A FINITE NEUTRINO REST MASS FROM GENERAL RELATIVITY

C. SIVARAM AND K. P. SINHA

Division of Physics and Mathematical Sciences, Indian Institute of Science, Bangalore-560012, India

ABSTRACT

The weak interaction having the universal Fermi coupling constant and mediated by a massive intermediate boson is related to the metric of the surrounding space-time through a line element analogous to the well-known Reissner-Nordström line element for the electromagnetic interaction. Together with a fundamental beta-decay length this predicts a finite rest mass for the neutrino which is well within experimental upper limits. The corresponding intermediate boson mass is also obtained. Further, making use of the line element for a particle with 'Yukawa charge', the pion-proton mass ratio is obtained.

INTRODUCTION

RECENTLY it was pointed out that the appropriate line element for a mass point with 'Yukawa charge' g is given as[1,2]:

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{\alpha' \exp[-2\mu r]}{2r^2}\right) c^2 dt^2 - \left(1 - \frac{2m}{r} + \frac{\mu\alpha' \exp[-2\mu r]}{r} + \frac{\alpha'}{r^2}\exp[-2\mu r]\right)^{-1} dr^2 - r^2 d\Omega^2, \quad (1)$$

where $m = GM/c^2$ (M being the mass and G the Newtonian gravitational constant), and $d\Omega^2 = r^2 d\theta^2 + \sin^2\theta \, d\phi^2$. The Yukawa potential is:

$$\phi(r) = \frac{g \exp(-\mu r)}{r}. \quad \text{In Eq. (1), } \alpha' = Gg^2/c^4.$$

The dimensionless strong (Yukawa) interaction coupling constant (pion-nucleon coupling constant is given by: $g^2/\hbar c = 14 \cdot 8$. In analogy with the electromagnetic coupling constant, $\alpha \equiv e^2/\hbar c = 1/137$ (e being the electric charge), this suggests that g can be considered as some kind of charge for strong interactions or a Yukawa charge. The exponential factor in Eq. (1) takes into account the short range of the strong interactions, with $\mu = m_\pi c/\hbar$, m_π being the pion mass, the pion mediating the interaction. In the long-range approximation, (*i.e.*, $\exp[-2\mu r] \approx 1$), the line-element given in Eq. (1) becomes similar to the Reissner-Nordström line-element for the electromagnetic interaction. Now the weak interaction (or beta-decay interaction) is also a short-range interaction. By analogy with strong interactions we could write a corresponding Yukawa potential, given by:

$$\phi_w(r) = \frac{g_w \exp(-\mu_w r)}{r},$$

where g_w is the weak interaction 'charge' corresponding to the dimensionless coupling constant $g_w^2/\hbar c$; and $\mu_w = m_w c/\hbar$, m_w being the mass of the mediating intermediate boson. This enables us to write the line-element which relates this potential to the metric of the surrounding space time as:

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{\alpha_w \exp[-2\mu_w r]}{2r^2}\right) c^2 dt^2 - \left(1 - \frac{2m}{r} + \frac{\mu_w \alpha_w \exp[-2\mu_w r]}{r} + \frac{\alpha_w}{r^2}\exp[-2\mu_w r]\right)^{-1} dr^2 - r^2 d\Omega^2, \quad (2)$$

where:

$$\alpha_w \equiv \frac{Gg_w^2}{c^4}.$$

THE FUNDAMENTAL BETA-DECAY LENGTH AND THE NEUTRINO REST MASS

The universal Fermi coupling constant for beta decay is given by

$$G_F = 1 \cdot 5 \times 10^{-49} \text{ erg. cm}^3. \quad (3)$$

It is interesting that using G_F we can construct a unique length as

$$l_w = \left(\frac{G_F}{\hbar c}\right)^{\frac{1}{2}} = 6 \times 10^{-17} \text{ cm}. \quad (4)$$

