vhere $\xi = 180°$ defines the planarity of the ieptide unit at the nitrogen atom. This illustrates he great importance of the newly defined dihedral ngle for biopolymer conformation. Calculations .re under way to work out the contribution to the nergies of molecules of interest in biology, assoiated with both the dihedral angles of the type ; and of the type ξ'.

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THE ROLE OF 'COSMOLOGICAL CONSTANT' AND *f*-GRAVITY IN REMOVING GRAVITATIONAL SINGULARITIES

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ABSTRACT

In Trautman's recent model of a universe with 10^{80} spin-aligned neutrons, the usual space-time singularity, which is an inherent feature of isotropic general relativistic cosmologies, and the occurrence of which was hitherto considered inevitable in gravitational collapse, is averted by incorporating torsion effects in the usual Einstein field equations through the Einstein-Cartan gravitational theory. Also in another recent paper Salam *et al.* have investigated the effect of taking *f*-gravity into account in Trautman's model. However, the severe difficulty of finding a suitable mechanism for aligning perfectly all the nuclear spins of the 10^{80} collapsing hadrons still persists. We suggest an alternative way which does not suffer from this drawback by invoking a "cosmological" constant Λ suitably defined to incorporate short-range *f*-gravity. It is shown that the singularity is then avoided and the results obtained by the above authors can be reproduced.

IT is generally accepted, especially after the work of Penrose and Hawking[1], that gravitational :ollapse inescapably leads to space-time singularities. The singularity theorems imply the inevitability of unlimited collapse to a singular state. It was ilso known from the much earlier work of Oppenleimer and Snyder[2] that the spherically symmetric gravitational collapse of a dust cloud gives rise to ı similar singularity. The occurrence of such singularities is an inherent feature of Einstein's general relativity. Again, the usual isotropic evolutionary models of the universe (Robertson-Walker models) are singular, *i.e.*, at some instant of time they pass through a singular state (interpreted sometimes as the 'initial' state in big-bang models) when matter is collapsed to a physically meaningless infinite density. Modified versions of general relativity such as the Brans-Dicke theory also predict such singularities.

However, Trautman[3] has recently indicated that such singularities may be avoided by directly including the influence of spin on the space-time geometry, *i.e.*, by incorporating the effects of Cartan's torsion in the Einstein-Cartan theory of gravitation, which is a generalization of Einstein's general relativity. This theory which essentially involves the addition of torsion terms to the usual Einstein equations was originated by Cartan[4] and independently worked out by Sciama[5] and Kibble[6]. Here the geometry of space-time is determined by a metric tensor and linear connection fields which are independently varied in the usual Palatini form of the action integral. In the absence of sources the connection reduces to the ordinary Christoffel symbol but with sources included the resulting equations imply an additional torsion term apart from the Riemannian connection. Unlike Einstein's theory the torsion tensor $\Omega_{\cdot\rho}{}^{\mu}$ is not required to vanish but is related to the density $S_{\nu\rho}{}^{\mu}$ of an intrinsic angular momentum source. The field equations then become[6,7]:

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} g^{\rho\sigma} R_{\rho\sigma} = 8\pi G c^{-4} T_{\mu\nu}, \tag{1}$$

$$\Omega_{\nu\rho}{}^{\mu} + \delta_{\nu}{}^{\mu} \Omega_{\sigma\rho}{}^{\sigma} - \delta_{\rho}{}^{\mu} \Omega_{\sigma\nu}{}^{\sigma} = 8\pi G c^{-4} S_{\nu\rho}{}^{\mu}. \tag{2}$$

Here $R_{\mu\nu\rho\sigma}$ is the curvature tensor formed from the connection, $R_{\mu\nu} = g^{\rho\sigma} R_{\rho\mu\nu\sigma}$, $g_{\mu\nu}$ is the metric tensor and $T_{\mu\nu}$ is the asymmetric energy momentum tensor. If the intrinsic angular momentum density $S_{\nu\rho}{}^{\mu}$ is neglected, Eqs. (1) and (2) reduce to the usual Einstein field equations.

