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**RESPONSE OF A TWO PHASE SYSTEM SUBJECT TO
OSCILLATIONS INDUCED BY THE MOTION OF ITS SUPPORT STRUCTURE**

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ABSTRACT

In many parts of the world, seawater desalination offers one of the most promising alternatives for supplying the required potable water. The desalination is an energy intensive process and use of nuclear reactors as the energy source for desalination is an attractive option. To ensure reactor safety for a barge-mounted plant, multiple systems for removing heat from the core even after the nuclear reactor is shutdown, need to be provided. One of the Residual Heat Removal Systems (RHRS) has to be energy independent and hence has to be based on the principles of natural circulation. A three-loop energy independent RHRS, in which the second loop is working in two-phase, is described. The barge will be invariably subjected to oscillations due to seawater movements. The RHRS should be able to perform its intended functions even when the barge is oscillating. If the natural frequency of oscillations of the RHRS is close to the frequency of oscillations of the barge, resonance may occur and lead to deterioration of heat removal capability. Hence, it is necessary to determine the natural frequency of oscillation of such a system. Temperature and quality distribution of the working fluid required for the estimation of natural frequency of the system is calculated by solving momentum balance and energy balance equations for the loops of the RHRS. Various terms of the governing equation describing oscillatory behaviour of non-isothermal, two-phase closed loop system are derived. The overall equation is solved using fourth order Runge-Kutta method. The response of the system for various initial conditions is computed. © 2002 Elsevier Science Ltd

Introduction

It has been estimated that on a global basis, about 12% of the world population lacks safe water supplies and 56% has no adequate sanitation. Matching the ever-growing demands with the limited

supply of finite resources has led to tremendous stress on natural fresh water. In many parts of the world, seawater desalination offers one of the most promising alternatives for supplying the required potable water.

The desalination is an energy intensive process and use of nuclear reactors as the energy source for desalination is an attractive option. A barge-mounted nuclear reactor desalination plant can provide drinking water in many coastal regions of our country. A number of Nuclear Desalination plants are operating worldwide and one land based plant is being built at Kalpakkam in India. For barge mounted desalination plants, various reactor system concepts have been examined. Pressurised Water Reactors of small rating are found to be the best choice for a barge-mounted reactor, which is coupled to a desalination plant [1]. A barge mounted desalination plant is likely to be an islanded system. Therefore, to ensure reactor safety [2], it is necessary to provide passive systems for removing residual heat from the core, which is generated after the nuclear reactor is shutdown. A possible Residual Heat Removal System (RHRS) based on three loops in natural circulation; in which one of the loops is working in two-phase is shown in Fig. 1 [3]. The first loop of the RHRS circulates reactor coolant between the core and the heat exchanger, which is located at a higher elevation than the core. In the second loop, the working fluid is converted to steam in the heat exchanger and rises up. The steam is condensed in a condenser, which is at

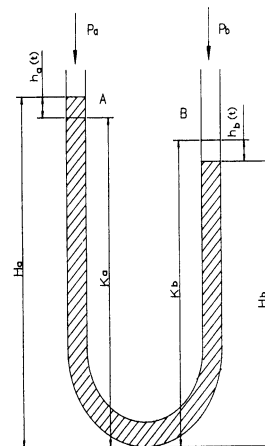
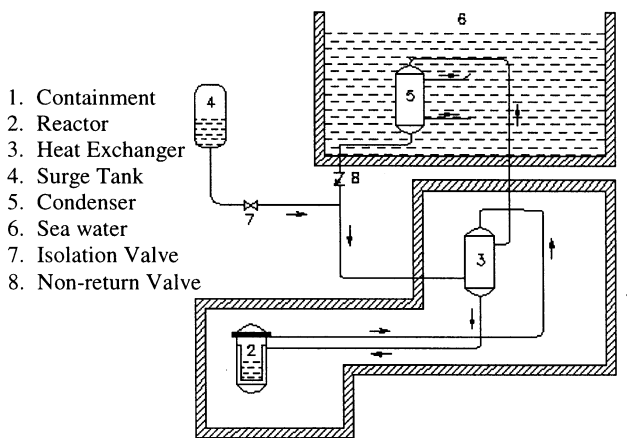


