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Theory of CO_2-laser induced x-ray emission from high density gaseous targets

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1. Introduction

In addition to the x-ray emission (Artsimovitch *et al* 1956, Andrianov *et al* 1959, Quin 1959, Lee and Elton 1971, Fukai and Clothiaux 1975) through vacuum discharge, increasing interest is being taken in the laser induced x-ray emission from solid targets (Nagel *et al* 1974 and Young 1974), when a CO_2 laser interacts with a high density plasma. It is generally believed that the energy absorption takes place in a narrow layer of density where the plasma frequency equals the laser frequency. Various theories have been given for the physical mechanisms involved. Nonlinear parametric instability (Kaw *et al* 1971, Rosenbluth *et al* 1972) and linear resonance model (Godwin 1972, Friedberg *et al* 1972) have been given as possible mechanisms. It has also been proposed (Krishan *et al* 1976), and the results compared with an experiment (Fabre and Stenz 1974), that a linearly growing instability very soon enters the nonlinear domain when it is saturated by the perturbation in the particle orbits. To settle the issue as to which mechanism is the best candidate, Yablonovitch (1975) controlled the density at and near the critical density. For obvious reasons gaseous targets were preferred over the solid targets. The experiment was done for pure H_2, He-H_2 mixtures, He nitrogen and argon. In pure samples of H_2 the x-ray emission rose sharply from zero at the critical to a maximum when the pressure was 25% more than the critical pressure. The same behaviour was more or less true for He. In this paper we shall present the theory that explains the experimentally observed sharp rise in the x-ray emission in H_2.

2. Dispersion relation

We shall first derive the dispersion relation for the electro-magnetic modes propagating in the plasma system. In the dipole approximation the vector potential of the laser field is given by

$$\mathbf{A}(t)=A_0(\hat{e}_x \cos\omega t+\hat{e}_y \sin\omega t), \tag{1}$$

where $\hat{e}_x$ and $\hat{e}_y$ are the unit vectors and ω the laser frequency. The Hamiltonian for a particle in the presence of the laser is given by

$$H = \frac{1}{2m}\left(\mathbf{p} - \frac{e}{c}\,\mathbf{A}\,(t)\right)^2, \tag{2}$$

where m is the mass of the electron, e the electric charge and c the velocity of light. The solution of the Schrödinger equation for the above Hamiltonian is given by

$$\psi = \frac{1}{\sqrt{v}} \sum_{n=-\infty}^{\infty} J_n\left(\beta_{p_\perp}\right) \exp\left[-\frac{i}{\hbar} E_{n,\mathbf{p}} + \frac{i}{\hbar}\,\mathbf{p}\cdot\mathbf{x} - i\delta_{p_\perp} n + \beta_{p_\perp} \sin\delta_{p_\perp}\right] \tag{3}$$

where

$$E_{n,\mathbf{p}} = \left[\frac{p^2}{2m} + \frac{e^2 A_0^2}{2m\,c^2} - \hbar\, n\, \omega\right],\ \beta_{p_\perp} = \frac{p_\perp e A_0}{mc\hbar\omega},$$

$$p_\perp^2 = p_x^2 + p_y^2,\ \tan\delta_{p_\perp} = \frac{p_y}{p_x}. \tag{4}$$

It is clear from the form of equation (3) that in the limit of long time averages the state of an electron in the presence of a laser can be regarded as a superposition of states labelled by index n. The energy of the state is $E_{n,\,\mathbf{p}}$ and the corresponding probability for being in that state is given by $J_n^2\left(\beta_{p_\perp}\right)$.