We shall characterize l_w as a fundamental length for beta-decay, as it is constructed solely from the universal constants G_F, $\hbar$ and c. For g_w we can now write:

$$g_w^2 = G_F \left(\frac{m_e c}{\hbar}\right)^2, \quad (5)$$

m_e being the electron rest mass. As we are using m_e on the right side of Eq. (5) we shall replace all subscripts W by W_e. Thus g_{w_e} is a kind of electron-neutrino 'charge'. Replacing m_e in Eq. (5) by the muon rest mass m_μ, we get the corresponding muon-neutrino charge g_{w_μ}. Then the dimensionless constants are $g_{w_e}^2/\hbar c$ and $g_{w_\mu}^2/\hbar c$.

The current idea[3,4] about the weak and electromagnetic interactions is that they are not merely analogous phenomena but different aspects of a unified dynamical mechanism. Our approach in the present problem will be in conformity with this idea. We shall consider the electron-neutrino as the counterpart of the electron in weak interactions with its characteristic weak charge g_{w_e}. The muon-neutrino then plays the corresponding role for the muon. Earlier we had shown[1,5] that the Reissner-Nordström metric exhibits an equilibrium point at r_0, given by:

$$r_0 = \frac{e^2}{m_e c^2} = 2\cdot 8 \times 10^{-13}\,\text{cm.}, \tag{6}$$

(for the electron rest mass m_e).

Now in an exactly similar manner we can show that in the long-range limit, *i.e.*, exp $[-2\mu_w r] \sim 1$, the metric given by Eq. (2) has an equilibrium point at r_{0w_e} given by:

$$r_{0we} = \frac{g_w{}^2}{2m_{\nu_e} c^2}, \tag{7}$$

where m_{ν_e} is the electron-neutrino rest mass (*i.e.*, we replace the mass M occurring in the metric by m_{ν_e}). Now in the electromagnetic case the equilibrium point occurs at the classical electromagnetic radius r_0 which is a kind of 'characteristic length' for electromagnetic interactions. By analogy one would expect the equilibrium radius r_{0e} to be the characteristic beta-decay length l_w given by Eq. (4).

Thus:

$$m_{\nu_e} = \frac{g_{we}{}^2}{2l_w c^2}, \tag{8}$$

Substituting for l_w and g_{we} from Eqs. (4) and (5) we have:

$$m_{\nu e} = \frac{1}{2}\left(\frac{G_F c}{\hbar^3}\right)^{\frac{1}{2}} m_e{}^2, \tag{9}$$

This gives $m_{\nu_e} \simeq 0\cdot 7$ electron-volts.

This is consistent with the experimental upper limit[6] of $m_{\nu_e} < 60$ eV. Similarly for the muon-neutrino we have by analogy

$$m_{\nu_\mu} = \frac{1}{2}\left(\frac{G_F c}{\hbar^3}\right)^{\frac{1}{2}} m_\mu{}^2, \tag{10}$$

giving $m_{\nu_\mu} = 0\cdot 028$ MeV consistent with the experimental upper limit[6] of $m_{\nu_\mu} < 1\cdot 5$ MeV. Also from the uncertainty principle, the 'range' r_w of the intermediate W boson is given by:

$$r_w = \frac{\hbar}{m_w c},\quad m_w \text{ being the boson mass.}$$

Equating this to the characteristic length l_w gives:

$$m_w = \left(\frac{\hbar^3}{G_F c}\right)^{\frac{1}{2}} \simeq 250\,\text{BeV}. \tag{11}$$

This is to be compared with other estimates such as those of Schwinger[7] who obtains:

$$m_w = \left(\frac{4\pi a}{2^{3/2} G_F}\right)^{\frac{1}{2}} = 53\,\text{BeV}.$$

There is as yet no reliable experimental evidence for the existence of the W-boson although many different estimates of its mass are predicted by various theories! The prevailing opinion now is that it should be a heavy boson (if it exists) with a mass perhaps several times the proton mass (~ 1 BeV) as there is no evidence for it with the energies available in present-day accelerators.

The Pion-Proton Mass Ratio

It can easily be shown that in the long-range limit the line element given by Eq. (1) for a particle having the proton mass m_p with Yukawa 'charge' g_s has an equilibrium point r_{0s} given by:

$$r_{0s} = \frac{g_s{}^2}{2m_p c^2}. \tag{12}$$

As was done in the case of weak interactions above, we can equate this to the range (or characteristic length) of the strong interaction mediated by the pion, *i.e.*, $\hbar/m_\pi c$, m_π being the pion rest mass.

Thus

$$\frac{g_s{}^2}{2m_p c^2} = \frac{\hbar}{m_\pi c}, \tag{13}$$

or

$$\frac{m_\pi}{m_p} = 2\frac{\hbar c}{g_s{}^2}.$$

Using the experimental value for the strong interaction coupling constant, *i.e.*,

$$\frac{g_s{}^2}{\hbar c} \approx 14\cdot 5,$$

$$\frac{m_\pi}{m_p} \simeq \frac{1}{7}. \tag{14}$$

Using $m_p = 938$ MeV; we have $m_\pi = 134$ MeV, close to the observed value $m_\pi = 139$ MeV.

Einstein Field Equations with Different Coupling Constants

The Einstein field equations[8]

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \tag{15}$$

relate a geometrical invariant quantity (*i.e.*, the Einstein tensor $G_{\mu\nu}$) on the left side to an invariant physical quantity (*i.e.*, the conserved energy-momentum tensor $T_{\mu\nu}$) on the right side through a proportionality constant κ. It must be emphasized that the derivation of Eq. (15) places no restriction whatsoever on the value of the constant κ. As Einstein used Eq. (15) as the basis for his relativistic theory of gravitation, he had to choose $\kappa = -8\pi G/c^4$, G being the Newtonian constant of gravitation, so as to be consistent with the Newtonian gravitational theory and its related Poisson equation. Following him it has become customary to always relate κ to the Newtonian gravitational constant G. Almost all applications of general relativity have hitherto been in the realm of astronomy and astrophysics and hence this choice of κ was suitable as gravitational forces are predominant in the macrocosmos. In principle, however, all interactions have their own energy-momentum sources and can give rise to gravitation. These interactions have their own characteristic coupling constants. We point out below some interesting physical consequences of identifying these coupling constants with κ, *i.e.*, by putting different values for κ. From quantum geometrodynamical considerations, Wheeler[9] has pointed out that at distances of the order of the Planck length, *i.e.*,

$$\left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}} = 1\cdot 6 \times 10^{-33}\,\text{cm},$$

quantum effects become important in general relativity. The uncertainty principle then shows that at such distances quantum gravitational effects proceed by exchange of quanta of masses of the order of the Planck mass,

$$\left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} = 2\cdot 2 \times 10^{-5}\,\text{g}$$

which in turn implies density fluctuations of the absurdly high value of 10^{93} gm/cc or 10^{114}erg/cc! This would also be the density (it would be infinity if quantum effects were ignored) of the inevitable singularity (*e.g.*, Penrose and Hawking[10]) occurring in gravitational collapse or the initial density of the singularity in big-bang cosmological models, *e.g.*, Harrison[11]). This absurdly high density could be scaled down by invoking *f*-gravity or the strong gravity mediated by massive spin-2 *f*-meson. This has a coupling constant G_f shown to be[1,12], 10^{38} times the Newtonian value G_N. With this value of $G = G_f$, the Planck length becomes $\sim 10^{-14}$ cm, the Planck mass of the order of the proton mass $(10^{-24}\,\text{g})$ and the quantum gravitational density fluctuations take on a much more reasonable value of $\sim 10^{17}$ g/cc. As most theories of superdense matter[13] (*e.g.*, in neutron stars) picture the densest state to consist predominantly of hadrons, it is natural that *f*-gravity must be invoked to discuss quantum gravitational effects. The quantities characteristic of quantum gravitation then come much closer to values encountered in elementary particle physics. With G_f the exchange quanta will have masses of the order of the *f*-meson mass which is precisely what we expect. We shall now relate κ to the Fermi coupling constant G_F. From G_F we can construct a quantity having the same dimensions as the gravitational constant G as

$$G_w \rightarrow G_F\left(\frac{c}{\hbar}\right)^2 = 1\cdot 4 \times 10^{26}.$$

Then the characteristic Planck length

$$\left(\frac{\hbar G}{c^3}\right)^{\frac{1}{2}},$$

with G_w replacing G becomes:

$$L = \left(\frac{G_F}{\hbar c}\right)^{\frac{1}{2}},$$

which is precisely the characteristic beta-decay length l_w! The mass of the exchange quanta then become

$$\left(\frac{\hbar c}{G_w}\right)^{\frac{1}{2}} = \left(\frac{\hbar^3}{G_F c}\right)^{\frac{1}{2}},$$

precisely the value obtained for the mass of the intermediate boson in Eq. (11). Thus l_w is the equivalent of the 'quantum gravitational length' (*i.e.*, Planck length) in weak interactions and m_w is the corresponding quantum gravitational mass (*i.e.*, Planck mass) for weak interactions. Similarly by using the strong interaction coupling constant we can show that the Planck length scales up to the pion Compton wavelength and the Planck mass scales down to the pion rest mass. The intermediate boson and the pion are therefore the corresponding 'quantum gravitational mass' for weak and strong interactions respectively. The virtual quanta mediating quantum gravitational interactions have their gravitational radius as the Planck length, *i.e.*, the same as their interaction range. These virtual quanta can be likened to 'quantum black holes' as their mass is always confined within their gravitational radius. From the above we can see that the 'virtual' pions and intermediate bosons which mediate strong and weak interactions are also, similar entities with their corresponding masses and 'gravitational radii' scaled by the respective interaction coupling constants.

CONCLUDING REMARKS

We see that a considerable amount of physics is hidden in the Einstein field equations if new interpretation of the 'cosmological constant' and κ were given[1]. From rather elementary considerations we have been able to present a unified picture of various interactions by incorporating the coupling constants of these different interactions into the field equations and their solutions. This is consistent with recent work[12] of the authors where it was shown that using the coupling constant G_f for *f*-gravity instead of the Newtonian constant would make general relativity far more relevant to particle physics. The calculation of the neutrino rest mass given in this paper is also in conformity with another recent work[14] of the present authors where the masses of stable elementary particles were tried to be understood solely in terms of the interactions they undergo.

1. Sivaram, C. and Sinha, K. P., *Lettere al Nuovo Cimento*, 1973, 8, 324.

2. Ross, D. K., *Nuovo Cimento*, 1972, **8 A**, 603.
3. Schwinger, J., *Ann. Phys. (N.Y.)*, 1957, **2**, 407.
4. Weinberg, S., *Phys. Rev. Letters*, 1971, **27**, 1688.
5. Sinha, K. P. and Sivaram, C., *Curr. Sci.*, 1973, **42**, 4.
6. Shrum, E. V. and Ziock, K. O. H., *Phys. Lett.*, 1971, **37 B**, 115.
7. Schwinger, J., *Phys. Rev. D*, 1973, **7**, 908.
8. Einstein, A., *The Meaning of Relativity*, Methuen and Co., London, 1960.
9. Wheeler, J. A., *Geometrodynamics*, Academic Press (N.Y.), 1962.
10. Penrose, R. and Hawking, S. W., *Proc. Roy. Soc. Lond.*, 1966, **295 A**, 490.
11. Harrison, E. R., *Phys. Today*, 1968, **21**, 31.
12. Sivaram, C. and Sinha, K. P., *Lettere al Nuovo Cimento*, 1974 (In press).
13. Zeldovich, Ya. B. and Novikov, I. D., *Relativistic Astrophysics*, Vol. I, Univ. of Chicago Press, 1971.
14. Sivaram, C. and Sinha, K. P. (to be published), 1974.

