

Influence of time step in the simulation modelling of evapotranspiration

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Abstract. Three simulations of evapotranspiration were done with two values of time step, *viz* 10 min and one day. Inputs to the model were weather data, including directly measured upward and downward radiation, and soil characteristics. Three soils were used for each simulation. Analysis of the results shows that the time step has a direct influence on the prediction of potential evapotranspiration, but a complex interaction of this effect with the soil moisture characteristic, rate of increase of ground cover and bare soil evaporation determines the actual transpiration predicted. The results indicate that as small a time step as possible should be used in the simulation.

Keywords. Evapotranspiration; bare soil evaporation; simulation modelling; radiation balance

1. Introduction

Calculation of evapotranspiration is a necessary step in assessing the water budget of a catchment, storage and conveyance capacities in an irrigation project, size of farm ponds etc. The major part of evaporation over large land areas takes place from the leaves of plants or from bare soil, and the rest from open water surfaces like reservoirs. While evaporation from open water can be computed in a relatively straightforward manner that from vegetation and bare soil is more complex because of its dependence on soil properties and moisture supply and being controlled in addition by plant-physiological behaviour. A common approach to solving the latter problem is by first determining the potential evaporation, *i.e.* assuming ample soil moisture supply, and then applying corrections for actual soil moisture, growth stage of plants etc.

1.1 Review of some evapotranspiration models

1.1a Potential evaporation

(i) *Penman's formula* The energy for the latent heat of vaporization comes from the sun, and the vapour collecting above the evaporating surface is removed by molecular and convective diffusion. The energy for evaporation is the balance of incident shortwave solar radiation and the reflected shortwave radiation, the longwave radiation of the atmosphere and from the evaporating surface itself, and the energy used for photosynthesis. This energy balance H is used in evaporation E , heating of the air K , heating of the material (plant or soil) S and heating of the surroundings of the material C (Penman 1948),

$$H = E + K + S + C. \quad (1)$$

Penman argued that S and C are negligible over a period, and reduced (1) to

$$H = E + K = E(1 + \beta), \quad (2)$$

where $\beta = K/E$, the Bowen ratio, and is given by

$$\beta = \gamma(T_s - T_a)/(e_s - e_a), \quad (3)$$

where γ is the psychrometric constant, T_s is the temperature of the evaporating surface, T_a is the temperature of air, e_s is the saturated vapour pressure at T_s and e_a is the partial pressure of vapour in the air. In the absence of direct measurement of radiation balance, Penman used Brunt's equation to determine H from the incident shortwave radiation, the reflection coefficient of the surface, sunshine duration, and the longwave radiation balance dependent on the air temperature, relative humidity and cloud cover. Determination of E from (2) and (3) involves T_s , the surface temperature, which is difficult to measure. Penman therefore eliminated T_s between (2) and Dalton's formula for evaporation (in terms of windspeed, e_s and e_a) and arrived at his "combination" formula:

$$E = (H\Delta + E_a \cdot \gamma)/(\Delta + \gamma), \quad (4)$$

where Δ is the slope of the saturated vapour pressure-temperature curve at T_a ,

$$E_a = (e_a - e_d)f(u), \quad (5)$$

e_a is the saturated vapour pressure at T_a , and $f(u)$ is a function of the windspeed. It has been found by several researchers that the following equation given by Taylor and Priestley serves equally well to calculate E :

$$E = 1.26 \frac{H\Delta}{\Delta + \gamma}. \quad (6)$$

Equation (6) is similar to (4), without, however, the wind function.

Penman's formula (as applied to crops) is generally taken to give the evapotranspiration from an extensive surface of vigorously growing and amply watered, short (about 10 cm tall) grass. A crop coefficient (multiplier) is necessary to get the potential evapotranspiration of a crop at various stages of its growth. In addition, if the day-time to night-time ratio of wind speeds deviates considerably from 2.0, it needs a wind adjustment coefficient (when daily mean values of weather parameters are used).

Doorenbos and Pruitt (1977) give values of the wind adjustment coefficient (used again as a multiplier of E) as a function of day-time wind speed and the ratio of day-time to night-time speeds, ranging in value from 0.27 to 1.19.

