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HEAT AND MASS TRANSFER IN UNSTEADY MAGNETO-HYDRODYNAMIC FLOW OVER A SEMI-INFINITE FLAT PLATE

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The unsteady laminar incompressible boundary-layer flow of an electrically conducting fluid past a semi-infinite flat plate with an aligned magnetic field has been studied when the free stream velocity and mass transfer vary with time. The effects of heat and mass transfer have been included in the analysis. Also it has been found that when the free stream velocity, mass transfer, square of magnetic field and square root of wall temperature vary inversely as a linear function of time, the foregoing problem admits a self-similar solution. In both the cases, the governing equations have been solved numerically using an implicit finite-difference scheme in combination with the quasilinearization technique. Both the skin friction and heat transfer are found to be strongly dependent on the free stream velocity, magnetic field and mass transfer. On the other hand, only the heat transfer is (strongly) affected by the dissipation parameter and Prandtl number.

1. INTRODUCTION

The steady laminar incompressible flow of a viscous electrically conducting fluid with constant properties past a semi-infinite flat plate with aligned magnetic field without heat transfer has been studied by several authors (Greenspan and Carrier 1959, Glauert 1961, Meksyn 1962, Davies 1963, Gribben 1965, Na 1970). The heat-transfer problem has been considered by Tan and Wang (1968) and Afzal (1972). All the above studies were confined to steady flows. It is well known that almost all the flow problems encountered in practice are unsteady. The velocity field for the unsteady boundary layer without mass transfer has been considered by Das (1970) where the unsteadiness has been introduced due to the impulsive change in the free stream velocity. Solutions for large and small times have been obtained using series expansion method.

The aim of the present study is to consider the unsteady problem with heat and mass transfer when the free stream velocity and mass transfer vary arbitrarily with time. Also, it has been found that when the free stream velocity, mass transfer, square of the magnetic field and square root of the wall temperature vary inversely as a linear function of time, the self-similar solution to the foregoing problem exists. In both the cases, the governing equations (semi-similar and self-similar equations) have been solved numerically using an implicit finite-difference scheme in combination with quasilinearization technique. The results have been compared with those available in the literature (Glauert 1961, Tan and Wang 1968, Na 1970, Das 1970).

2. Governing Equations

We consider the unsteady laminar incompressible viscous electrically conducting fluid of constant properties past a fixed semi-infinite unmagnetized and nonconducting plate. The applied magnetic field H_0 and the fluid flow velocity u_e are parallel to the plate. The velocity at the edge of the boundary layer u_e varies arbitrarily with time, but the applied magnetic field H_0 is constant. We assume that both the viscous and magnetic Reynolds numbers are sufficiently large for momentum and magnetic boundary layers to have developed. The Hall effect has been neglected. Under the aforementioned assumptions, the boundary layer equations for the velocity, magnetic and temperature fields in the absence of electric field are expressed as (Greenspan and Carrier 1959, Glauert 1961, Gribben 1965, Tan and Wang 1968, Das 1970)

$$u_x + v_y = 0 \qquad \text{...(1a)}$$

$$(H_1)_x + (H_2)_y = 0 \qquad \text{...(1b)}$$

$$u_t + uu_x + vu_y = (u_e)_t + \nu u_{yy} + (\mu_0/\rho)\,[H_1\,(H_1)_x + H_2\,(H_1)_y] \qquad \text{...(1c)}$$

$$(H_1)_t + u\,(H_1)_x + v\,(H_1)_y - H_1\,u_x - H_2\,u_y = \alpha_1\,(H_1)_{yy} \qquad \text{...(1d)}$$

$$T_t + uT_x + vT_y = \nu Pr^{-1}\,T_{yy} + (\nu/c_p)\,u_y^2 + (\rho\,c_p\,\sigma)^{-1}\,[(H_1)_y]^2. \qquad \text{...(1e)}$$

The boundary conditions are given by

$$\left.\begin{aligned} &\text{At } \; y = 0 : u = 0,\, v = v_w,\, (H_1)_y = H_2 = 0 \\ &\text{As } \; y \to \infty : u \to u_e\,(t),\, H_1 \to H_0 \end{aligned}\right\}\; t \geqslant 0. \qquad \text{...(2)}$$

The initial conditions are given by the corresponding steady-state equations obtained by putting $t = u_t = (u_e)_t = (H_1)_t = T_t = 0$ in the set of equations (1).

Here x and y are, respectively, the distances along and perpendicular to the plate; u and v are the velocity components along x and y directions, respectively; t is the time; H_1 and H_2 are the components of the induced magnetic field in the x and y directions, respectively; T is the temperature; ρ, ν and σ are, respectively, the density, kinematic viscosity and electrical conductivity; c_p and Pr are, respectively, the specific heat at a constant pressure and Prandtl number; μ_0 is the magnetic permeability; α_1 is a dimensional parameter associated with the magnetic Prandtl number; the subscripts t, x and y denote differentiation with respect to t, x and y, respectively; and the subscripts e and w denote conditions at the edge of the boundary layer and on the surface, respectively.