FIG. 1

Schematic of Natural Circulation Residual Heat Removal System

FIG. 2

Non-isothermal Open Loop System

a higher elevation and the condensate flows back to the heat exchanger. The third loop is a seawater loop. The cold seawater enters the shell side through the lower nozzle of the condenser, takes away heat from the condensing steam and flows out through the upper nozzle. The flow of seawater in the condenser is also by natural circulation. There are three regimes of heat transfer viz., pre-heating zone, boiling zone and superheating zone, in the heat exchanger. Condensing and Subcooling zones are present in the Condenser.

A barge will be subjected to various oscillations due to seawater movements. The RHRS should be able to perform its intended functions even when the barge is oscillating. It is necessary to find out the natural frequency of oscillation of such a system. If the natural frequency of oscillations of the RHRS is close to the frequency of oscillations of the barge, resonance may occur and lead to deterioration of heat removal capability. In the worst case, even flow reversal might happen. Thus at the design stage it is necessary to determine natural frequency for flow oscillations of the system and ensure that possibility of resonance does not exist. This paper outlines a method for determining natural frequency of any fluid system.

Analysis

To develop method of analysis, we followed a stepwise approach. This involved studying the following.

- i) Isothermal fluid in an open loop of constant area of cross section and flow being laminar,
- ii) Isothermal fluid in an open loop of constant area of cross section flow being laminar or turbulent,
- iii) Isothermal fluid in an open loop with step changes in the area of cross section,
- iv) Isothermal fluid in an open loop system having parallel paths as well as area changes,
- v) Non-isothermal fluid in an open loop system having parallel paths as well as area changes,
- vi) Non-isothermal fluid in a closed loop having parallel paths as well as area changes.

The cases (i) to (iv) have been dealt in detail in [1]. We concentrate here on the last two cases since they are of direct practical relevance.

Variable Temperature Systems

The behaviour of an open loop system (Fig. 2) can be described by the generalised conservation equation of energy. The energy balance equation for such a system is [4],

$$\frac{d}{dt}(U + K + \Phi) = \Delta \left[\left(H + \frac{1}{2} \frac{v^3}{v} + \phi \right) w \right] - E_v - W. \quad (1)$$

In a practical residual heat removal system, temperature distribution and flow rates are inter-linked. Fluid oscillations would also influence heat transfer. However, to simplify the analysis, this influence has been neglected. The temperature distribution of the fluid in the tube is assumed to be known. In addition, any change in fluid temperature due to viscous dissipation is neglected. Because of this, the momentum balance equation and the energy balance equations are treated separately. Due to these assumptions, change in U is neglected. Since there is no flow to or from the system, $w=0$. The individual terms of the equation (1) are derived hereafter.

Kinetic Energy Term

The first term on the Left-hand side is,

$$\frac{d}{dt} K = \frac{d}{dt} \left(\int \frac{1}{2} \rho v^2 dV \right) = \frac{d}{dt} \int_0^L \int_0^R \frac{1}{2} \rho v^2 2\pi r dr dl = \frac{d}{dt} \sum_i \frac{1}{2} \pi R_i^2 \int_0^{L_i} \rho_i v_i^2 dl.$$

Here the velocity is position dependent due to variation of temperature along the length of the pipe. From continuity it follows that,

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2 = \rho_{av} v_{av} A.$$

Hence the kinetic energy term is,

$$\frac{d}{dt} K = \frac{1}{2} \frac{d}{dt} \sum_i \pi R_i^2 \int_0^{L_i} \rho \left(\frac{\rho_{av,i} v_{av,i}}{\rho} \right)^2 dl = \frac{1}{2} \sum_i \pi R_i^2 \rho_{av,i}^2 \left(\int_0^{L_i} \frac{1}{\rho} dl \right) \left(\frac{d}{dt} v_{av,i}^2 \right). \quad (2)$$

Potential Energy Term

Since the system is non-isothermal, the steady state heights would be different. Let the steady state heights of the liquid columns be K_a and K_b . When the column is set into motion, at any instant of time, let H_a and H_b be heights of the liquid columns (Fig. 2). The potential energy term can then be calculated as,

$$\begin{aligned} \frac{d}{dt} \Phi &= \frac{d}{dt} \int_0^L \int_0^R \rho g h 2\pi r dr dh = g \frac{d}{dt} \left(\int_0^{H_a} \pi r^2 \rho_a h dh + \int_0^{H_b} \pi r^2 \rho_b h dh \right) \\ &= g \frac{d}{dt} \left(\int_0^{K_a} \pi r^2 \rho_a h dh + \int_{K_a}^{K_a+h_a} \pi r^2 \rho_a h dh + \int_0^{K_b} \pi r^2 \rho_b h dh + \int_{K_b}^{K_b+h_b} \pi r^2 \rho_b h dh \right). \end{aligned}$$

Here it is assumed that $H_a+H_b=L$ and the amplitude of oscillation is much smaller than the length of the pipe sections. Since derivative of the first and the third term with respect to time is equal to zero,

$$\begin{aligned} \frac{d}{dt} \Phi &= g \frac{d}{dt} \left[\pi R_a^2 \rho_a \left[\frac{h^2}{2} \right]_{K_a}^{K_a+h_a} + \pi R_b^2 \rho_b \left[\frac{h^2}{2} \right]_{K_b}^{K_b+h_b} \right] \\ &= \frac{g}{2} \frac{d}{dt} \left[\pi R_a^2 \rho_a \left((K_a + h_a)^2 - K_a^2 \right) + \pi R_b^2 \rho_b \left((K_b + h_b)^2 - K_b^2 \right) \right] \\ &= \pi g \left(R_a^2 \rho_a h_a \frac{dh_a}{dt} + R_b^2 \rho_b h_b \frac{dh_b}{dt} - R_b^2 \rho_b K_b \frac{dh_b}{dt} + R_a^2 \rho_a K_a \frac{dh_a}{dt} \right). \end{aligned} \tag{3}$$

Viscous Dissipation Term

The dissipation due to pipe friction and pipe fittings is calculated by [4],

$$E_v = \sum_i \frac{1}{2} v_{av,i}^2 \frac{L_i}{R_h} f_i + \sum_i \frac{1}{2} v_{av,i}^2 e_{v,i} \tag{4}$$

where, f_i is the fanning friction factor and $e_{v,i}$ is the loss coefficient of the pipe section.

Work Term

Work done by the fluid against external pressure is given by,

$$W = \pi R_b^2 v_b p_b - \pi R_a^2 v_a p_a \tag{5}$$

Here v_a and v_b are the velocities and p_a and p_b are the pressure exerted at the end of the tube. These values are indicated in Fig. 2.

The Overall Equation

Using the equations (2) through (5), the overall equation (1) can be written as,

$$\begin{aligned} & \frac{1}{2} \sum_i A_i \rho_{av,i}^2 \left(\int_0^L \frac{1}{\rho} dl \right) \frac{d}{dt} v_{av,i}^2 + \frac{1}{2} \sum_i \left(\frac{L_i f_i}{R_{h,i}} + e_{v,i} \right) v_{av,i}^2 \\ & + g \left(A_a \rho_a h_a \frac{dh_a}{dt} + A_b \rho_b h_b \frac{dh_b}{dt} - A_b \rho_b K_b \frac{dh_b}{dt} + A_a \rho_a K_a \frac{dh_a}{dt} \right) \\ & = A_b v_b \rho_b - A_a v_a \rho_a \end{aligned} \tag{6}$$

In any pipe section, the velocity $v_i = \frac{dh_i}{dt}$. Also, $h_1 A_1 = h_2 A_2 = h_i A_i = h A_r$, where A_r is a reference area. Then,

$$\frac{dh_1}{dt} = \frac{A_r}{A_1} \frac{dh}{dt} \quad \text{or, } v_1 = \frac{A_r}{A_1} v \quad \text{and} \quad \frac{dh_2}{dt} = \frac{A_r}{A_2} \frac{dh}{dt} \quad \text{or, } v_2 = \frac{A_r}{A_2} v,$$

in general,
$$\frac{dh_i}{dt} = \frac{A_r}{A_i} \frac{dh}{dt} \quad \text{or, } v_i = \frac{A_r}{A_i} v.$$