The dielectric function for a plasma in the absence of any external field is given by

$$\mathscr{E}\,(\mathbf{q},\,\Omega) = 1 + \sum_{S,\,\mathbf{k}} \frac{\omega_{ps}^2 m_s}{q^2 V \eta_0}\left[\frac{f_s\,(\mathbf{k}+\mathbf{q}) - f_s\,(\mathbf{k})}{\hbar\,\Omega - E_s\,(\mathbf{k}+\mathbf{q}) + E_s\,(\mathbf{k})}\right]. \tag{5}$$

Here the particles occupy a volume V with density η_0, S is the species index, $E_s(\mathbf{k})$ is the energy and $f_s(\mathbf{k}) = e^{-E_{s,\,k}/T}$, ω_{ps} is the plasma frequency. In the presence of the laser, equation (5) can be modified by observing the following correspondence $f_s(\mathbf{k}) \to f_{s,n}(\mathbf{k}),\ f_s(\mathbf{k}+\mathbf{q}) \to f_{s,n'}\,(\mathbf{k}+\mathbf{q}),\ E_s(\mathbf{k}) \to E_{s,n}(\mathbf{k}),\ \sum_{\mathbf{k}} \to \sum_{n,\,n',\,\mathbf{k}}$,

$$\frac{4\pi e^2}{Vq^2} \to \frac{4\pi e^2}{q^2 V} J_n^2\left(\beta_{p_\perp}\right) J_{n'}^2\left[\beta_{(\mathbf{p}+\hbar\mathbf{q})\perp}\right],$$

where the last change occurs because of change in the interparticle vertex and $f_{s,n} = \exp\left[-E_{s,n}(\mathbf{k})/T\right]$. The dispersion relation for the electromagnetic wave therefore becomes

$$1 + \sum_{S} \frac{\omega_{ps}^2\, m_s}{q^2\,\hbar} \int d^3p \sum_{n,\,l} J^2_{n-l}\left(\beta_{p_\perp}\right) J_n^2\left[\beta_{(\mathbf{p}-\hbar\mathbf{q})_\perp}\right] \left[\frac{f_{n-l}\,(\mathbf{p}) - f_n\,(\mathbf{p}-\hbar\mathbf{q})}{\Omega - l\,\omega - \dfrac{\mathbf{p}.\mathbf{q}}{m} + \hbar q^2/2m}\right], \tag{6}$$

where $n'=n-l$ has been used. Expanding (6) in powers of $\frac{\hbar \mathbf{q}}{p}$ and $\frac{l\hbar\omega}{T}$ and noting that the total number of particles are equal $\sum_{\mathbf{k},n} J_n^2(\beta_{p\perp}) f_n(\mathbf{k})$ we get

$$1-\sum_S \omega^2_{psf} \sum_l \sum_{a=+1}^{-1} \left[\frac{1}{(\Omega-l\omega+a\,q_x\,u_{xs})^2}+\frac{a_s}{\Omega-l\omega+a\,q_x\,u_{sx}}\right], \qquad (7)$$

where

$$\omega^2_{psf}=\tfrac{1}{2}\,\omega^2_{ps}\,J^2_{N_s-l}(Z_s)\quad a_s=\frac{l\omega m_s}{Tq^2},\quad u_s=\frac{eA_0}{\sqrt{2}m_s c},\quad \mathbf{q}\text{ is along }\hat{e}_x$$

$N_s \simeq Z_s = \frac{e^2 A_0^2}{m_s c^2 \hbar\omega}$, and derivatives of $J_n(Z)$ with respect to Z have been neglected, as they are small.

In the above derivation the fact that the largest maxima of the Bessel function occurs for order $|n| \approx$ argument of the Bessel function, has been used. The limit $\hbar\mathbf{q}/p<1$ is quite obvious for collective oscillations, but $l\hbar\omega/T<1$ is not so obvious. However if one is looking for oscillation with $\Omega \simeq \omega l$, then from Heisenberg's uncertainity principle it becomes obvious, in fact, that $l\hbar\omega/T$, is less than 1. We shall be looking for collective modes with frequence $\Omega \simeq l\omega$.

When the plasma is formed there is a strong tendency for it to avalanche down towards the low field region. However the electrons being much lighter than the ions, cascade away much faster, so that at any instant of time, they see a lower field than the ions. Let us assume for a moment the electrons and the ions see the same field and that we are dealing with a situation where the laser is strong enough so that $Z_e \simeq 10^4$. Such a situation is found in the experiment of Yablonovitch. We are, in particular, interested in modes for which $l \sim 10^3$. Then it is easy to see from the definition of ω_{psf} that ions will not then contribute. But the experiment shows that ions also play an equally important role. As the electrons cascade away from the focal spot faster than the ions, we choose $Z_e = Z_i$.

In order to solve the dispersion relation (7) we shall follow the method of independent stream model. Then the ions give rise to a positive energy mode under the condition (Krishan *et al* 1975, 1977)

$$F_i=\left\{\frac{1}{\Omega}\frac{\partial}{\partial\Omega}\left[\Omega^2\epsilon_i(\mathbf{q}_i,\Omega)\right]\right\}\Bigg|_{\Omega=\omega_i} \qquad (8)$$

$$=\left[\frac{2q_i^2c^2}{\omega_i^2}+\frac{2\omega^2_{plf}\,\omega_i}{(\Omega_i-l\omega-q_iu_i)^3}+\frac{2\omega^2_{plf}\,\omega_i}{(\Omega_i-l\omega+q_iu_i)^3}+\frac{\omega^2_{plf}\,a_i\omega_i}{(\Omega_i-l\omega-q_iu_i)^2}\right.$$

$$\left.+\frac{\omega^2_{plf}\,a_i\omega_i}{(\Omega_i-l\omega+q_iu_i)^2}\right]>0 \qquad (8a)$$

and the electrons give a negative energy mode for

$$F_e = \frac{1}{\Omega}\frac{\partial}{\partial\Omega}\{[\Omega^2\epsilon_i(\mathbf{q}_e, \Omega]\}\big|_{\Omega=\omega_e} \tag{9}$$

$$\equiv \frac{2q_e^2c^2}{\omega_e^2} + \frac{2\omega^2_{pef}\omega_e}{(\Omega_e - l\omega - q_eu_e)^3} + \frac{\omega^2_{pef}\alpha_e\omega_e}{(\Omega_e - l\omega q_eu_e)^2}$$

$$+ \frac{2\omega^2_{ef}\omega_e}{(\Omega_e - l\omega + q_eu_e)^3} + \frac{\omega^2_{pef}\alpha_e\omega_e}{(\Omega_e - l\omega + q_eu_e)^2} < 0. \tag{9a}$$

In the above ϵ_i is the dielectric function for the independent stream of ions and ϵ_e that for the electrons, (Ω_i, q_i) and (Ω_e, q_e) are the corresponding solutions of the independent ion and electron systems. The expressions ϵ_e and ϵ_i can be easily identified by looking at equation (7). For two mode coupling, the energy and momentum conservation require

$$(\text{sgn}F_e)\,|\,\Omega_e| + \text{sgn}F_i\,|\,\Omega_i| = 0, \ \mathbf{q}_e + \mathbf{q}_i = 0. \tag{10}$$

The two independent dispersion relations for electrons and ions after some straight forward algebraic steps and using (10) yield

$$\Omega_0 \equiv \Omega_e = \Omega_i = l\omega \pm qu\left(1 + \tfrac{1}{8}\frac{m}{M}\right), \tag{11}$$

$$q^2 = q_i^2 = q_e^2 = \left(\frac{\Omega_0}{c}\right)^2 \pm \frac{\Omega}{c}\left[\left(\frac{\Omega_0}{c}\right)^2 - \frac{128}{\frac{m^2}{M^2}u_e^2}\,\omega^2_{pe}J^2{}_{N-l}(Z_e)\right]^{\frac{1}{2}},$$

$$F_i = \frac{2q^2c^2}{\Omega_0^2} - \frac{2\omega^2_{pif}\Omega_0}{(qu)^3} + \frac{\omega^2_{pif}\alpha_i\Omega_0}{(qu)^2}, \tag{12}$$

$$F_e = \frac{2q^2c^2}{\Omega_0^2} - \frac{2\omega^2_{pcf}\Omega_0}{\left(\frac{m}{M}\right)^{\frac{3}{2}}(qu)^3} + 64\omega^2_{pcf}\frac{\alpha_e\Omega_0}{(qu)^2}\frac{M}{m}. \tag{13}$$

Taking $u \simeq 2\times10^9$ cm/sec (from the experiment) and $l=3000$ we find that the dominant term in (12) is the first one while the dominant term in (13) is the second one. This ensures that $F_i>0$ and $F_e<0$.

Now we shall find the growth rate due to interaction between one negative and one positive energy mode. The important contribution is given by the diagram of figure 1. The matrix element for interaction which is also the growth rate is calculated out to be

$$\gamma = \tfrac{1}{2}\frac{1}{\sqrt{(F_iF_e)}}\frac{\omega^2_{pe}}{\Omega_0}. \tag{14}$$

We mentioned before the dominant term in F_i is the first one which is independent of density and l; but the dominant term in F_e which is the second one depends on density and l. Although the density remains of the order of $10^{17}/cm^3$, the Bessel function is the one that fluctuates with l. We can uniquely find a value of l close to 3000 that makes F_e small which turns out to be l=3002. Thus at critical density

$$\gamma = 3{\cdot}43\times10^{9}/\text{sec} \tag{15}$$

2.1. *Saturation level of the mode*

We shall now determine the saturation level of the mode which can be done by using orbit perturbation theory. This can be calculated by introducing the self energy correction in the Green's function for particles. One finds that the important contribution is given by the diagram (Harris 1969, 1975) in figure 2. Thus the non-linear dispersion relation turns out to be

$$1-\sum_{a=+1,\,-1}\sum \omega^2_{psf}\left\{\frac{1}{\left[\Omega-l\omega+aqu_s-\frac{5}{2}\dfrac{\eta}{(\Omega-l\omega+aqu_s)^3}\right]^2}+\frac{a_s}{\left[\Omega-l\omega+aqu_s-\dfrac{\eta}{(\Omega-l\omega+aqu_s)^3}\right]}\right\}=\frac{q^2c^2}{\Omega^2} \tag{16}$$

where

$$\eta = 2\pi\mathscr{E}\left(\frac{qu}{\Omega_0}\right)^2\left(\frac{q^2e^2}{m^2}\right)[J^2{}_{N_e-l-1}((Z_e)+J^2{}_{N_e-l+1}(Z_e)],\ N_e\simeq Z_e \tag{17}$$

$\mathscr{E}$ being the energy of electromagnetic field. In the above the correction to ion Green's function has been neglected as it is small. The solution of equation (16) can be found and is given by

$$\Omega=\Omega_0+\Delta\Omega+i\gamma \tag{18}$$

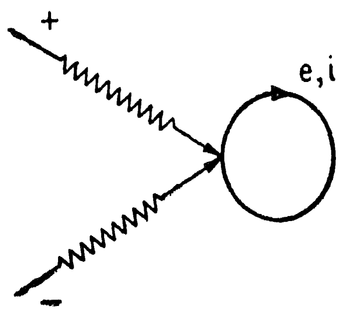

Figure 1.

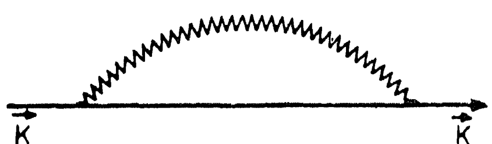

Figure 2.

where $\Omega_0 = |\Omega_e|$ and γ are given by (11) and (14) respectively and

$$\Delta\Omega = -\frac{5}{2}\frac{\eta}{\left[\left(\frac{m}{M}\right) qu_e\right]^3}\left[1 + \frac{\left(\frac{m}{8M}\right)^3 (qu)^3}{\omega^2_{pef}\,\Omega_0} + \frac{3i\gamma_0}{\frac{m}{8M} qu}\left(1 + \frac{2\left(\frac{m}{8M}\right)^3 (qu)^3}{\omega^2_{pef}\,\Omega_0}\right)\right], \; u_e \equiv u. \tag{19}$$

Saturation occurs when the imaginatory part of (18) vanishes. This gives

$$\mathscr{E} = \frac{1}{2\pi}\left(\frac{2}{15}\right)\left(\frac{m}{8M}\right)^4 \frac{m^2 u^2 \Omega_0^2}{e^2}\,[J^2{}_{N_e - l - 1}(Z_e) + J^2{}_{N_e - l + 1}(Z_e)]^{-1}\left[1 + \frac{2\left(\frac{m}{M}\right)^3 (qu)^3}{\omega^2_{pef}\,\Omega_0}\right]. \tag{20}$$

Evaluating the Bessel functions we find

$$\mathscr{E} = 1.43 \times 10^5 \text{ ergs/cm}^3.$$

Hence the fraction of laser energy gone into x-ray production is 7×10^{-6}.

3. Conclusions

We note from the density dependence of the last term in (20) that the saturation energy increases with increasing density. At the critical density ($\omega_{pe} \simeq \omega$) the explicit value of the density dependent term is just

$$\left[1 + \frac{2\left(\frac{m}{M}\right)^3 (qu)^3}{\omega^2_{pef}\,\Omega_0}\right]^{-1} \approx [1 + 0.32]^{-1}.$$

Moreover with increasing density the above expression increases until it saturates off to unity. For densities lower than the critical the saturation energy will correspondingly decrease. However, this decrease is much more sharp than one might tend to think. The reason for this is that for densities slightly below the critical, the growth rate drops sufficiently so as to yield characteristic times of growth greater than the lasing period. We notice that already at the critical density the characteristic growth rate time is 3×10^{-10} sec whereas the laser pulse time is 5×10^{-10} sec. Thus for densities lower than critical the wave will not even get chance to grow so that saturation for them will not be possible to attain. The energy emitted in the form of x-rays from these densities, will then be almost unobservable.

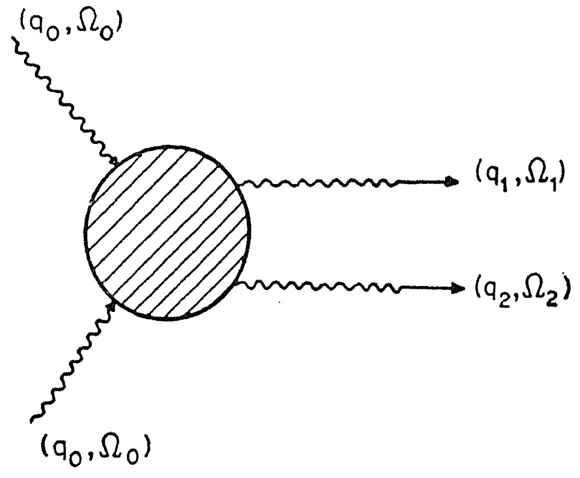

Figure 3.

Thus the essential behaviour of the saturation energy would be something like this: for low densities (until about the critical) there is virtually no x-ray generation. Somewhere close to critical density the emission rises rapidly. After this initial sharp rise, its further increase is relatively slow with tendency to saturate off. In the experiment of Yablonovitch an initial sharp rise was observed at densities very close to the critical. However instead of increasing slowly to some limiting value thereafter, the x-ray emission exhibits a peak (at densities close to the critical) before it saturates off at higher densities. Our theory fails to predict this peaking behaviour. The initial sharp rise and the final levelling-off which were experimentally observed are in accordance with our theory. The aspect in which our theory does not agree with the experiment could be because of the following reasons:

1. The crucial physical parameter u—the quiver velocity—may not be the same at all densities.
2. There may be density gradients.
3. The dipole approximation may loose its validity.

4. Generation of short wavelengths

It is possible to have wavelengths shorter than 15 Å through nonlinear interactions. The nonlinear mechanism is shown in figure 3, where (q_0, Ω_0) refer to the primary emission of x-rays which has been predicted earlier on the basis of linear theory; Ω_2 is about $2\Omega_0$ but $q_2 \ll q_0$; $\Omega_1 \ll \Omega_0$ but $q_1 \simeq 2q_0$. It can be seen that (Ω_1, q_1), (Ω_2, q_2) are both solutions of linear dispersion relation.

The growth rate due to the nonlinear interaction is of the order of 10^3/sec. Thus to detect short wavelength x-rays, one has to expose the film many times as was done in the experiment. The behaviour of the short x-rays with respect to the plasma density will obviously be the same as for the long wavelengths.

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