Applying the following transformations

$$\left.\begin{array}{l} \eta = (u_s/2\nu x)^{1/2}\, y,\ \psi = (2u_s\, \nu\, x)^{1/2}\, \varphi\,(t^*) f\,(\eta, t^*) \\ t^* = (u_s/2x)\, t,\ A = (2\nu\, x/u_s)^{1/2}\, H_0\, g\,(\eta, t^*) \\ u = \psi_y,\ v = -\psi_x,\ H_1 = A_y,\ H_2 = -A_x \end{array}\right\} \quad \text{...(3a)}$$

$$\left.\begin{array}{l} G\,(\eta, t^*) = (T-T_\infty)/(T_w-T_\infty),\ u/u_e = f',\ u_e = u_s\, \varphi\,(t^*) \\ v = -\varphi\,(t^*)\,(u_s\, \nu/2x)^{1/2}\,(f-\eta f'),\ H_1 = H_0\, g' \\ H_2 = -(\nu/2xu_s)^{1/2}\, H_0\,(g-\eta g') \end{array}\right\} \quad \text{...(3b)}$$

$$\left.\begin{array}{l} f_w = A_1/\varphi\,(t^*),\ A_1 = -(2\,\mathrm{Re}_x)^{1/2}\,(v_w/u_\infty) \\ \mathrm{Re}_x = u_s\, x/\nu \end{array}\right\} \quad \text{...(3c)}$$

to the set of equations (1), we find that (1a) and (1b) are identically satisfied and (1c) to (1e) reduce to

$$f''' + \varphi f f'' + \varphi^{-1}\varphi_{t^*}(1-f') - \beta\varphi^{-1} g g'' - f'_{t^*} = 0 \quad \text{...(4a)}$$

$$\alpha g''' + \varphi\,(f g'' - f'' g) - g'_{t^*} = 0 \quad \text{...(4b)}$$

$$Pr^{-1}\, G'' + \varphi f G' + Br\,[\varphi^2 f''^2 + \alpha\beta\,(g'')^2] - G_{t^*} = 0 \quad \text{...(4c)}$$

where

$$\left.\begin{array}{l} \alpha = \alpha_1/\nu,\ \alpha_1 = 1/\mu_0\,\sigma,\ \beta = \mu_0\, H_0^2/\rho\, u_s^2 \\ Br = u_s^2\,/\,[c_p\,(T_w - T_\infty)]. \end{array}\right\} \quad \text{...(5)}$$

The boundary conditions can be written as

$$\left.\begin{array}{l} f = f_w, f' = g = g'' = 0,\ G = 1 \text{ at } \eta = 0 \\ f' = g' = 1,\ G = 0 \text{ as } \eta \to \infty \end{array}\right\} \text{for } t^* \geqslant 0 \quad \text{...(6)}$$

The initial conditions are the steady-state equations (i.e. at $t^* = 0$) obtained by putting $\varphi = 1$, $\varphi_{t^*} = f'_{t^*} = g'_{t^*} = G_{t^*} = 0$ in (4a) to (4c) and they are

$$f''' + f f'' - \beta g g'' = 0 \quad \text{...(7a)}$$

$$\alpha g''' + (f g'' - f'' g) = 0 \quad \text{...(7b)}$$

$$Pr^{-1}\, G'' + f G' + Br\,(f''^2 + \alpha\beta g''^2) = 0. \quad \text{...(7c)}$$

Here η is the dimensionless variable; ψ and f are the dimensional and dimensionless stream functions, respectively; t^* is the dimensionless time; A and g are the dimensional and dimensionless magnetic stream functions, respectively; G is the dimensionless temperature; f' and g' are the dimensionless velocity and magnetic field in the x direction, respectively; Re_x is the local Reynolds number; α and β are, respectively, the reciprocal of the magnetic Prandtl number and magnetic force number; Br is the Brinkman number (dissipation parameter); and the subscripts s and ∞ denote, respectively, steady-state case and free stream conditions. f_w is the surface mass transfer parameter and A_1 is a constant associated with the mass transfer parameter on the surface provided the normal velocity at the wall v_w/u_∞ and the local Reynolds number Re_x are assumed to be constant. Also $A_1 \gtrless 0$ according to whether there is suction or injection. φ is an arbitrary function of t^* having first continuous derivative for $t^* \geqslant 0$.

Here, we have taken the fluid as finitely conducting and the plate as non-conducting. Therefore, there will be no surface current sheet or equivalently, the tangential component of the magnetic field (H_1) is continuous across the interface. The condition is expressed by $(H_1)_y = 0$ when $y = 0$ (i.e. in dimensionless form $g'' = 0$ when $\eta = 0$). Most of the investigators have used the condition $H_1(0) = 0$ (i.e. $g'(0) = 0$) which has been found to be not correct by Gribben (1965) for the aforementioned reason.