Now the equation (6) can be written as,

$$\begin{aligned} & \sum_i A_i \rho_{av,i}^2 \left(\int_0^L \frac{1}{\rho} dl \right) v_{av,i} \frac{d}{dt} v_{av,i} + \frac{1}{2} \sum_i \left(\frac{L_i f_i}{R_{h,i}} + e_{v,i} \right) v_{av,i}^2 \\ & + g (A_a \rho_a h_a v_a + A_b \rho_b h_b v_b - A_b \rho_b K_b v_b + A_a \rho_a K_a v_a) = A_b v_b \rho_b - A_a v_a \rho_a \end{aligned}$$

Expressing velocities in terms of velocity based on the reference diameter,

$$\begin{aligned} & \sum_i A_i v_i \rho_{av,i}^2 \left(\int_0^L \frac{1}{\rho} dl \right) \frac{A_r}{A_i} v_r \frac{d}{dt} \left(\frac{A_r}{A_i} v_r \right) + \frac{1}{2} \sum_i \left(\frac{L_i f_i}{R_{h,i}} + e_{v,i} \right) \left(\frac{A_r}{A_i} v_r \right)^2 \\ & + g \left(A_a \rho_a h_a \frac{A_r}{A_a} v_r + A_b \rho_b h_b \frac{A_r}{A_b} v_r - A_b \rho_b K_b \frac{A_r}{A_b} v_r + A_a \rho_a K_a \frac{A_r}{A_a} v_r \right) \\ & = A_b \rho_b \frac{A_r}{A_b} v_r - A_a \rho_a \frac{A_r}{A_a} v_r \end{aligned}$$

After simplifying and re-arranging we get,

$$\begin{aligned} & \sum_i \frac{A_r}{A_i} \rho_{av,i}^2 \left(\int_0^L \frac{1}{\rho} dl \right) \frac{d}{dt} \left(\frac{dh}{dt} \right) + \frac{1}{2} \sum_i \frac{A_r}{A_i^2} \left(\frac{L_i f_i}{R_{h,i}} + e_{v,i} \right) \frac{dh}{dt} + g A_r \left[\frac{\rho_a}{A_a} + \frac{\rho_b}{A_b} \right] h \\ & = (\rho_b K_b - \rho_a K_a) + \rho_b - \rho_a \end{aligned} \tag{7}$$

Let

$$T_1 = \sum_i \frac{A_r}{A_i} \rho_{av,i}^2 \left(\int_0^L \frac{1}{\rho} dl \right); \quad T_2 = \frac{1}{2} \sum_i \frac{A_r}{A_i^2} \left(\frac{L_i f_i}{R_{h,i}} + e_{v,i} \right);$$

$$T_3 = gA_r \left[\frac{\rho_a}{A_a} + \frac{\rho_b}{A_b} \right]; \quad T_4 = g(K_b \rho_b - K_a \rho_a) + (p_b - p_a).$$

Hence the equation (7) takes the form,

$$T_1 \frac{d^2 h}{dt^2} + T_2 \frac{dh}{dt} + T_3 h = T_4. \tag{8}$$

This equation is solved by Runge-Kutta 4th order method [5].

Initial level of liquid in the limbs will be different due to difference in temperature of the liquid along the length. The level of liquid in both the limbs are estimated and are used in calculations. Three types of initial conditions are possible, viz.,

- (i) at $t=0, h=h_0$ and $\frac{dh}{dt} = 0$ $p_a = p_b$ for all values of t ,
- (ii) at $t=0, h=0$ and $\frac{dh}{dt} = 0$ $p_a \neq p_b$ for all values of t ,
- (iii) or a combination of both of the above.

Results for these initial conditions are analysed and given hereafter.

TABLE 1
Temperature Distribution of the Open Loop Variable Temperature System

Pipe No.	Location	Temperature	Pipe No.	Location	Temperature
1	0.00	20.0	3	0.80	40.0
	0.20	30.0		1.00	35.0
	0.30	33.0	4	0.00	35.0
	0.50	35.0		0.10	32.0
	0.80	32.0		0.30	38.0
	1.00	30.0		0.40	35.0
2	0.00	30.0		0.75	38.0
	0.30	32.0		1.00	40.0
	0.50	38.0	5	0.00	40.0
	0.75	40.0		0.20	35.0
1.00	45.0	0.40		30.0	
3	0.00	45.0		0.50	25.0
	0.30	49.0		0.80	20.0
	0.70	45.0		1.00	20.0

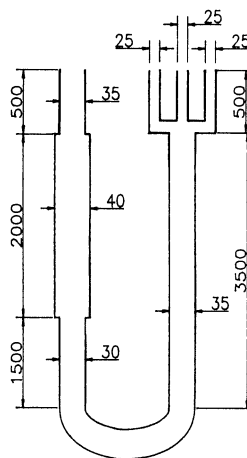


FIG.3
Variable area variable temperature system

It is assumed that due to an external perturbation, some difference between the steady state levels of the liquid column is created and then it is set free to oscillate. The geometry of the piping system is given in the Fig 3. The temperature distribution of the fluid in the pipes used is given in the Table 1. This distribution is one of the many used for analysis.

With these conditions, the system was analysed. Fig. 4 gives the change in the height of the liquid column with respect to the initial height for left column (column 'a'). For column 'b', results differ only slightly and on the plot they will appear the same and so have not been plotted.

In the second case, an external pressure is applied (at $t = 0$) to one of the limbs of the system in steady state and is maintained thereafter. On application of the differential pressure, the fluid starts oscillating and attains a new steady state. This behaviour is depicted in the Fig. 5.

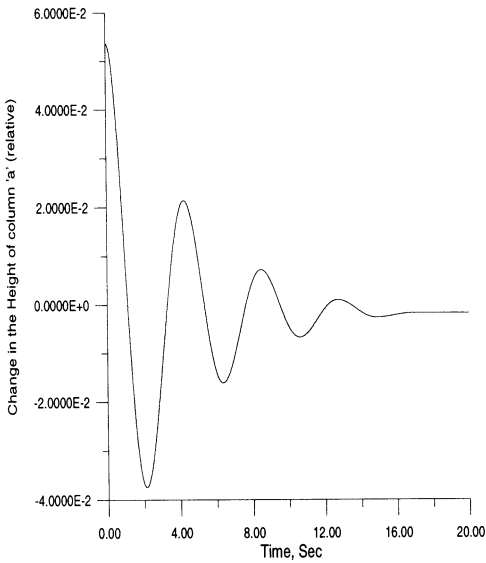


FIG. 4
Results for the case, where
initial height of the limbs are different

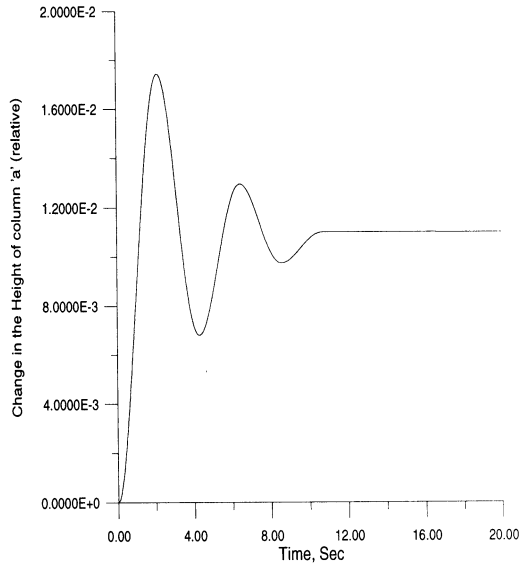


FIG. 5
Results for the case, where
initial pressure on the limbs are different

Thus, the response for the case, when the temperature distribution of the fluid is specified is computed. Here temperature distribution was assumed. In order to analyse the response of the RHRS when subjected to oscillations, the temperature and quality distribution of the working fluid in the loop has to be calculated. The method of this calculation has been reported earlier [8,9].