Earlier work of Kopczyn'ski[7] showed that the usual Friedmann singularity in cosmological models is avoided with a solution of the Einstein-Cartan equations with a spherically-symmetric distribution of spins. Kopczyn'ski also constructed a class of non-singular cosmological models based on the Einstein-Cartan theory which provide a lower bound for the minimum radius of the universe. In Trautman's model, the universe is filled with spinning dust characterized by its four-velocity u^{μ}, mass density ρ and spin density $S_{\nu\rho}{}^{\mu} = u^{\mu}\bar{S}_{\nu\rho}$, $u^{\rho}\bar{S}_{\nu\rho} = 0$, these assumptions being compatible with the isotropy of the Robertson-Walker line element. Now assuming that the spins of all the 10^{80} neutrons in the universe are aligned along the x-axis, Trautman reduces Eq. (1) to the modified Friedmann equation :

$$\frac{1}{2}\dot{R}^2 - \frac{GM}{R} + \frac{3G^2S^2}{2c^4R^4} = 0, \qquad (3)$$

where M is the total conserved mass, $\frac{4}{3}\pi\rho R^3$, and S is the total spin (as all spins are assumed to be perfectly aligned) given by $S = \frac{4}{3}\pi\sigma R^3$, σ being a spin density. The last term in Eq. (3) modifies the usual Friedmann relation and is equivalent to a "repulsive potential" dominant at small values of R. This term prevents the solution from approaching zero. Thus Trautman is able to show that at $t = 0$, the minimum radius of a sphere containing N particles of mass $m = M/N$ and spin $\frac{1}{2}\hbar = S/N$ is :

$$R_{min} = (3NG\hbar^2/8m_N c^4)^{1/3}. \qquad (4)$$

m_N being the neutron mass. For $N = 10^{80}$, believed to represent the total number of baryons in the accessible universe, R_{min} is of the order of 1 cm ! The corresponding density of matter at $t = 0$, is of the order of $m_N{}^2c^4/G\hbar^2 \simeq 10^{55}$ g.cm.$^{-3}$. Trautman points out that this low value for R_{min} is much greater than the Planck length $(\hbar G/c^3)^{\frac{1}{2}} \simeq 10^{-33}$ cm., at which stage quantum fluctuations of the gravitational field become significant and quantum gravitational effects are supposed to play an important role[8-10]. Also the density at $t = 0$ in Trautman's model is much smaller than the absurdly high density $c^5/G^2\hbar \simeq 10^{94}$ g.cm.$^{-3}$ at which quantum effects of the gravitational field dominate. Thus, inclusion of quantum gravitational effects in classical general relativistic collapse and cosmology avoids the infinite density of the singularity but gives it the absurdly high density of 10^{94} g.cm^{-3}. But by including torsion in classical general relativity, Trautman has brought down the cosmological density at $t = 0$ still further to 10^{55} g.cm^{-3} corresponding to the size of 1 cm. But the serious difficulty with Trautman's model is the need for a suitable mechanism to effect the perfect spin alignment necessary to get his results. A possible mechanism suggested by him is the presence of a cosmic (intergalactic) magnetic field—which may successfully compete with the increasing temperature T as the collapse proceeds, provided the flux is conserved and $HR^2 =$ constant. As $RT =$ constant, (*i.e.*, black-body radiation), $\mu H/kT$ behaves like $1/R$ and might have been initially large enough to align all spins. However, the presence of such an intergalactic magnetic field is highly doubtful and there are difficulties associated with the exclusion principle in case such a perfect alignment at such high temperatures (near $t = 0$) is ever possible. Moreover, the intense magnetic field necessary for the model will also significantly contribute to the energy-momentum tensor. This has not been taken into account by Trautman. As the collapse proceeds increasing alignment of spins is necessary to provide sufficient torsion to brake the collapse.

In a recent work, Narlikar[11] has suggested that the problem of singularities should be resolved by a proper consideration of matter creation through a negative energy C-field. By incorporating the C-field energy tensor in the usual Einstein field equations he obtains a non-singular Friedmann model with the same $R(t)$ as obtained by Trautman.

This can be understood as follows : In general relativity the presence of the negative energy C-field amounts to repulsion, and this causes one of the main requirements of Penrose and Hawking to break down thus avoiding the singularity. However, the evidence for a C-field and the actual occurrence of creation of matter in the universe is highly controversial.