In order to apply Penman's formula for tall crops also, it can be modified to include the resistance to transfer of vapour from within the substomatal cavities to the atmosphere (Thom and Oliver 1977). This modified formula is, however, not in widespread use. Penman's formula is known to underestimate evapotranspiration in arid regions. It is necessary to add an advective term to it, *e.g.* in the western United States (Chang 1968). Sometimes errors of upto 40% are reported (Doorenbos and Pruitt 1977) when daily mean values of the weather parameters are used. A great deal of work has been done on the effect of wind and the wind function (Stanhill 1962; Rijtema 1965; Wright and Jensen 1972; Doorenbos and Pruitt 1977; Cuenca and Nicholson 1982) to be used in (5). The form of $f(u)$ differs from one location to another and changes with the method used to calculate $(e_a - e_d)$ in (5), which is done in at least six

different ways (Cuenca and Nicholson 1982). Doorenbos and Pruitt (1977) have made a detailed analysis of this question and concluded that no single wind function valid for all climates and seasons can be found. They adopted the strategy of using a single linear wind function and applying corrections by means of their wind adjustment factor. An extreme view also exists, based on a regression analysis of lysimeter and weather data, that radiation and temperature account for the variation in potential evapotranspiration to the extent of 94%, wind, humidity etc. having no significant influence (Hargreaves 1983).

Despite all these uncertainties, Penman's formula is considered to be the best among all available formulae. But the fact that it needs data such as sunshine duration, wind and humidity, which are measured normally only at a relatively few important weather stations, has led to the development of several empirical formulae. Three well-known ones among such formulae are discussed in the following section.

(ii) *Empirical formulae: Thornthwaite's formula:* This formula relates the potential evapotranspiration to the mean air temperature. The underlying assumption, in common with many other empirical formulae, is that mean temperature is a measure of other weather parameters like radiation. It was developed by Thornthwaite (1948) from lysimeter and watershed water loss data in central and eastern USA, and reads:

$$E' = 1.6 (10 T_i / I)^a, \quad (7)$$

where E' is the total evapotranspiration in a standard month of 360 daylight hours, T_i the mean monthly temperature for the i th month in °C,

$$I = \sum_{i=1}^{12} [T_i / 5]^{1.514}, \quad (8)$$

$$\text{and } a = 0.000000675I^3 - 0.0000771I^2 + 0.01792I + 0.49239, \quad (9)$$

E for any given month is obtained from E' by correcting for the actual number of daylight hours of the month.

As with other similar formulae, the Thornthwaite formula works well in regions where temperature and radiation are strongly correlated. In other regions, its predictions are poor (Chang 1968). In central USA, where it was developed, it is found to be more accurate than even the Penman formula. Regions where temperature is not well correlated with radiation (Doorenbos and Pruitt 1977) are, *e.g.* (i) equatorial regions, (ii) small islands and coastal areas where temperature is influenced by the sea rather than by radiation changes, (iii) at high altitudes, where nights are cold despite high day-time radiation and (iv) regions with a wide variability of sunshine duration in some months, such as those with monsoon climates. In such regions, the Thornthwaite formula may be in error by as much as 100%. Unless verified by lysimeters or other more accurate methods, it is not recommended for use even as a guide to agricultural planning (Chang 1968).

The Blaney-Criddle formula: Based on measurements in western USA, Blaney and Criddle (1950) gave the following formula:

$$E = KT_i p, \quad (10)$$

where E is the monthly evapotranspiration in mm, K a crop coefficient, T_i the mean monthly air temperature in °C, and p the monthly percentage of day-time hours in the year.

Like the Thornthwaite formula, (10) also relates E to temperature, in addition to allowing for the day length. This formula also suffers from the same drawbacks as Thornthwaite's formula and is not recommended for use where temperature is poorly correlated to radiation (Doorenbos and Pruitt 1977). The variability of K with crop stage is a unique feature of (10) among all empirical formulae. Experiments have been conducted at Pantnagar in India (Dakshinamurthy *et al* 1973) to determine monthly K for wheat during the season December–April (outside the monsoon period). Fortnightly K values have been determined for a number of crops in Arizona, USA (Erie *et al* 1965). The formula gives accurate results for the semiarid lands of western USA. There is a very wide variability in the K values in the reported literature, even for places with similar T and p but with differing climates. This led Doorenbos and Pruitt (1977) to modify it in the form:

$$E = cp (0.46 T_i + 8), \quad (11)$$

where the crop coefficient K has been dropped, and an adjustment factor c is introduced, which depends on the minimum relative humidity, sunshine duration and day-time wind. They use another crop coefficient K_c to allow for the effect of the crop stage.

