A cryptographic system based on finite field transforms

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Abstract. In this paper, we develop a cipher system based on finite field transforms. In this system, blocks of the input character-string are enciphered using congruence or modular transformations with respect to either primes or irreducible polynomials over a finite field. The polynomial system is shown to be clearly superior to the prime system for conventional cryptographic work.

Keywords. Ciphers; computer data protection; cryptography; finite field transforms; irreducible polynomials; military communication; polynomial congruences; prime congruences; public crypto-systems.

1. Introduction

A cryptographic system [7], [11] consists of a set of transformations T_i , each of which can act on an input message M to produce a corresponding enciphered message E_i , i.e.,

$$E_i = T_i(M).$$

Each transformation T_i is specified by an associated key k_i . The enciphered message is transmitted to the receiver over an interceptable medium. At the receiving end, the original message M is recovered by applying the inverse transformation T_i^{-1} on the received (enciphered) message E_i . It is clear that the existence of T_i^{-1} is a necessary condition for T_i to be a valid encoding transformation.

Example :

Substitution cipher—In this cipher, each input character is transformed into another character. The transformed character set may or may not be the same as the input character set. In the former case, the transformation is a simple permutation of the input character set. Let the (ordered) input characters be a, b, c, d . If the transformed character set is $\{0, 1, 2, 3\}$, any permutation of these 4 characters represents a key to a particular transformation. If $2, 1, 3, 0$ is the specified key k_i , T_i is given by $a \rightarrow 2, b \rightarrow 1, c \rightarrow 3, d \rightarrow 0$, and T_i^{-1} is given by $0 \rightarrow d, 1 \rightarrow b, 2 \rightarrow a$, and $3 \rightarrow c$. There are $4! = 24$ different transformations possible.

It is assumed that the enemy (i.e., the persons from whom the message M is protected) knows the set of transformations T_i being used. He also has available,