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The three faces of Maxwell's equations

N MUKUNDA and E C G SUDARSHAN*

Centre for Theoretical Studies and Department of Physics, Indian Institute of Science, Bangalore 560 012, India

*Institute of Mathematical Sciences, Madras 600 113, India

Abstract. In dealing with electromagnetic phenomena and in particular the phenomena of optics, despite the recognition of the quanta of light people tend to talk of the amplitudes and field strengths, as if the electromagnetic field were a classical field. For example we measure the wavelength of light by studying interference fringes. In this paper we study the interrelationship of three ways of looking at the problem: in terms of classical wave fields, wave function of the photon; and the quantized wave field. The comparison and contrasts of these three modes of description are carefully analyzed in this paper. The ways in which these different modes complement our intuition and insight are also discussed.

Keywords. Maxwell's equations; photon wavefunction; coherent states; quantized Maxwell field.

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1. Introduction

The concept of the field as an autonomous dynamical entity independent of ponderable matter, first intuitively realized by Michael Faraday, found precise expression at the hands of James Clerk Maxwell. With the advent of the equations of classical electromagnetism, it was possible to endow the field with the dynamical attributes of energy and momentum; and electromagnetic radiation could be seen as the direct expression of its independent degrees of freedom.

Maxwell's electromagnetic theory has served as the window through which both special relativity and quantum theory were first seen. These were later realized to be general features of physical phenomena. While the special theory of relativity springs from a physical understanding of the invariance group of Maxwell's equations, and its clash with Galilean-Newtonian relativity, quantum theory arose from a study of the properties of black body radiation which indicated a failure of classical statistical mechanics when combined with Maxwell's theory. Thus the former has its origins in a property of the Maxwell equations, and the latter in a breakdown of its classical interpretation.

The existence of the quantum of action was thus first discerned by Planck via the statistical properties of radiation in 1900, and it was soon put to use by Einstein in 1905 to arrive at the concept of quanta of radiation. Its relevance for matter came later, through Einstein's inadequate theory of specific heats in 1907 followed by Bohr's

The authors felicitate Prof. D S Kothari on his eightieth birthday and dedicate this paper to him on this occasion.