Robust Pole Placement Stabilizer Design Using Linear Matrix Inequalities

P. Shrikant Rao and I. Sen

***Abstract*—This paper presents the design of robust power system stabilizers which place the system poles in an acceptable region in the complex plane for a given set of operating and system conditions. It therefore, guarantees a well damped system response over the entire set of operating conditions. The proposed controller uses full state feedback. The feedback gain matrix is obtained as the solution of a linear matrix inequality expressing the pole region constraints for polytopic plants. The technique is illustrated with applications to the design of stabilizers for a single machine and a 9 bus, 3 machine power system.**

***Index Terms*—Linear matrix inequalities, power system dynamic stability, robustness.**

I. Introduction

POWER system stabilizers (PSS) are now commonly used by utilities for damping the low frequency oscillations in power systems. The conventional lead compensation type of PSS [1] is quite popular with the industry due to its simplicity. However, the performance of these stabilizers can be considerably degraded with the changes in the operating condition during normal operation.

Power systems continually undergo changes in the operating condition due to changes in the loads, generation and the transmission network resulting in accompanying changes in the system dynamics. A well designed stabilizer has to perform satisfactorily in the presence of such variations in the system. In other words, the stabilizer should be robust to changes in the system over its entire operating range.

The nonlinear differential equations governing the behavior of a power system can be linearized about a particular operating point to obtain a linear model which represents the small signal oscillatory response of the power system. Variations in the operating condition of the system result in corresponding variations in the parameters of the small signal model. A given range of variations in the operating conditions of a particular system thus generates a set of linear models each corresponding to one particular operating condition. Since, at any given instant, the actual plant could correspond to any model in this set, a robust controller would have to impart adequate damping to each one of this entire set of linear models.

In recent years there have been several attempts at designing power system controllers using H_∞ based robust control techniques [2], [3]. In this approach, the uncertainty in the chosen system is modeled in terms of the bounds on the frequency response. A H_∞ optimal controller is then synthesized which guarantees robust stability of the closed loop. Other performance specifications such as disturbance attenuation criteria are also imposed on the system. However, it should be noted that the main objective of using a PSS is to provide a good transient behavior. Guaranteed robust stability of the closed loop, though necessary, is not adequate as a specification in this application. In addition, the problems of poorly damped pole-zero cancellations and choice of weighting functions used in the design limit the usefulness of this technique for PSS design. H_∞ design, being essentially a frequency domain approach, does not provide much control over the transient behavior and closed loop pole location. It would be more desirable to have a robust stabilizer which, in addition, guarantees an acceptable level of small signal transient performance. This can be achieved by proper placement of the closed loop poles of the system. This paper proposes the design of a stabilizer fulfilling these requirements based on linear matrix inequalities.

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The authors are with the Department of Electrical Engineering, Indian Institute of Science, Bangalore, India, 560012.

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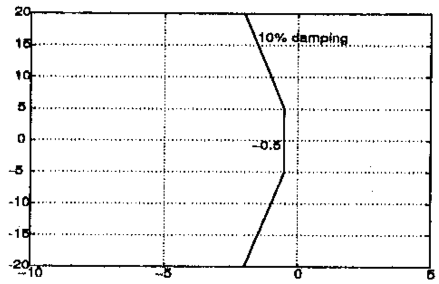

Fig. 1. The $\mathcal{D}$ contour.

II. Performance Requirements of Power System Stabilizers

In power systems, a damping factor, ζ, of at least 10% and a real part, σ, not greater than -0.5 for the troublesome low frequency electromechanical mode, guarantees that the low frequency oscillations, when excited, will die down in a reasonably short time. Such a restriction on all the system eigenvalues would imply that all the poles of the system lie to the left of the $\mathcal{D}$-contour shown in Fig. 1. This property will be referred to as $\mathcal{D}$-stability. If this condition is satisfied over a given range of operating conditions, a well damped response is guaranteed over the specified range and the controller which achieves this can be said to be robust, i.e., it guarantees acceptable transient small signal performance in spite of the variations in the plant.

The above objective should be achieved without excessively large controller gains, since these could lead to controller output saturation and a poor large disturbance response of the system.