Makkink's formula: Makkink (1957) gave a formula relating E to incoming radiation R_s , weighted according to air temperature:

$$E = 0.61 \Delta R_s / (\Delta + \gamma) - 0.12, \quad (12)$$

Δ and γ have the same meanings as in (4), which (12) resembles. The formula underestimated E in Israel by a factor of 1.49, although the correlation was high (Chang 1968). The discrepancy is attributed to the fact that the formula was derived for a humid region, and did not allow for advection of energy which would occur in an arid region.

However, radiation being the primary source of energy for evaporation, Makkink's formula is inherently superior to formulae which depend primarily on temperature. Where measured values of R_s are not available, as is frequently the case, it can be calculated from observations of sunshine duration which are recorded at weather stations:

$$R_s = \left(a + b \frac{n}{N} \right) R_a, \quad (13)$$

where a and b are constants at a given place, n the actual sunshine hours, N the maximum possible sunshine hours, and R_a the Angot radiation. R_a is tabulated in meteorological tables for different latitudes. a and b have respectively the values 0.31 and 0.46 for New Delhi and 0.31 and 0.49 for Madras.

Doorenbos and Pruitt (1977) recommend a modified version of (12):

$$E = c \Delta R_s / (\Delta + \gamma), \quad (14)$$

where c is an adjustment factor which depends on mean humidity and day-time wind. To use (14), therefore, one needs the air temperature and either the measured R_s or sunshine duration, and an estimate of the general humidity and wind levels using which c can be read off from graphs.

1.1b *Actual evapotranspiration when moisture is not limiting* All the formulae discussed above give the potential evapotranspiration for grass, except the original

Blaney-Criddle formula which uses K values specific for crops. When applied for crops, the crop stage, which has a very great influence on the crop evapotranspiration, has to be considered. The model used in the present work (described later in greater detail) accomplishes this by dividing the evapotranspiration into bare soil evaporation (from the unshaded ground area) and transpiration, and estimating the ground cover on each day. Doorenbos and Pruitt (1977) have developed crop coefficients K_c (different from the Blaney-Criddle crop coefficient, though bearing the same name) to obtain E_{crop} , the crop evapotranspiration:

$$E_{\text{crop}} = K_c E. \quad (15)$$

Four crop stages are identified, namely initial, crop development, mid-season and late stages. K_c depends on the stage as well as the crop, and varies from 0.2 to 1.25, the lower values occurring at the initial and late stages.

1.1c Influence of soil moisture The influence of soil moisture on the ratio of the actual evapotranspiration (AET) to E_{crop} is complex, and depends on the soil characteristics, crop type, crop stage and the magnitude of the potential evapotranspiration (PET) itself. As the soil dries, its hydraulic conductivity decreases, and water moves more slowly towards the roots. When the moisture tension becomes too high, AET becomes less than PET. This break-off tension is higher for lower values of PET and vice-versa. It is high for some crops like horsegram (which is why they are "drought-resistant") and low for sugarcane. For a given crop it is higher during the initial and late stages than during the midseason stage.

It is common practice to specify the break-off point in terms of the depletion of the available soil moisture ASM (defined as the difference between field capacity and permanent wilting point). Some crops are assumed to suffer water stress when 25% of ASM is depleted, some others at 50% and the drought-resistant crops at 75% (Dakshinamurthy *et al* 1973). The British Meteorological Office incorporates the influence of soil moisture through the stomatal resistance term in the Penman equation itself (Thompson *et al* 1981). This also simultaneously takes care of the crop height and stage of growth, AET being thus calculated directly from the modified Penman formula. The first 40% of ASM is assumed to be available freely, and AET becomes less than E_{crop} only after this is exhausted. The soil moisture is apportioned conceptually into two reservoirs, the first one containing 40% of ASM and the second the rest. Water from the second reservoir is drawn only after the first becomes empty. Rainfall or irrigation fills the first reservoir to start with, and the excess after it becomes full goes to the second. Doorenbos and Kassam (1979) have attempted to take into account several parameters and have classified crops into four groups. For each group of crops, they specify values of the soil water depletion fraction p depending on the magnitude of E_{crop} . For a range of E_{crop} from 2 to 10 mm/day, p varies from 0.875 to 0.175 depending on the crop group. It is assumed that AET and E_{crop} are equal as long as

$$\text{CAM} \geq (1 - p) \text{ASM}, \quad (16)$$

where CAM is the current available moisture (defined as the difference between the actual soil moisture content and permanent wilting point). Once CAM becomes less than $(1 - p) \text{ASM}$, the following relation is assumed to hold, based on work by Rijtema and Abukhaled:

$$\text{AET} = \frac{\text{CAM}}{(1 - p) \text{ASM}} E_{\text{crop}}, \quad (17)$$

i.e. the ratio of AET to E_{crop} is proportional to the available water content in the root zone. In other words, AET varies linearly with the moisture content of the soil. By a simple integration, it can be shown that in the absence of replenishment of soil moisture through irrigation or rainfall, CAM will decrease exponentially with time t :

$$\text{CAM} = (1 - p) \text{ASM} \cdot \exp \left[-\frac{tE_{\text{crop}}}{(1 - p) \text{ASM}} \right]. \quad (18)$$

This approach is subject to the criticism that p is independent of the soil characteristics. The assumption of linear relationship between AET and soil moisture content should also be investigated further. The present work uses a model based on soil moisture tension as described later.

1.1d Water balance models A complete water balance model uses the above models to provide an output which can be used in irrigation scheduling, water budget calculation, planning of water resources projects, crop yield predictions, etc. The model used in the present work is described in the succeeding sections. In this section, two other recent models are summarised.

(i) **MORECS.** The British Meteorological office uses this model (*viz* The Meteorological Office Rainfall and Evaporation Calculation System) (Thompson *et al* 1981) to estimate evaporation and soil moisture deficit in Great Britain. The country is divided into square grids 40×40 km each. Weather parameters from stations within each grid, which are received at Bracknell around 9 A.M. each day, are averaged and used in a modified Penman equation to calculate PET for a range of vegetation covers from bare soil to forest. AET is then obtained considering the soil moisture for three magnitudes of ASM. Daily water balance is then calculated under the various types of cropped surfaces, and also under the average land use for each square, using the relative proportions of the various surfaces in each grid. Finally computer-drawn maps are produced showing the weekly grid averages of weather parameters, PET, AET, soil moisture deficit and the combined surface run-off and groundwater recharge. Printouts are also produced showing the same quantities for individual surface types. The output is distributed by post and to some extent *via* telex, facsimile and Prestel. MORECS outputs of soil moisture deficit were compared with neutron probe measurements at several sites and the agreement was found to be generally good.

(ii) **Water.** This model was developed by Burt *et al* (1981). It uses the Penman formula and generally follows the approach of Doorenbos & Pruitt (1977). Mean monthly values of weather parameters are used as input, and daily values are interpolated from them by fitting spline polynomials. Sowing dates and initial moisture, if not user-prescribed, are estimated based on air temperature and a 1.5 year water balance run respectively. E is then calculated each day by Penman's formula as modified by Doorenbos and Pruitt. The different growth stages are determined for each crop by the accumulated degree-day method, based on experimental results for the crop. Good agreement was found between predicted E_{crop} and lysimeter measurements.

1.2 Objective and approach of the present work

Reliable instruments are currently available for direct measurement of shortwave and longwave radiation at a given point in the upward as well as downward directions. A few meteorological stations in the world already have records of mean radiation over

short intervals of time for several years, along with other meteorological data. With such data, H can be calculated directly simply as a balance of the upward and downward radiations without having to use a Brunt-type formula. This gives great flexibility in choosing the time step for simulation.

The work reported here was undertaken to study the effect of the time step chosen in modelling the evapotranspiration of a crop through the growing season, on the results obtained. An evapotranspiration model was run using two time steps, viz 10 minutes and one day. The simulations extended over one calendar year, starting about five months before sowing and ending two months after harvest. Two runs were made with the one-day time step for reasons described later. No attempt has been made to examine the merits of the model itself, which was adapted from an existing one. No conclusions regarding the model itself are to be drawn, attention being confined to the effects of the time step used.

2. The data

Ten years of meteorological data recorded at the Nuclear Research Centre at Karlsruhe, West Germany, from 1973 to 1982 were available on magnetic tapes. The data were recorded every 10 minutes by instruments mounted at various heights on a tower, from midnight to the next 24 hr, from 1 January to 31 December of each year. Although many more items of data were available on the tape, only the following were used in the computations: (i) year, month, day, hour and minute to which the record corresponded; (ii) dew point at 2 m height above ground; (iii) air temperature 2 m above ground; (iv) rainfall; (v) 10-minute mean of the combined downward shortwave and longwave radiation, designated as $(S + L)_d$; and (vi) 10-minute mean of the combined upward shortwave and longwave radiation, designated as $(S + L)_u$. There were many gaps in the individual items of data, apparently because the instruments or recording equipment were out of order. The gaps extended over one or more days in certain years. In this respect, the year 1978 was the best, there being relatively fewer missing data. Accordingly the data for 1978 were selected for the computations. Figure 1 shows month-wise the percentage of the time the data on rainfall, temperature and either $(S + L)_d$ or $(S + L)_u$ or both were missing. Table 1 gives the individual dates on which one

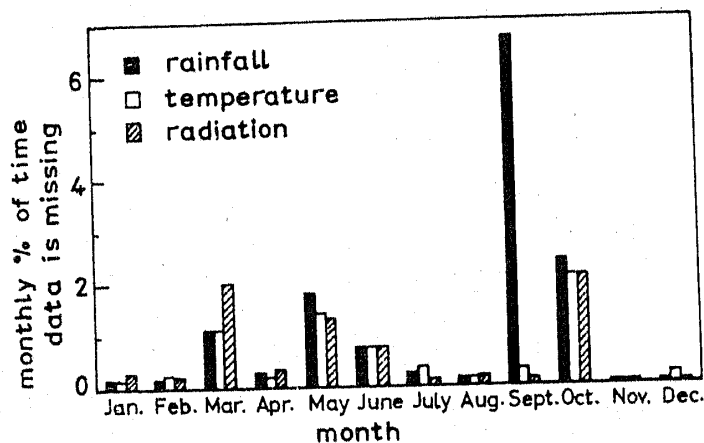


Figure 1. Monthwise extent of missing data

Table 1. Dates on which data are missing for 144 minutes or more

Date	Duration for which data are missing (minutes)
10 March	360
4 May	390
9 May	200
9 June	170
1 September	340
11 September	360
12 September	1090
28 September	210
29 September	500
3 October	240
4 October	170
11 October	650

or more items of data were missing for more than 10% of the time (*i.e.* 144 minutes). Taking the year as a whole, temperature data is missing 0.58% of the time, radiation 0.62% and rainfall 1.2%. These figures are very small. With the gaps in data filled in as described later (§ 3.2), it can be reasonably assumed that the data is complete.

2.1 Temperature and dew point

The air temperature as well as dew point differ by an order of magnitude from summer to winter, as is typical of high latitudes. Figure 2a shows the variation of temperature and dew point (both at 2 m height) on a winter day (1 January) and figure 2b, on a summer day (2 July), from 0 hr to 24 hr. The daily mean value of the air temperature, calculated by averaging all the 144 values of each day, varies through the year as shown in figure 3. The minimum value of the daily mean is -6°C (18 February and 7 December) and the maximum 23.5°C (29 July).

2.2 Radiation

Radiation was measured over successive periods of 10 minutes each and the mean value over each period was recorded, in $\text{mcal/cm}^2/\text{min}$. The radiation data consist of four values (although only two are used in the computations), *viz* downward shortwave and longwave combined $(S+L)_d$, downward shortwave (S_d) , upward shortwave and longwave combined $(S+L)_u$ and upward shortwave (S_u) . S_d is the incident solar radiation (direct as well as diffuse), and S_u is that part of S_d which is reflected by the surface. The ratio S_u/S_d thus gives the albedo of the surface (which is grass, under the tower), and $(S_d - S_u)$ is the absorbed shortwave radiation. Both S_u and S_d would be zero during the night.

The absorbed radiation heats the surface, which consequently emits longwave radiation. The atmosphere, which gets heated by convection, also emits longwave radiation, part of which is reflected by the surface and the rest absorbed. There is thus a longwave radiation exchange between the evaporating surface and the atmosphere which also plays a role in the heat budget. $(S+L)_d$ is the total radiation incident on the