_2 \tilde{d}(z)}{[\tilde{C}_2(z) - 1]}.$$

It follows from (A.6) and (A.7) that

$$\tilde{x}(z) = \frac{z EC_2 \tilde{x}^-(z)}{[\tilde{C}_2(z) - 1]} \quad (\text{A.8})$$

where

$$\begin{aligned} \tilde{C}_2(z) &= \sum_{k=0}^{\infty} z^k \sum_{j=k}^{\infty} c_2(j) \\ &= \frac{1 - z \tilde{c}_2(z)}{1 - z} \end{aligned}$$

Substituting (A.5) in (A.8) and simplifying, we get

$$\tilde{x}(z) = \frac{EC_2(1 - p_2 - E\Gamma)(1 - z)[1 - \tilde{\gamma}(z)]}{E\Gamma[1 - \tilde{c}_2(z)][\tilde{\gamma}(z)(\bar{p}_2 + p_2 z) - z]}. \quad (\text{A.9})$$

By applying Little's Law, we obtain the mean sojourn time S_L of the low priority burst in the HOL process, i.e., the mean HOL contention time of a low priority burst. Observe, however, from the way $\{X_k, k \geq 1\}$ is embedded in the HOL

contention process, that a burst that sojourns for m slots in the process, contributes to $\{X_k\}$ only for $m - 1$ slots. Thus Little's Law yields

$$\frac{EX}{EC_2} = E(S_L - 1) \\ = ES_L - 1 \quad (\text{A.10})$$

where EX is obtained from the pgf of X (A.9).

This result for ES_L is perhaps the most complete and accurate closed form result we can hope to get for this problem. It only involves the (asymptotically accurate) assumption of Poisson arrivals. This result is used to check the accuracy of numerical results to be obtained from the analysis in Section II-B1.

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