Reference [4] presents a method for synthesizing state feedback for polytopic plants i.e. plants whose state space descriptions vary over polytopes, which guarantees pole placement in any desired region of the complex plane for all plants in the specified polytope. The formulation is based on expressing the pole region constraints as linear matrix inequalities (LMI's) which can be easily solved using available semidefinite programming methods [5], [6]. This paper uses the technique of [4] to synthesize a robust state feedback controller for single and multi-machine systems. The performance of these controllers are then evaluated over the entire range of operating conditions through eigenvalue analysis and time response simulation.

III. LMI Formulation of Pole Placement Objectives

A subset $\mathcal{D}$ of the complex plane is called an LMI region if there exist a matrix $\alpha \in \mathcal{R}^{m\times m}$ and a matrix $\beta \in \mathcal{R}^{m\times m}$ such that

$$\mathcal{D} = \{z \in C;\ f_{\mathcal{D}}(z) < 0\}$$

with

$$f_{\mathcal{D}} := \alpha + z\beta + \overline{z}\beta^{T} = [\alpha_{kl} + \beta_{kl}z + \beta_{lk}\overline{z}]_{1\le k,\, l\le m}$$

where $\overline{z}$ is the complex conjugate of z, C is the complex plane, ^{T} denotes transpose and <0 stands for negative definite. Many convex regions in the complex plane which are symmetric with respect to the real axis including half planes, horizontal strips, circles and sectors can be expressed as LMI regions. The intersection of a number of LMI regions is also an LMI region. Complicated regions can be constructed as the intersection of a number of individual LMI regions.

Let $z = x + jy$. The half plane $x < -0.5$ can be represented by the relation

$$z + \overline{z} < 2(-0.5) \ \Rightarrow\ 1.0 + z + \overline{z} < 0$$

i.e. $\alpha_1 = [1]$ and $\beta_1 = [1]$.

The conic sector making an angle θ with the imaginary axis can be represented as

$$\begin{bmatrix} \cos\theta(z+\overline{z}) & -\sin\theta(z-\overline{z}) \\ \sin\theta(z-\overline{z}) & \cos\theta(z+\overline{z}) \end{bmatrix} < 0$$

i.e.,

$$\alpha_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix};\ \beta_2 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}.$$

Appending the α and β matrices of the half plane and the conic sector (with $\sin\theta = 0.1$ for $\zeta = 10\%$) together, the region to the left of the $\mathcal{D}$-contour shown in Fig. 1 can be represented by the matrices:

$$\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\quad \beta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.995 & -0.1 \\ 0 & 0.1 & 0.995 \end{bmatrix}.$$

Given a plant with state matrix A and an LMI region represented by α and β, the following result from [4] expresses the required pole placement constraint as an LMI.

The matrix A is $\mathcal{D}$-stable if and only if there exists a symmetric, positive definite matrix X such that

$$M_{\mathcal{D}}(A, X) := \alpha\otimes X + \beta\otimes(AX) + \beta^{T}\otimes(AX)^{T} < 0 \qquad (1)$$

where $\otimes$ represents the Kronecker product of matrices. A symmetric, positive definite matrix X generates a quadratic Lyapunov function and hence its existence ensures the D-stability of the system [4].

A state feedback controller of the form $u = Kx$ results in a closed loop state matrix $A_{CL} = A + BK$. Applying the above result for the matrix A_{CL} gives the following

$$\begin{aligned} M_{\mathcal{D}}(A_{CL}, X) < 0 &\Rightarrow \alpha\otimes X + \beta\otimes(A_{CL}X) \\ &\quad + \beta^{T}\otimes(A_{CL}X)^{T} \\ &< 0 \Rightarrow \alpha\otimes X + \beta\otimes(AX + BKX) \\ &\quad + \beta^{T}\otimes(AX + BKX)^{T} < 0. \end{aligned}$$

Introduction of a new variable $L = KX$ restores the linearity of the above expression giving

$$\alpha\otimes X + \beta\otimes(AX + BL) + \beta^{T}\otimes(AX + BL)^{T} < 0. \quad (2)$$

The above expression is a matrix inequality, linear in the variables X and L, and can be solved using standard optimization techniques. Once a feasible solution (X, L) satisfying (2) is found, the required state feedback gain matrix can be computed as $K = LX^{-1}$.

Thus, given the state space realization of a linear system, the above formulation generates a state feedback which places the closed loop poles in any desired LMI region.