In a recent paper, Isham, Salam and Strathdee[12] have investigated the effect on Trautman's results when f-gravity is taken into account, *i.e.*, Cartan's formulation is taken into account for both f and g fields. They have derived the field equations used by Trautman from Lagrangian field theory, *i.e.*, from a variational principle. By considering the interaction of a Dirac spinor with the vierbein gravitational field and spinor connection it is indicated that the torsional effect in Trautman's model manifests itself by the appearance in the Lagrangian of an effective spin-spin contact interaction term $\kappa^2(\bar{\psi}\gamma_{\mu}\gamma_5\psi)^2(-g)^{\frac{1}{2}}$ proportional to the Newtonian gravitational constant. Previously, the appearance of such a non-minimal term in the Palatini-type Lagrangian when spinor fields are coupled to gravity was emphasized by various authors such as Weyl[13], Sciama[5] and Kibble[6]. This corresponds to Trautman's term which arrests collapse to a singularity as discussed before. Earlier Salam *et al.* had suggested[14] that the natural vehicle for taking hadronic short-range forces into consideration in gravitational physics was through a two-tensor (*f-g*) theory of gravity, *i.e.*, in addition to Einstein's massless graviton field $g_{\mu\nu}$ (mediated by massless gravitons) we have to consider the massive strong gravity field $f_{\mu\nu}$ mediated by massive spin 2^+ *f*-mesons. The gravitational interaction between hadrons can proceed *via f*-mesons with *f*-gravity coupling to hadronic matter. They have shown that the interposition of *f*-gravity on Trautman's work has a profound effect and their chief result is that the spin-aligned hadronic matter will now collapse to a minimum radius of $\simeq 10^{13}$ cm, rather than 1 cm which is the minimum radius of the Trautman universe. This implies that the maximum matter density in Trautman's universe of 10^{80} neutrons with their spins aligned is $\simeq 10^{17}$ gm cm^{-3} rather than 10^{55} gm cm^{-3}. Thus hadronic matter can collapse to densities only one or two orders of magnitude higher than nuclear densities. As most astrophysical and cosmological models of superdense matter[15] involve a hadron gas composed predominantly of hadrons, it seems reasonable to invoke *f*-gravtiy with its coupling constant G_f rather than the Newtonian constant G. However the difficult problem of perfect alignment of all hadron spins still remains a severe drawback as in the case of the Trautman model. We now suggest a way out of the difficulty. We shall obtain both the results of Trautman and Salam *et al.*, without this requirement.

In an earlier work[16] we had reinterpreted the 'cosmological constant' Λ occurring in Einstein's equations in terms of the inverse Compton length of the *f*-meson. This seemed to be the most natural way to incorporate the short-range *f*-gravity *via f*-mesons into Einstein's field equations.

Thus

$$\Lambda \simeq \left(\frac{m_f c}{\hbar}\right)^2. \tag{5}$$

Then for a spherically-symmetric collapse we can write for the potential: (Λ is positive)

$$V = c^2(g_{00} - 1) = c^2\left(-\frac{2GM}{Rc^2} - \frac{\Lambda R^2}{3}\right).$$

To find the equilibrium value of the radius (R_{eq}) we require the potential gradient to vanish and thus all forces to vanish. This gives

$$\ddot{R} = 0 = \frac{2GM}{R^2 c^2} - \frac{2\Lambda R}{3} = 0 \tag{6}$$

or

$$R_{eq} = \left(\frac{3GM}{\Lambda c^2}\right)^{1/3} = \left(\frac{3GM^2\hbar'^2}{m_f^2\, c^4}\right)^{1/3}. \tag{7}$$

More precisely, the usual Friedmann equation with a cosmological term Λ, (for an Einstein-DeSitter universe with zero pressure and Euclidean 3-space) is:

$$\dot{R}^2 = \frac{8\pi G\rho}{3} R^2 - \frac{2\Lambda c^2}{3} R^2. \tag{8}$$

Putting $M = (4/3)\pi R^3\rho$, $\dot{R} = 0$, at $t = 0$, gives:

$$R(0) = \left(\frac{3GM}{\Lambda c^2}\right)^{\frac{1}{3}},$$

the same as in Eq. (7), where M is the total mass of the 10^{80} baryons in the universe (each baryon having a mass $\simeq 10^{-24}$ g) and Λ is as defined in Eq. (5), where $m_f \simeq 1500$ MeV. Substitution of these values in Eq. (7) yields: $R(0) \simeq 1$ cm (!) and the corresponding density of matter is given by $\rho = \Lambda c^2/4G \simeq 10^{55}$ g. cm^{-3}. These values are the same as those in Trautman's model. The Λ-term appearing in Eqs. (6) and (8) is of the same order of magnitude as the torsion term in Trautman's model. In both cases these additional terms (torsion in Trautman's, the redefined Λ in ours) in the Friedmann equation are the ones which avert the singularity. For $\Lambda = 0$, *i.e.*, for the massless gravity field, of infinite range, we recover the old singularity, $R = 0$, from Eq. (8).

If we invoke *f*-gravity, the Newtonian constant G must necessarily be replaced by the corresponding coupling constant G_f for *f*-gravity, which was estimated in an earlier work[17] to be of the order of 10^{39}G. Therefore replacing G in Eq. (7) by G_f we get $R(0) \simeq 10^{13}$ cm and the corresponding matter density turns out to be 10^{17} g. cm^{-3}; these results being the same as those obtained by Salam *et al.*[12]!

We note that in our formulation the difficult hurdle of finding a suitable mechanism for the alignment of all hadron spins is not encountered.

We can try to understand these results as follows: The Hawking-Penrose[1] singularity theorems for the Einstein field equations $G_{\mu\nu} + \Lambda g_{\mu\nu} = KT_{\mu\nu}$ which can be derived from the Lagrangian

$$\mathcal{L} = (-g)^{1/2}\kappa(R - 2\Lambda) + \mathcal{L}_{\text{matter}}, \tag{9}$$

do not necessarily hold if the cosmological constant is positive. Indeed it is remarked in Ref. (13) that for all spins aligned in the same direction, the effective spin-spin interaction term in the Lagrangian for the Einstein-Cartan theory which forms the essence of Trautman's torsion term can be regarded as being of the form $L = -(-g)^{\frac{1}{2}}\Lambda_{eff}(x)$, where

$\Lambda_{eff}(x)$ is a positive effective "cosmological" contribution to the Lagrangian. As we remarked earlier the Λ term in Eqs. (6) and (8) is of the same order of magnitude as Trautman's torsion term.

Also it is interesting to note that by using G_f instead of G, the absurdly high density $c^5/G^2\hbar \simeq 10^{94}$ g. cm.$^{-3}$, at which quantum effects of the gravitational field and of space-time curvature are expected to become important is reduced (scaled down) to $\simeq 10^{17}$ g. cm^{-3}, the same value as quoted above. This enables us to explain the somewhat paradoxical result of our getting a much larger radius R(0) and a much lower density ρ (0) on using the higher coupling constant G_f when one would have expected the reverse to happen, *i.e.*, lower radius and higher densities due to the larger gravitation constant, (and hence stronger collapse). The answer is that for *f*-gravity the quantum effects which avert the singularity and halt the collapse (essentially through the Λ term) become important at much lower densities ($\simeq 10^{17}$ g. cm^{-3}) and hence larger radii of the collapsing matter than in classical GTR when these effects become important only at densities $\simeq 10^{94}$ g. cm^{-3} corresponding to much lower initial radii. For classical GTR with $\Lambda = 0$ and without consideration of quantum effects, the density of the singularity is infinity corresponding to R(0) = 0. That the Λ term can avert total collapse is indicative of a repulsive interaction effect. In fact, in a recent work it has been shown that the theory with Λ is related to massive scalar mesons and spin 2+ tensor mesons, with the former being more massive. Also, the former seems to have a repulsive shorter-range interaction[18]. A brief account of the work reported here appears in Nature[19].

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KAEMMERERITES FROM CHROMITE DEPOSITS OF BYRAPUR, HASSAN DISTRICT, KARNATAKA STATE

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ABSTRACT

Two physically distinct varieties of chromium chlorites occur in the chromite deposits of Byrapur, which form a part of the Naggehalli schist belt. Thty differ in their habit, one being flaky and the other lamellar to fibrous. Cr_2O_3 in the flaky chlorite is 3·9% and in the lamellar it is 4%. Based on the spacing d_{007} and intensity d_{002} reflections (Lapham, 1958), the Cr is believed to be in the octahedral coordination in both the varieties and hence identified as kaemmererites.

THE Nuggehalli schist belt of Karnataka State extending over a length of 40 km. (Lat. 12° 58′ 30″–13° 17′ 43″ and Long. 76° 17′ 20″–76° 29′ 30″) and a width of about 1·5 km. is composed of green schists, amphibolites, serpentinites, and lenses of dunite transected by siliceous and carbonate veins. The ultramafic lenses contain rich chromite deposits. In association with these chromite deposits, the occurrence of chromium chlorites has been reported earlier from Hulikere chromite mines near Jambur,