Proc. Indian Acad. Sci., Vol. 85 A, No. 5, 1977, pp. 332–337.

The effect of suction on boundary layer for rotating flows with or without magnetic field

G. NATH AND B. J. VENKATACHALA

Department of Applied Mathematics, Indian Institute of Science, Bangalore 560012

MS received 22 September 1976; in revised form 12 November 1976

ABSTRACT

The effect of suction on the steady laminar incompressible boundary-layer flow for a stationary infinite disc with or without magnetic field, when the fluid at a large distance from the surface of the disc undergoes a solid body rotation, has been studied. The governing coupled nonlinear equations have been solved numerically using the shooting method with least square convergence criterion. It has been found that suction tends to reduce the velocity overshoot and damp the oscillation.

1. INTRODUCTION

ROTATING flows about stationary surfaces are of considerable interest to engineers in their study of flows inside vortex reactors, cylindrical chambers, rockets, magnetohydrodynamic vortex power generators, etc., and to meteorologists in their studies of hurricanes, tornadoes, etc. An extensive survey of this type of flows and their various applications have been given by Lewellen[1] and Rott and Lewellen.[2]

Bödewadt[3] studied the laminar boundary-layer flow produced by the viscous, steady incompressible fluid which undergoes a solid body rotation at a large distance from an infinite stationary disc. The case when the fluid is electrically conducting and interacting with a magnetic field in the absence of mass transfer was considered by King and Lewellen.[4] The interesting aspect of the Bödewadt's solution was that it showed oscillations in the velocity profiles before reaching their asymptotic values at the edge of the boundary layer which led to the controversy whether a physically realizable flow could contain the velocity oscillations without separation or transition. However, recent theoretical and experimental studies of Rogers and Lance,[5] Bein and Penner,[6] Kuo[7] and Millsaps and Nadahl[8] indicate the validity of his solution. On the other hand, King and Lewellen[4] found that the magnetic field tends to damp the oscillations in the velocity profiles.

The aim of the present analysis is to study the effect of suction on the surface of a stationary insulated infinite disc with or without a magnetic field, when the fluid at a large distance from the surface undergoes a solid body rotation and to examine the effect of suction on the oscillation of velocity profiles. The governing coupled nonlinear equations have been solved numerically using the shooting method with least square convergence criterion.[9] It has been found that the oscillations in the velocity profiles are damped by a suction or a magnetic parameter.

2. GOVERNING EQUATIONS

We consider the steady laminar boundary-layer flow over a stationary insulated infinite disc with a magnetic field B_0 applied perpendicular to its plane, when the electrically conducting fluid at a large distance from it undergoes a solid body rotation. It is assumed that the fluid has constant physical properties and that the flow is axisymmetric. The magnetic Reynolds number is assumed to be small. Hence the magnetic field is independent of the fluid motion. Under these conditions, the boudary-layer equations under similarity conditions taking into account the effect of suction can be expressed as[4]

$$H''' - HH'' + H'^2/2 - 2(G^2 - 1) - MH' = 0 \tag{1}$$

$$G'' - HG' + H'G - M(G - 1) = 0 \tag{2}$$

$$P = -(HH' - H''), \qquad F = -H'/2. \tag{3}$$

The boundary conditions are

$$H(0) = -a, \qquad H'(0) = G(0) = H'(\infty) = 0, \qquad G(\infty) = 1 \tag{4}$$

where H' (or F), G and H are the dimensionless radial, tangential and axial velocity components respectively, M and a are the dimensionless magnetic and suction parameters respectively, P is the dimensionless static pressure and prime denotes differentiation with respect to the similarity variable η.

Equations (1) to (4) are same as those of ref. 4 if we put $H = -2F$, $G = \Gamma$, $M = s$, $a = 0$, ($n = 1$ in ref. 4). They reduce to those of ref. 3 when $M = a = 0$ (*i.e.*, no magnetic field or suction). The governing eqs (1) and (2) under conditions (4) have been solved numerically using shooting method with least square convergence criterion developed by Nachtsheim and Swigert.[9] Since the method is given in complete detail in ref. 9, it is not repeated here for the sake of brevity.

Table 1. Radial and tangential shear stress parameters at the wall.