IV. Robust Pole Placement for Polytopic Plants

Consider an uncertain linear time invariant system

$$\dot{x} = Ax + Bu \qquad (3)$$

where the matrices A and B take values in a matrix polytope, i.e.

$$(A, B) \in \left\{ \left(\sum_{i=1}^{N} piAi,\ \sum_{i=1}^{N} piBi \right) : \sum_{i=1}^{N} pi = 1,\ pi \ge 0 \right\}.$$

Here, $(Ai, Bj), i = 1 \cdots N$ are the N vertices of the polytope.

Synthesis of a common $\mathcal{D}$-stabilizing feedback for this entire polytope requires the LMI (2) to be written for each of the vertex systems. A common solution to the resulting set of N LMI's, if one exists, will guarantee closed loop pole location in the desired region for all points in the polytope.

V. Application to PSS Design

Linearization of the power system equations about an operating point results in a small signal model of the form (3). A specified range of operating conditions for the system generates a set of A and B matrices. To apply the result from Section IV to synthesize a state feedback controller for guaranteed robust pole placement for the given system, it is required to obtain a matrix polytope that contains this set. This problem is nontrivial and is not attempted here. Instead, it is possible to choose a set of external operating conditions, in terms of the system loading, power transfers and network parameters, and use the polytope generated by the A and B matrices of this set of plants for the

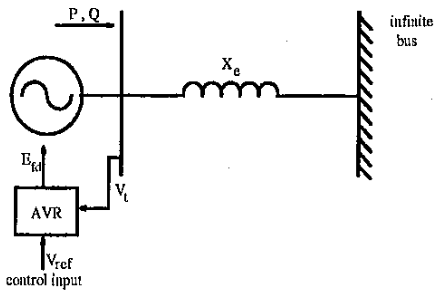

Fig. 2. Single machine infinite bus system.

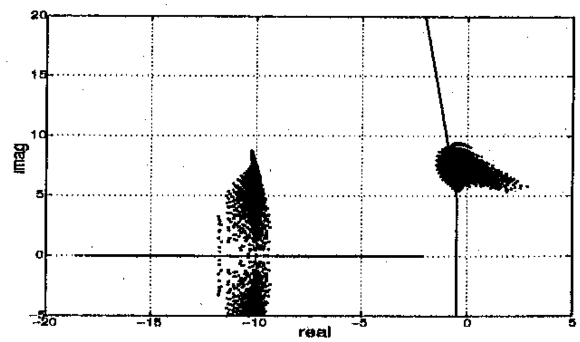

Fig. 3. Open loop poles (single machine system).

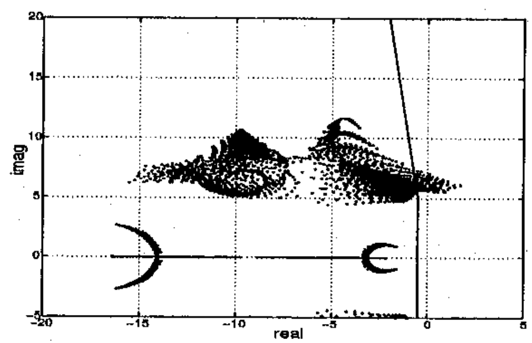

Fig. 4. Closed loop poles with the H_∞ optimal PSS.

design. This is the procedure followed here. The nonlinear dependence of the entries of the A and B matrices on the operating condition parameters results in a loss of the sufficiency of the result in Section IV. It cannot be guaranteed that such a choice of the external operating conditions covers all the cases. Hence, the synthesized feedback gains have to be checked by computing the closed loop eigenvalues of the system for the entire range of operating conditions. In case the robust D-stability requirement is not satisfied, the one or more plants violating the requirement can be added to the set of vertices and the synthesis can be repeated. Such an iteration was not found necessary in the examples given here.

A. Single Machine System

A single line schematic diagram of a single machine infinite bus system is shown in Fig. 2. The generator is fitted with an automatic voltage regulator (AVR) and a static excitation system. Neglecting stator transients and the effect of damper windings, the generator and exciter can be modeled as a 4th order system [7]. The system data is given in the Appendix.

The operating condition for the above system is completely defined by the values of the real power, P, the reactive power, