

LETTER TO THE EDITOR

Is one-parameter scaling theory of localisation consistent? Comment on Schuh's comment

N Kumar

Physics Department, Indian Institute of Science, Bangalore 560012, India

Received 1 June 1983

Abstract. It is maintained that the one-parameter scaling theory is inconsistent with the physics of Anderson localisation.

This comment is in response to the comments by Schuh (1983) on my recent note (Kumar 1983) concerning the question of consistency of the one-parameter scaling theory of localisation (Abrahams *et al* 1979). In my note it was (i) proved that the assumption of one-parameter scaling, namely that the β function ($\equiv \partial \ln G / \partial \ln L$) scale with the block conductance G alone, subtends a definite and non-trivial restriction on the functional dependence of conductance G on the block size L and disorder Δ , as given by equation (5) of Kumar (1983); and then (ii) argued and concluded that the functional dependence, together with the assumed regularity condition, does not admit smooth interpolation between the asymptotic behaviours of conductance expected on the two sides of the mobility edge, when the latter is known to exist, e.g. for $d = 3$. Schuh agrees with (i) but contends that (ii) is based on an incomplete analysis of the equation (5) in that the function χ was assumed to be single-valued and φ monotonic. He argues that one could admit a multivalued function χ , the two branches of which are joined smoothly at the mobility edge. Then φ need not be monotonic in Δ , and one could interpolate between the metallic and the insulating behaviours without any inconsistency.

In the following I will now (A) first clarify how the suggestion of Schuh is already contained implicitly in my earlier treatment, and then (B) show why one still has the inconsistency. It is convenient to re-write equation (5) as

$$\ln G = \chi(nL + \varphi(\Delta)) \equiv \psi(Ll^{\varphi(\Delta)}) \dots \quad (1a)$$

with

$$\varphi(\Delta) = \int^{\Delta} \frac{d\Delta'}{f_1(\Delta')} \dots \quad (1b)$$

The non-trivial physical requirements are that (a) G is a monotonically and strictly decreasing function of strength of disorder Δ and that (b) for finite L there can be no non-analytic behaviour. Now to see (A) it is sufficient to note that if $f_1(\Delta)$ has a zero at $\Delta = \Delta_c$, as indeed it must if the β function is to have one (equation (4) of Kumar 1983), then the integral in equation (1b) is undefined unless we specify how the pole in the

integrand is to be treated. I had implied the principal value. One could, however, use the other possible indentations, namely

$$\frac{1}{f_1(\Delta)} = P \frac{1}{f_1(\Delta)} \pm i\pi \delta(f_1(\Delta)) \dots \quad (2)$$

Then $\exp(\varphi(\Delta))$ can indeed be arranged so as to change sign at $\Delta = \Delta_c$. Thus ψ can tend to two different asymptotic forms as $L \rightarrow \infty$ according as $\Delta > \Delta_c$ or $\Delta < \Delta_c$. This would amount to uniformly matching the two branches of the multivalued function that Schuh has in mind.

I will now show why equation (1) still cannot interpolate smoothly between the known asymptotic behaviours on the two sides of the mobility edge. I will confine myself to the case of $d = 3$, where the existence of a mobility edge separating the metallic and the insulating states is not in question. The point to note is that the factorisation of the argument of ψ in equation (1a) implies that Δ_c is not a function of L : also, that the function $f_1(\Delta)$ may have at the most one zero (at $\Delta = \Delta_c$) only, or the condition (a) will be violated. Now let us compare the following two situations, one for $\Delta \leq \Delta_c$ (metallic) and the other for $\Delta \geq \Delta_c$ (insulating), with the sample size L held much smaller than the localisation length for $\Delta \geq \Delta_c$ and yet macroscopic enough, i.e. sufficiently greater than the mean free path for backscattering, say, for these considerations to apply. Now, for $\Delta < \Delta_c$ the conductance must increase with L , while for $\Delta > \Delta_c$ it must decrease since the argument of ψ changes sign at $\Delta = \Delta_c$. But, with the sample size much smaller than the localisation length, there is no physical distinction between the 'metallic' and the 'insulating' regimes. This simply means that $f_1(\Delta)$ (and hence the β -function) can have no zero, or that the functional form (1a) is physically inadmissible. I must confess that this point was not made very explicit in my earlier note, except for a general reference to Mott and Kaveh (1981), who seem to have noted this point essentially.

I would like to thank Dr J Heinrichs for discussion and hospitality during the course of this work at the Institute of Physics of the University of Liège.

References

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