	$a = 0$		$a = 1$		$a = 2$	
M	$H''(0)$	$G'(0)$	$H''(0)$	$G'(0)$	$H''(0)$	$G'(0)$
0	1·8839	0·7729	1·6700	1·3863	1·2924	2·1550
1	1·2198	1·1119	1·1634	1·7237	0·9870	2·4794
2	0·9183	1·4602	0·9062	2·0481	0·8152	2·7682
3	0·7604	1·7580	0·7601	2·3306	0·7062	3·0230

3. RESULTS AND DISCUSSION

The radial, tangential and axial velocity profiles H', G and H respectively and static pressure profiles P for various values of magnetic parameter M and suction parameter a are given in figures 1–4. The radial shear stress parameter $H''(0)$ and the tangential shear stress parameter $G'(0)$ are given in table 1. The tangential and radial velocity profiles in the absence of the magnetic field and suction (*i.e.*, $M = a = 0$) exhibit velocity overshoot and approach their asymptotic limits at the edge of the boundary layer in an oscillatory manner. Figures 3 and 4 reveal that axial velocity H and

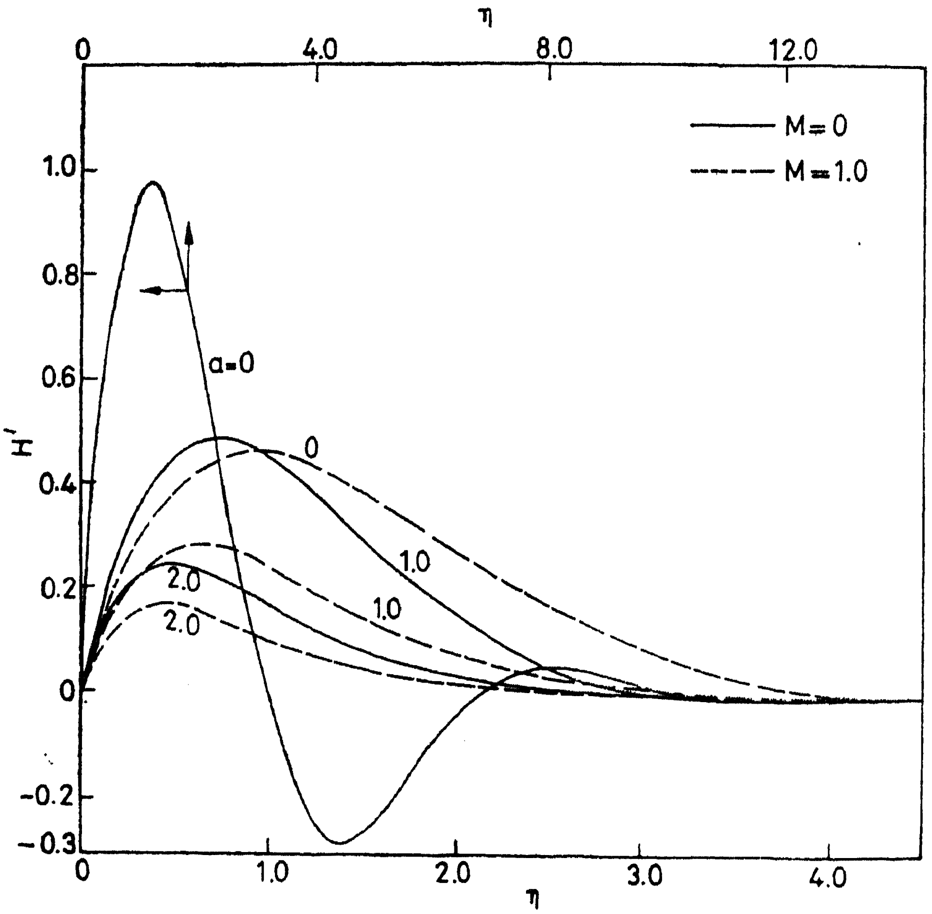

Figure 1. Radial velocity profiles,

pressure P for $M = a = 0$ also show oscillatory characteristics. In all these cases, asymptotic values are reached near $\eta = 12 \cdot 5$. The physical basis for the velocity overshoot and oscillations is the surplus convection of angular momentum present in the boundary layer.[4] The detailed explanation for this phenomenon is given in refs. 1, 4, 8, hence it is not repeated here. From figures 1–4, it is evident that the suction parameter a ($a = -[(w/v_0)(v_0 r_0/\nu)^{-1/2}]$, where w ($w < 0$) is the suction velocity, ν is the kinematic viscosity and v_0, r_0 are constants) and the magnetic parameter M tend to reduce the velocity overshoot and also damp the oscillation. The suction or magnetic parameter reduces the radial and axial velocities and pressure near the wall, whereas it increases the tangential velocity (the scale when $M = a = 0$ is different from that when $M = a \neq 0$ in figures 1–4). It is also observed that these profiles reach their asymptotic values at much lesser values of η (*i.e.*, at $\eta \simeq 4 \cdot 0$).