TEMPERATURE DEPENDENCE OF MAGNESIUM STIMULATED ADENOSINE TRIPHOSPHATASE ACTIVITY DURING AGING OF THE CENTRAL NERVOUS SYSTEM OF RAT

CHANDRA MOHAN AND E. RADHA

Department of Zoology, Bangalore University, Bangalore-560001, India

Introduction

TEMPERATURE dependence of Mg^{++} ATPase has been studied in mammalian brain homogenates[1-3] and microsomal preparations[4]. Comparisons of these homeotherms with heterothermic animals[4] and poikilothermic animals[5] have shown that ATPases in these animals are less sensitive to low temperatures. The properties of the total ATPase enzyme complex in mammalian brain preparations have been studied in detail[6-9] but more attention has been concentrated on the $Na^{+}-K^{+}$ ATPases because of their relationships with the oxidative phosphorylation and active transport processes. Mg^{++} ATPase is responsible for the control of passive permeability of excitable cells[10-11] and probably of all cells and cellular organelles[12-13]. Bowler and Duncan[14] have suggested that Mg^{++} ATPase of the excitable cells has a role in the control and maintenance of the excitability of these cells. It is also known that the development of the receptor potential during the excitation of sense organs reflects a change in the passive permeability of the receptor membrane to cations[15].

Studies on the effect of temperature on Mg^{++} ATPase activity in rat brain during postnatal development is very little, and no data is available on the temperature dependence of the enzyme activity in the aging central nervous system. This paper is a study of the temperature dependence of the magnesium stimulated ATPase activity during the aging of the central nervous system.

Material and Methods

Albino rats (Wistar strain) of 1 day, 3, 13, 44 and 87 weeks age used for the study were maintained at 28 ± 2° C on a commercial diet (Rat and mice feed purchased from Hindustan Lever Ltd., Bombay).

The animals were decapitated and different regions of the brain (Cerebrum, Cerebellum, Medulla and Optic lobes) excised immediately and weighed in precooled beakers of 5 ml capacity. Tissues were homogenised in glass homogenisers in ice-cold 0·13 M Tris-buffer (pH 7·4) in 0·25 M sucrose as described by Tirri *et al.*[1] to give a homogenate which contained, depending upon the age of the animal 14 to 20 mg of tissue per ml. This was centrifuged at 6000 r.p.m. for 30 minutes and the supernatant was used for the study of the enzyme activity.

Incubation medium contained 5 mM ATP in Tris-buffer, 5 mM $MgCl_2$ and 0·2 ml of the enzyme extract all in a volume of 1·8 ml. The above were incubated at 10°, 20°, 30°, 40° and 50° C for a period of 30 minutes and the reaction was stopped by the addition of 1 ml of 10% trichloroacetic acid. The tubes were cooled immediately in ice-cold water and centrifuged for 10 minutes at 3000 r.p.m. Inorganic phosphate liberated was estimated spectrophotometrically (Beckman DU-2) by the method of Fiske and Subba Row as described by Leloir and Cardini[16]. Protein content in the enzyme extract was estimated by Biuret method[17].

Energy of activation was calculated from the Arrhenius plots as described by Giese[18].

Results

An incubation period of 30 minutes was used for all extracts, regardless of their activity. This period was chosen on the basis of our earlier studies[3]. Figures 1–5 show the activity-temperature relationships of Mg^{++} ATPase in 1 day, 3, 13, 44 and 87 weeks old rats respectively. It is seen that Mg^{++} ATPase is temperature insensitive in cerebellum, medulla and optic lobes of 1 day old animals