The skin-friction and heat-transfer coefficients are given by

$$\left.\begin{aligned} C_f &= 2\tau_w/\rho\, u_s^2 = 2^{\frac{1}{2}}\, Re_x^{-\frac{1}{2}}\, \varphi(t^*)\, f_w'',\ \tau_w=\mu\,(\partial u/\partial y)_w \\ N_u &= -\,x\,(\partial T/\partial y)_w/(T_w-T_\infty) = -\,2^{-\frac{1}{2}}\, Re_x^{\frac{1}{2}}\, G_w' \end{aligned}\right\} \quad \ldots(8)$$

where C_f and Nu are the skin-friction and Nusselt number, respectively; τ_w is the shear stress at the wall f_w'' and G_w' are the skin-friction and heat-transfer parameters, respectively and μ is the coefficient of viscosity.

It may be noted that the steady-state equations (7a) to (7c) with boundary conditions (6) are the same as those of Tan and Wang (1968) and Na (1970) if we put $\eta = 2^{\frac{1}{2}}\bar{\eta}$, $f = \bar{f}/2^{\frac{1}{2}}$, $g = \bar{g}/2^{\frac{1}{2}}$, $G = 2\bar{G}$, $Br = 8\overline{Br}$ in (6) and (7) except the boundary condition $g''(0) = 0$ in (6) which should be replaced by $g'(0) = 0$. Furthermore, both Tan and Wang (1968) and Na (1970) have used the integral form of (7b) which is $\alpha g'' + (fg' - f'g) = 0$. Also, equations (7a) and (7b) reduce to those of Gribben (1965) and Das (1970) if we put $\eta = 2^{-\frac{1}{2}}\bar{\eta}$, $f = 2^{-\frac{1}{2}}\bar{f}$, $g = 2^{-\frac{1}{2}}\bar{g}$.

It may be remarked that for $\beta = 0$ (which implies the absence of magnetic field), eqns. (4a) and (4c) reduce to the classical Blasius equations for the unsteady hydrodynamic flow. In such a situation (4b) representing the equations of the magnetic field is no longer needed. Furthermore, when $\alpha \rightarrow \infty$ (i.e. for zero electrical conductivity) equation (4b) reduces to $g'''(\eta, t^*) = 0$ which on integration and using the condition $g''(0, t^*) = 0$. Consequently, eqns. (4a) and (4c) reduce to those of classical unsteady hydrodynamic flow problem over a flat plate. Similar arguments hold good for steady-state eqns. (7a) to (7c) which has been advanced by Tan and Wang (1968). As in steady case, for $\alpha = 0$ (i.e. infinite electrical conductivity) no interaction between magnetic field and flow field takes place (Tan and Wang 1968). Also, it has been shown that $\beta < 1$ $(0 \leqslant \beta < 1)$. Furthermore, for steady-state equations governed by equations (7a)-(7c), Wilson (1964) and Stewartson and Wilson (1964) have shown that the numerical procedure yields two solutions for $0 < \beta < 1$ and $\alpha > 1$. The lower branch solution is found to be of no physical significance. Therefore, we have considered here only the upper branch solutions.

It has been found that when the free stream velocity, square of the free stream magnetic field and mass transfer and square-root of wall temperature vary inversely as a linear function of time, the set of partial differential equations with three independent variables admits a similarity solution i.e., it reduces to a system of ordinary differential equations. Applying the following transformation.

$$\left.\begin{aligned} &\eta = (u_s/2\nu x)^{\frac{1}{2}} (1-\lambda t^*)^{-\frac{1}{2}}\, y,\; t^* = (u_s/2x)\, t \\ &u_e = u_s (1-\lambda t^*)^{-1},\; \psi = (2u_s \nu x)^{\frac{1}{2}} (1-\lambda t^*)^{-\frac{1}{2}} f(\eta) \end{aligned}\right\} \quad \ldots(9a)$$

$$\left.\begin{aligned} &A = (2\nu x/u_s)^{\frac{1}{2}} H_0\, g(\eta),\; H_0 = H_s (1-\lambda t^*)^{-\frac{1}{2}} \\ &u = \psi_y,\; v = -\psi_x,\; H_1 = A_y,\; H_2 = -A_x \end{aligned}\right\} \quad \ldots(9b)$$

$$\left.\begin{aligned} &u = u_s (1-\lambda t^*)^{-1} f'(\eta),\; -v = (2x)^{-1} (2u_s \nu x)^{\frac{1}{2}} (1-\lambda t^*)^{-\frac{1}{2}} (f-\eta f') \\ &G(\eta) = (T - T_\infty)/(T_w - T_\infty),\; T_w - T_\infty = (T_{w0} - T_\infty)(1-\lambda t^*)^{-2} \end{aligned}\right\} \quad \ldots(9c)$$

$$\left.\begin{aligned} &H_1 = H_s (1-\lambda t^*)^{-1} g'(\eta) \\ &H_2 = -(2\nu x/u_s)^{\frac{1}{2}} (2x)^{-1} (1-\lambda t^*)^{-\frac{1}{2}} H_s (g - \eta g') \end{aligned}\right\} \quad \ldots(9d)$$

to eqns. (1a) to (1e), we find that (1a) and (1b) are identically satisfied and (1c) to (1e) reduce to

$$f''' + ff'' - \beta g g'' + \lambda (1 - f' - \eta f''/2) = 0 \quad \ldots(10a)$$

$$\alpha g''' + (f g'' - f'' g) - \lambda (g + \eta g''/2) = 0 \quad \ldots(10b)$$

$$Pr^{-1} G'' + f G' - \lambda (G + \eta G'/2) + Br\,(f''^2 + \alpha \beta g''^2) = 0. \quad \ldots(10c)$$

The relevant boundary conditions are given by

$$\left.\begin{aligned} &f = f_w,\; f' = g = g'' = 0,\; G = 1 \quad \text{at } \eta = 0 \\ &f' = g' = 1,\; G = 0 \quad \text{as } \eta \rightarrow \infty \end{aligned}\right\} \quad \ldots(11)$$

where

$$\left.\begin{aligned} \beta &= \mu_0 H_s^2 / \rho u_s^2, \; Br = u_s^2 / [c_p (T_{w0} - T_\infty)] \\ f_w &= -(v_w/u_\infty)(2\text{Re}_x)^{\frac{1}{2}} (1-\lambda t^*)^{\frac{1}{2}} = A_1 \end{aligned}\right\} \qquad \ldots(12)$$

The mass transfer parameter f_w will be a constant (say A_1) if v_w/u_∞ varies as $(1-\lambda t^*)^{-\frac{1}{2}}$ and Re_x is taken as a constant.

The expressions for skin-friction and heat-transfer coefficients for this case are given by

$$\left.\begin{aligned} C_f &= 2\mu (\partial u/\partial y)_w / \rho u_e^2 = 2^{\frac{1}{2}} (\overline{\text{Re}_x})^{-\frac{1}{2}} f''_w \\ N_u &= x(\partial T/\partial y)_w / (T_w - T_\infty) = 2^{-\frac{1}{2}} (\overline{\text{Re}_x})^{\frac{1}{2}} G'_w \end{aligned}\right\} \qquad \ldots(13)$$

where

$$\overline{\text{Re}_x} = u_e x/\nu. \qquad \ldots(14)$$

It may be noted that for $\lambda = 0$, eqns. (10a) to (10c) reduce to those of steady-state case (Tan and Wang 1968). Also $\lambda \gtrless 0$ according to whether the flow is accelerating or decelerating.

3. Results and Discussion

Equations (4a) to (4c) have been solved numerically under boundary conditions (6) and initial conditions (7) using an implicit finite-difference scheme in combination with the quasilinearization technique. Since the detailed description of the method is given by Inouye and Tate (1974), it is not presented here. In order that the solution of the difference equations converges to the true solution, the effect of step sizes $\Delta\eta$ and Δt^* on the solution has been studied and their optimum values have been obtained. Consequently, we have taken $\Delta\eta = 0.05$ and $\Delta t^* = 0.1$ for computation. Also, we have taken the values of the edge of the boundary layer (η_∞) between 6 and 12 depending on the values of α, β and Pr. The results presented here are independent of step sizes and η_∞ at least up to 4th decimal place. For computation, the free stream velocity distribution has been taken to be of the form $\varphi(t^*) = 1+\epsilon t^{*2}$ and $\varphi(t^*) = (1+\epsilon \cos \omega^* t^*)/(1+\epsilon)$. A typical case takes 16.1 second CPU time on DEC-1090 computer. Here ϵ and ω^* (frequency parameter) are constants.

In order to assess the accuracy of our method, we have compared the skin friction and the x component of the induced magnetic field on the surface (f''_w, g'_w) for steady-state case with those of Glauert (1961), Na (1970) and Das (1970). Also, we have compared our heat-transfer result (G'_w) with those of Tan and Wang (1968).

TABLE I

Comparison of skin-friction parameter and x component of the induced magnetic field f''_w and g'_w for $t^* = A_1 = 0$, $\alpha = 0.1$

β	f''_w Present Calculation	f''_w Na	f''_w Glauert	g'_w Present Calculation	g'_w Na	g'_w Glauert
0.0060	1.3239	1.3239	1.3238	0.4903	0.4903	0.4915
0.0181	1.3149	1.3149	1.3151	0.4882	0.4882	0.4892
0.0602	1.2829	1.2829	1.2839	0.4811	0.4810	0.4808
0.1792	1.1891	1.1892	1.1918	0.4593	0.4592	0.4557
0.2960	1.0910	1.0911	1.0946	0.4358	0.4356	0.4284
0.3535	1.0404	1.0402	1.0438	0.4231	0.4229	0.4139
0.4103	0.9883	0.9880	0.9915	0.4099	0.4097	0.3986
0.4659	0.9343	0.9341	0.9379	0.3963	0.3960	0.3828

TABLE II

Comparison of skin-friction parameter and x component of the induced magnetic field f''_w and g'_w for $t^* = A_1 = 0$, $\beta = 0.3$

α	f''_w Present Calculation	f''_w Das	g'_w Present Calculation	g'_w Das
0.05	0.2459	0.2458	0.0191	0.0192
0.10	0.2432	0.2431	0.0406	0.0406
0.30	0.2374	0.2375	0.1645	0.1644
0.40	0.2402	0.2400	0.2903	0.2901
1.0	0.2931	0.2929	0.8611	0.8609
10.0	0.3078	0.3077	0.9889	0.9886
25.0	0.3087	0.3085	0.9958	0.9955
100.0	0.3089	0.3088	0.9991	0.9989
400.0	0.3090	0.3089	0.9999	0.9997

They are all found to be in excellent agreement. The comparison is shown in Tables I–II and Fig. 1. It may be noted that Das (1970) has studied the unsteady flow due to impulsive change of free stream velocity from u_0 to $u_0\,(1+\epsilon)$. Here free stream velocity has been assumed to vary continuously with time. Hence, we have not compared our unsteady results with his.

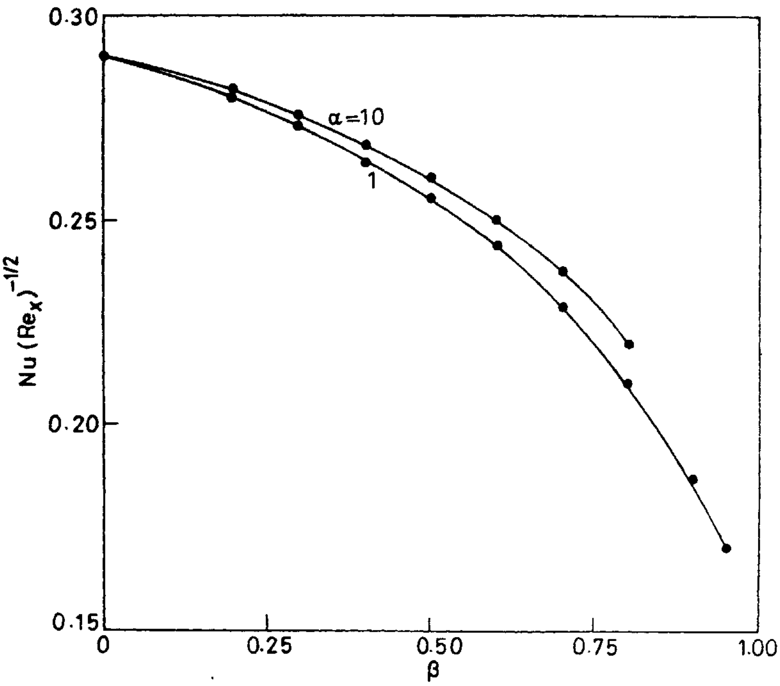

FIG. 1. Comparison of heat-transfer results ($Nu(Re_x)^{-\frac{1}{2}}$) for steadystate case ($t^*=0$) with those of Tan and Wang for $Pr=0.7$, $A_1=Br=0$.———, present calculation; 0, Tan and Wang.

Since the effects of the reciprocal of magnetic Prandtl number (α), magnetic force number β and dissipation parameter (Br) on the velocity, temperature and magnetic fields for the steady-state case have been throughly studied elsewhere (Glauert 1961, Gribben 1965, Tan and Wang 1968, Na 1970), here we present results mostly to show the effect of unsteadiness (i.e. time t^*) on them. The results for the accelerating free stream velocity ($\varphi\,(t^*) = 1+\epsilon\, t^{*2}$, $\epsilon = 0.25$) are given in Figs. 2-5. Also the case of oscillatory free stream velocity is studied but the results are not presented here as they merely respond to the free stream velocity almost in the same manner.

The variation of the skin-friction parameter (f''_w), the x component of the induced magnetic field at the wall (g'_w) and the heat transfer parameter ($-G'_w$)with time t^* for some representative values of the reciprocal of magnetic Prandtl number (α), magnetic force number (β) and mass transfer parameter (A_1) has been shown in Figs. 2 and 3. It is found that for a given α, β and A_1, f''_w, g'_w and $-G'_w$ increase with time t^*, because the free stream velocity is assumed to be increasing with time (i.e. $\varphi\,(t^*) = 1 + \epsilon\, t^{*2}$, $\epsilon > 0$) which in turn reduces the thicknesses of