From table 1, it is observed that the tangential shear stress parameter $G'(0)$ increases as a or M increases, but the effect of increasing a or M on the radial shear stress parameter $H''(0)$ is just the reverse. We have

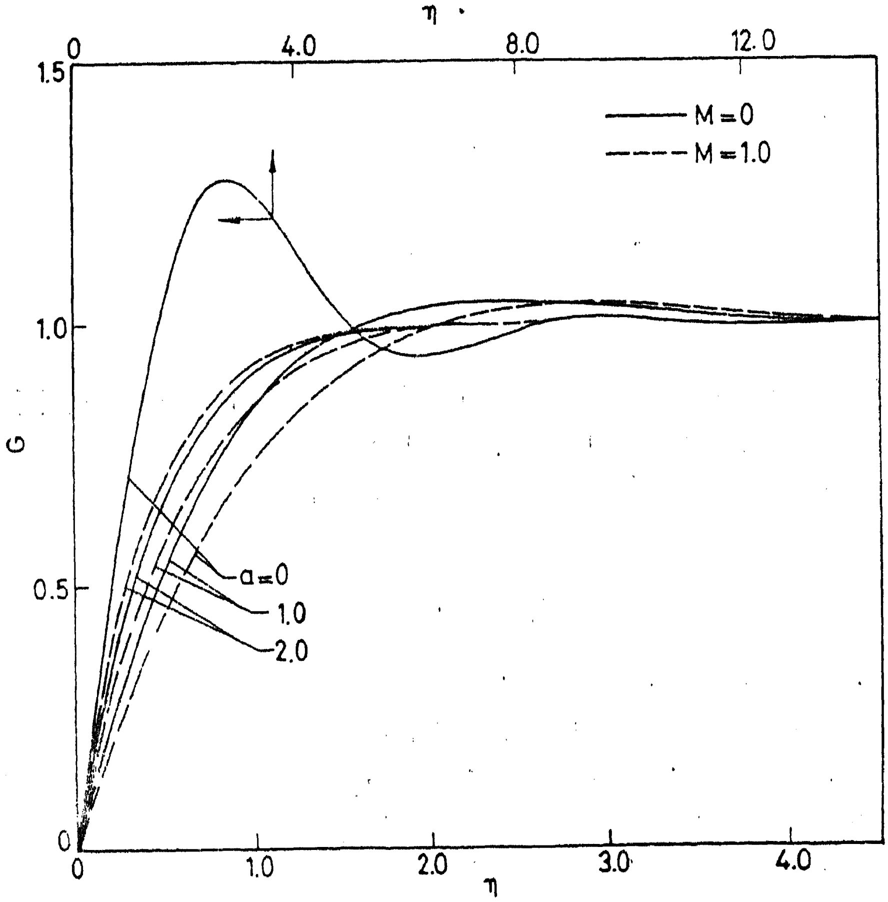

Figure 2. Tangential velocity profiles.

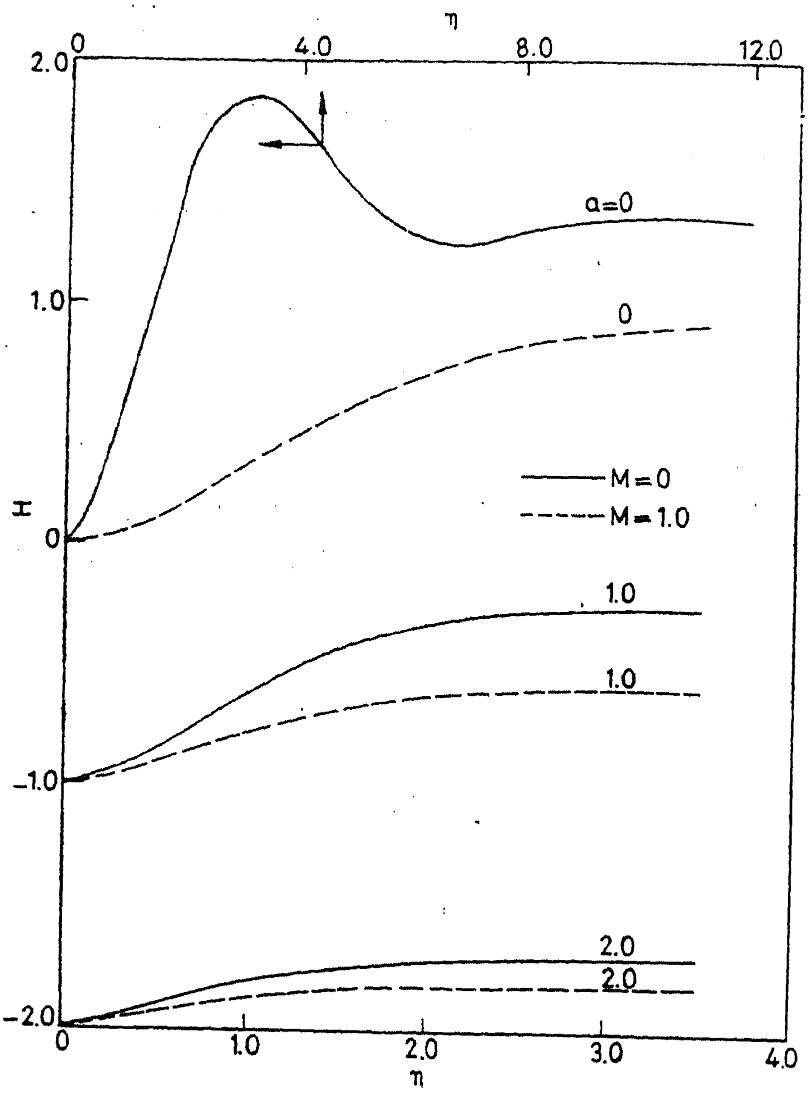

Figure 3. Axial velocity profiles.