Closed Loop Systems

A simplified scheme of the second loop of the natural circulation residual heat removal system is indicated in the Fig 6.

The mathematical model developed in previous section deals with an open loop, variable temperature system. The RHR system is a closed loop. The boiling region and the superheated region in the heat exchanger, the steam pipes and the condensing region of the condenser are also to be included for analysing the Natural Circulation System.

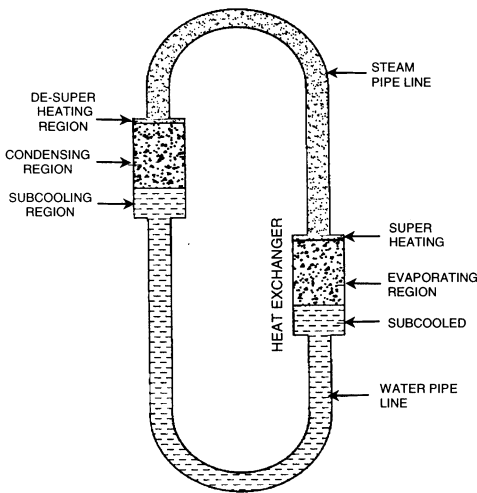


FIG. 6
Schematic of closed loop two phase residual heat removal system

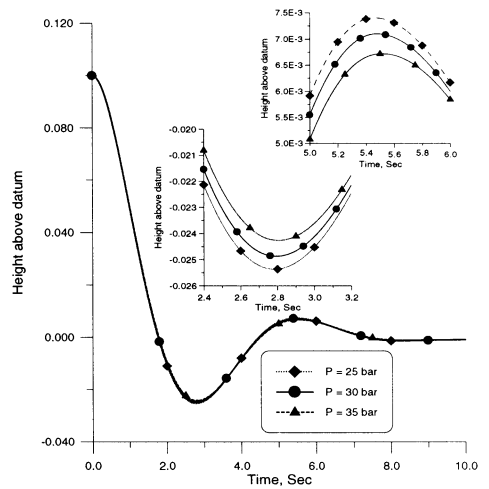


FIG. 7
Response of the RHR system

In the previous section, the density and viscosity of the working fluid were evaluated as functions of temperature. However in the present case, as two-phase mixture is involved in the heat exchanger and in the condenser, the density and viscosity are to be evaluated as a function of temperature and or quality as appropriate. In the open loop system considered in section 2.1, the external pressure acting on one of its limbs can be different from the external pressure acting on the other limb. In the present case, where a closed loop is being considered, movements of the liquid menisci are in phase and the change in the volume of the fluid due to oscillations is very small as compared to the total volume of the system [1]. Hence, in the closed loop system, the pressure acting on both the liquid menisci will be the same and it is assumed to remain constant. To maintain the flow rate through the loop, the liquid in the condenser is

at a higher elevation than that in the heat exchanger. The difference in the levels is calculated by equating the head generated in the two limbs as a result of difference in density. The work term in the present case turns out to be zero.

Viscous dissipation term E_v includes resistance to motion offered by the steam path and both the pipe friction and fitting losses are accounted for. The pressure drop in the pre-heating and superheating regions of the heat exchanger, sub-cooling region of the condenser and in the steam and water pipe lines are calculated [10]. Pressure drops in the boiling region of the heat exchanger and the condensing region of the condenser are evaluated using the Martinelli-Nelson method [6].

To evaluate the kinetic energy term, integration of specific volume of the working fluid over the entire loop is to be carried out. The ESSAP program [9] generates the temperature and quality distribution of the working fluid throughout the loop. Using this information, density of the fluid at various locations in the loop is calculated considering both temperature and quality distributions as appropriate. Numerical integration by Simpson's 1/3rd rule [5] is employed for each part of the loop to estimate the integral. The results of integration for each of the part is multiplied by $\rho_{r,i}^2 A_r / A_i$ and are summed up to get the value of the kinetic energy term.

Following the development in the previous section, the overall governing equation can be written as,

$$T_1 \frac{d^2 h}{dt^2} + T_2 \frac{dh}{dt} + T_3 h = T_4.$$

The coefficients T_1 , T_2 , T_3 and T_4 of the equation are given by,

$$T_1 = \sum_i \frac{A_r}{A_i} \rho_{av,i}^2 \left(\int_0^L \frac{1}{\rho} dl \right); \quad T_2 = \frac{1}{2} \sum_i \frac{A_r}{A_i^2} \left(\frac{L_i f_i}{R_{h,i}} + e_{v,i} \right);$$

$$T_3 = g A_r \left[\frac{\rho_a}{A_a} + \frac{\rho_b}{A_b} \right]; \quad T_4 = g (K_b \rho_b - K_a \rho_a) + (p_b - p_a).$$

Where the terms are evaluated for the case of two phase closed loop as discussed above.

Program Description

A program ORBITS (OscillatoRy Behavlour of residual heat removal System) has been developed for carrying out the analysis of oscillations of the Residual Heat Removal System. The program has an open structure. The components, which constitute the loop, are specified in a modular

manner. In this case the possible components are; (i) Pipelines (may be water pipeline or steam pipeline), (ii) heat exchanger and (iii) condenser. Different numbers of each of the components can be specified. The temperature and quality of the working fluid at various locations along the length of the pipe sections, the heights of pre-heating, evaporating and superheating regions of heat exchangers, and heights of the condensing and subcooling sections for condenser are inputs. Each of the components is subdivided into a number of segments during the calculations. The thermal and transport properties of water and steam are calculated by using the methodology of the property routine STH2Ox used in RELAP [7] program. The coefficients T_1 , T_2 , T_3 and T_4 are evaluated. This equation is then solved by Runge-Kutta 4th order method.

Results

The results of oscillatory behaviour of the system for three operating pressures are given in Fig. 7. The results indicate that the system is highly damped due to the presence of two-phase regions in the heat exchanger and the condenser. When the operating pressure is increased, the length of the boiling region and the condensing region increases. Due to this effect, the time period of oscillations increases. However, the changes are small. Hence, it can be stated that small changes in operating pressure of the system do not affect the frequency of oscillations significantly.

Conclusion

A program has been developed to compute natural frequency of fluid oscillations in a closed system. For the system studied, the natural period of oscillation is about 5.5 sec and is not significantly influenced by operating pressure. Due to seawater waves, the period of oscillation of a barge (located in the Arabian Sea / Bay of Bengal) on which a Nuclear Desalination Plant of the type described here may be mounted is about 12 to 15 sec. Since the natural frequency of oscillation of the system is away from the frequency of oscillations induced by sea waves, the chances of resonant oscillations of the residual heat removal system and its consequential inability to perform the intended function is remote.

Nomenclature

A	Area	m^2	R	Radius	m
C_p	Specific heat	$J\ kg^{-1}\ K^{-1}$	r	distance from the centre along the radius	m

D	Diameter of Tube	m	Re	Reynolds Number	--		
E_v	Viscous dissipation	J	t	Time	s		
e_v	Total Loss coefficient of pipe section	--	U	Internal Energy	J		
f	fanning friction factor	--	v	Velocity of fluid	$m\ s^{-1}$	SUBSCRIPTS	
g	Acceleration due to gravity	$m\ s^{-2}$	V	Volume	m^3	1	location 1
G	mass flux	$kg\ m^{-2}\ s^{-1}$	W	Work	J	2	location 2
h	height of liquid column	m	z	distance along the axis	m	a	Location a
H	Enthalpy	J	Φ	Potential Energy	J	b	Location b
h	heat transfer coefficient	$W\ m^{-2}\ K^{-1}$	μ	Viscosity	Pa. s	av	Average
K	Kinetic Energy	J	ρ	Density	$kg\ m^{-3}$	h	Hydraulic
k	Thermal conductivity	$W\ m^{-1}\ K^{-1}$	T	Period of oscillation	s	i	Section i
L	Length of Pipe Section	m	v	Specific volume	$m^3\ kg^{-1}$	n	natural
p	Pressure	Pa	ω	frequency of oscillation	Hz	T	total

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