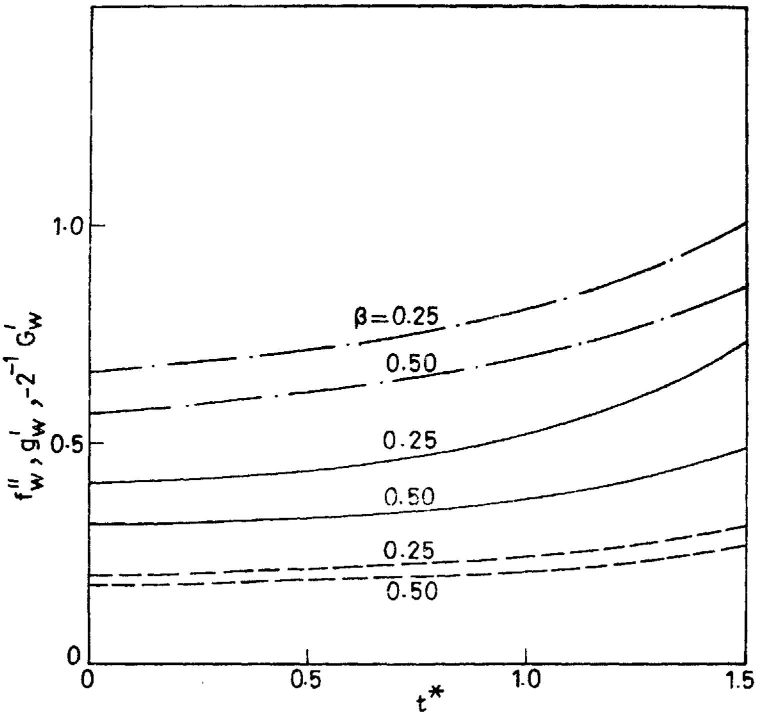

FIG. 2. Effect of the magnetic force number (β) on the skin friction, x component of the induced magnetic field and heat transfer (f_w'', g_w', $-2^{-\frac{1}{2}} G_w'$) for $\alpha = 10$, Pr $= 0.7$, Br $= A_1 = 0$. ———, f_w''; ——.——, g_w'; – – – –, $-2^{-\frac{1}{2}} G_w'$.

the momentum, magnetic and thermal boundary layers. Hence f_w'', g_w' and $-G_w'$ are increased due to increase in time t^*. If the free stream velocity is considered as decreasing with time t^*, the opposite effect is observed. However it is observed that the skin friction, x component of the induced magnetic field and heat transfer at the wall (f_w'', g_w', $-G_w'$) change very little with time t^* when t^* is small, because the steady-state skin friction, heat transfer and x component of the induced magnetic field at the wall dominate initially and restrict the penetration of unsteadiness to a thin boundary layer. As time progresses they get modified due to the interaction of steady flow and superimposed unsteady flow. However, the layer of fluid affected by the un-steady flow will remain very thin for small times. The change becomes more pronounced for large times. For all values of t^*, the parameters α and β produce opposite effects on f_w'', g_w' and $-G_w'$ i.e. f_w'', g_w' and $-G_w'$ decrease as β increases, but increases with α. This is due to the increase in momentum, magnetic and thermal

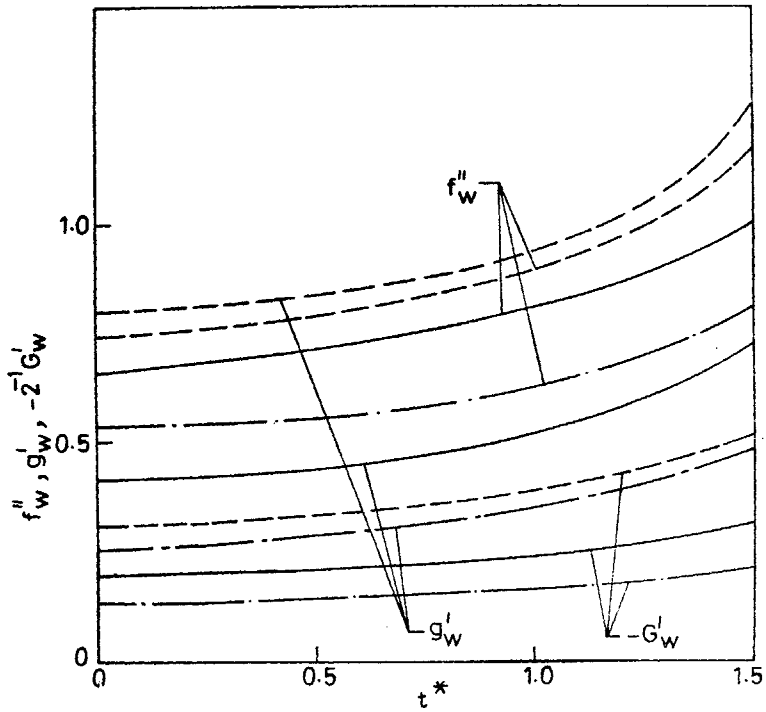

FIG. 3. Effect of the mass transfer parameter (A_1) on the skin friction, x component of the induced magnetic field and heat transfer (f''_w, g'_w, $-2^{-\frac{1}{2}} G'_w$) for $\alpha=10$, $\beta=0.25$, Pr$=0.7$, Br$=0$.———, $A_1=0$; ——.——, $A_1=-0.5$; ———, $A_1=0.5$.

boundary layer thicknessess as β increases and the effcct of α is just opposite. However, f''_w first decreases with α in the range $(0 < \alpha \leqslant 0.3)$ and then increases. The reason for such a behaviour can be explained as follows (Das 1970). Small values of α correspond to large electrical conductivity (σ) so that the magnetic lines of forces are more or less frozen into the fluid for small α. As α increases, the electrical conductivity (σ) is reduced which results in reduction in the y component of the induced magnetic field (H_2) and hence in pondermotive force which resists the motion parallel to the plate. This tends to increase the skin friction (f''_w). Also by increasing α, the magnetic diffusivity (α_1) becomes larger resulting in greater x component of the induced magnetic field (H_1 or g') near the wall. Since the induced y component of the magnetic field (H_2) is proportional to the x component of the induced magnetic field (H_1), the former also increases with α. This raises more resistance to fluid motion and reduces the skin friction at the wall (f''_w). It has been found that for small α the second effect dominates the first effect and for large α, it is the other way around. The above explanation holds good for both steady and unsteady flows. Similar effect has been observed by Glauert (1961), Na (1970) and Das (1970) for steady case. Here the effect of α is not shown in figures as it is qualitatively similar to the steady case which is given in Table II. As expected the suction ($A_1 > 0$) increases f''_w, g'_w and $-G'_w$ and the effect of injection ($A_1 < 0$) is just opposite (see Fig 3). The reason for such a behaviour is the reduction in thickness of momentum, magnetic and thermal boundary layers due to suction which results in higher skin friction, magnetic field and heat transfer. The effect of injection is just the reverse.

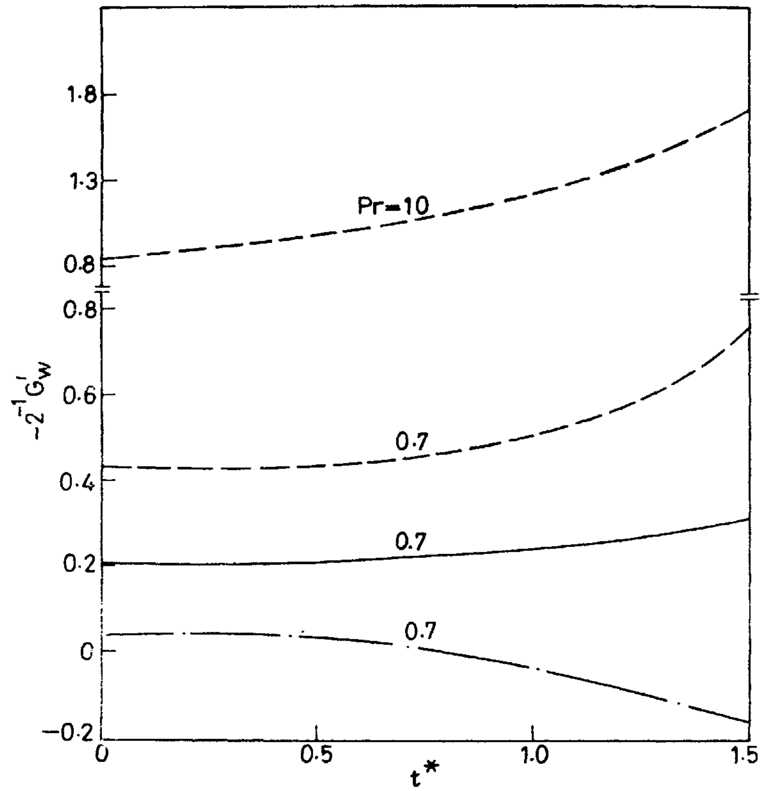

FIG. 4. Effect of dissipation parameter (Br) and Prandtl number (Pr) on the heat transfer ($-2^{-\frac{1}{2}} G'_w$) for $\alpha=10$, $\beta=0.25$, $A_1=0$. ———, Br=0; — — — -. ———, Br=0.5; — — — —, Br=0.5.

The effect of the Prandtl number (Pr) and dissipation parameter (Br) on the heat transfer ($-G'_w$) is shown in Fig 4. The skin friction (f''_w) and the x component of the induced magnetic field (g'_w) are unaffected by Pr and Br. It is observed that the heat transfer is increased due to the increase in the Prandtl number because thermal boundary layer thickness decreases with Pr which result in higher temperature gradient at the wall and hence higher heat transfer. The effect of dissipation parameter (Br) is seen to have strong influence on the heat transfer when t^* is large. It is found that for $\mathrm{Br} > 0$ ($T_w > T_\infty$), the heat transfer $(-G'_w)$ first slightly increases with t^* and then decreases to negative values at $t^* > t_1^*$. This implies that after a certain instant of time, the fluid near the wall gets heated due to viscous dissipation and its temperature becomes more than that of the wall. Therefore the wall gets heated instead of being cooled. When $\mathrm{Br} < 0$ ($T_w < T_\infty$), the heat transfer $(-G'_w)$ increases with t^* which implies that heat is transferred from the fluid to the wall for all values of time t^*. When $\mathrm{Br} = 0$, the heat transfer $(-G'_w)$ increases slowly with t^* in contrast to that for $\mathrm{Br} < 0$.

The velocity profile (f'), magnetic profile (g) and temperature profile (G) are shown in Fig. 5. We find that they become more steep as time progresses. This

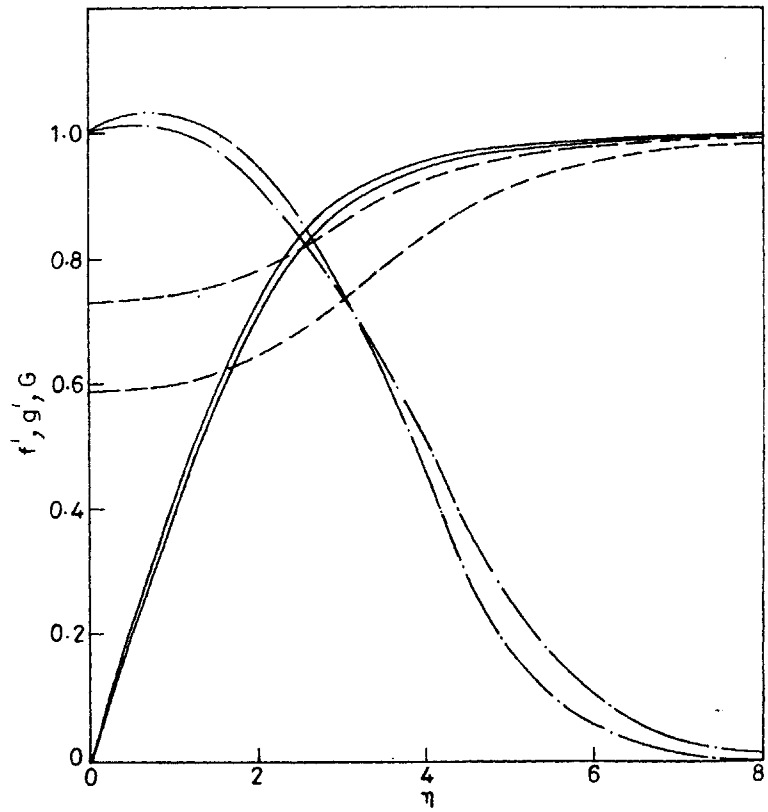

FIG. 5. Velocity, x component of the induced magnetic field and temperature profiles (f', g', G) for $\alpha=5$, $\beta=0.25$, $t^*=1.0$, $\mathrm{Pr}=0.7$, $\mathrm{Br}=-0.5$, $A_1=0$. ———, f'; – – – –, g'; ——·——, G.

steepness is due to the reduction in the thicknesses of the momentum, magnetic and thermal boundary layers.

The effect of oscillatory free stream velocity ($\varphi(t^*) = (1 + \epsilon \cos \omega^* t^*)/(1 + \epsilon_1)$) on the skin friction, x component of the induced magnetic field and heat transfer (f''_w, g'_w, $-G'_w$) has been studied but not presented here for the sake of brevity. It is observed that F'_w, g'_w, $-G'_w$ respond significantly to the free stream velocity i.e. they oscillate with t^*.

The results for self-similar case are given in Fig. 6. It is found that the skin friction, x component of the magnetic field and heat transfer (f''_w, g'_w, $-G'_w$) are more for accelerating flow ($\lambda > 0$) than for decelerating flow ($\lambda < 0$). This is

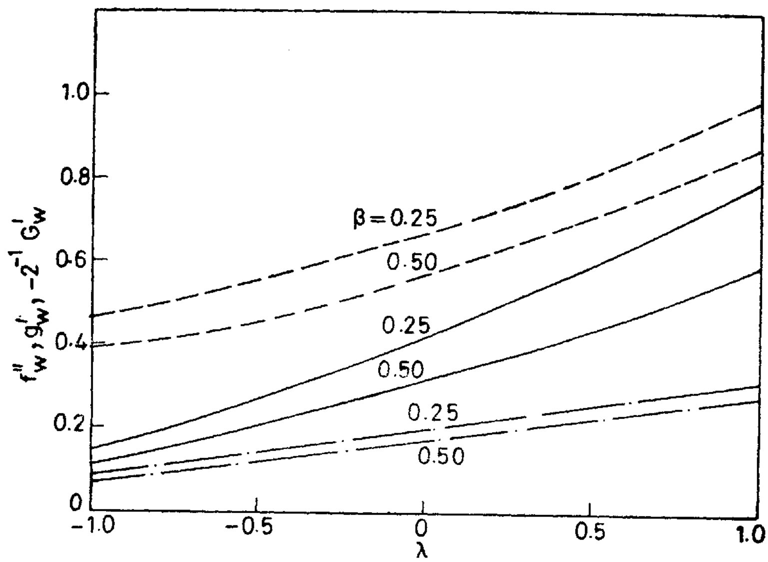

FIG. 6. Effect of the magnetic force number (β) on the skin friction, x component of the induced magnetic field and heat transfer (f''_w, g'_w, $-2^{-\frac{1}{2}} G'_w$) for $\alpha = 10$, Pr $= 0.7$, $A_1 =$ Br $= 0$. ————, f''_w; – – – –, g'_w; ——·——, $-2^{-\frac{1}{2}} G'_w$.

because the momentum, magnetic and thermal boundary layer thicknesses are thinner for $\lambda > 0$ and thicker for $\lambda < 0$.

4. CONCLUSIONS

The skin friction, x component of the induced magnetic field and heat transfer are affected by tne free stream velocity, mass transfer, reciprocal of magnetic Prandtl number, and magnetic force number. On the other hand, the Prandtl number and dissipation parameter affect only the heat transfer. After certain instant of time, the direction of the heat transfer is found to change due to viscous dissipation. The reciprocal of the magnetic Prandtl number, and magnetic force number produce opposite effect on the skin friction, x component of the induced magnetic field and heat transfer for all times.

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