compared our results for $M = a = 0$ with those of Bödewadt[3] and Millsaps and Nydahl[8] and for $a = 0$ and $M \neq 0$ with those of King and Lewellen[4] and they are found to be in excellent agreement.

4. CONCLUSIONS

The velocity profiles in the absence of the magnetic field and suction exhibit velocity overshoot and approach their asymptotic values in an oscillatory manner. It is observed that both suction and magnetic parameters tend to reduce the velocity overshoot and damp the oscillation. The tangential shear stress increases but the radial shear stress decreases as the suction or magnetic parameter increases.

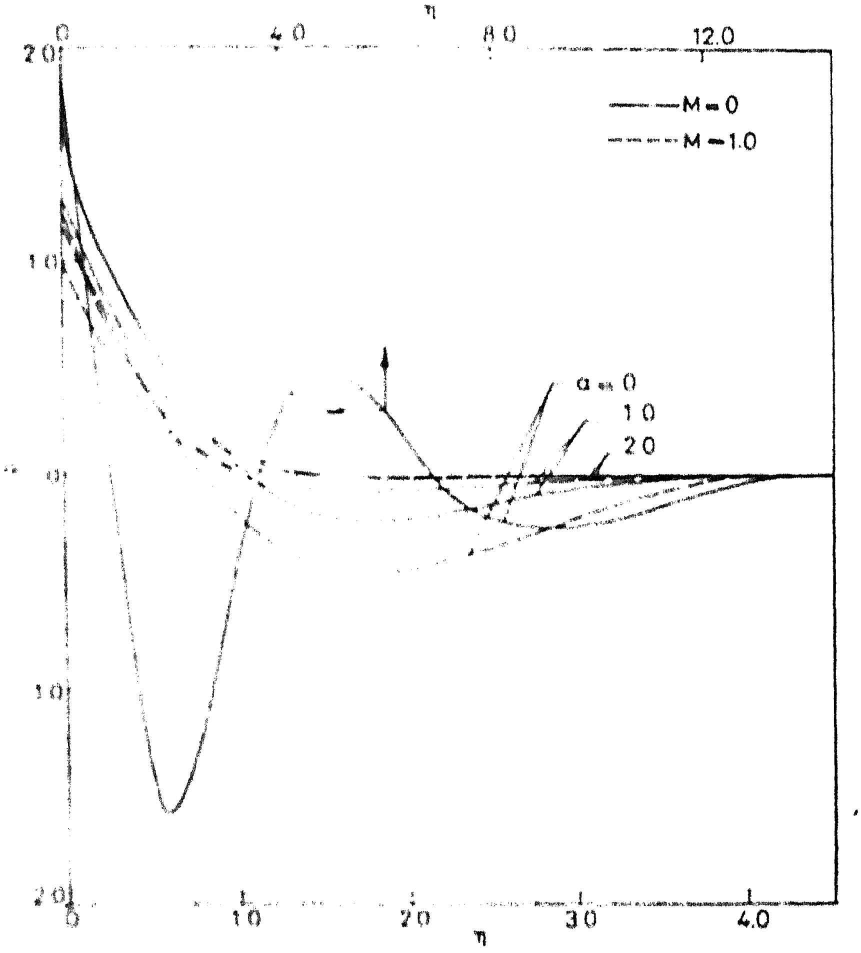

Figure 4. Pressure profiles.

REFERENCES

wellen, W. S., *NASA CR* 1712 (1971).

att, N. and Lewellen, W. S., *Boundary Layers and Their Interactions in Rotating Flows*, Progress Aeronaut. Sci. Vol. 7, Pergamon Press, p. 111 (1966).

dewadt, U. T., *ZAMM* 20 241 (1940).

ng, W. S. and Lewellen, W. S., *Phys. Fluids* 7 1674 (1964).

gers, M. H. and Lance, G. N., *J. Fluid Mech.* 7 617 (1960).

en, F. and Penner, S. S., *Phys. Fluids* 13 1665 (1970).

ao, H. L., *Atmos. Sci.* 28 20 (1971).

illsaps, K. and Nydahl, J. E., *ZAMM* 53 241 (1973).

chtsheim, P. R. and Swigert, P., *NASA TND* 3004 (1965).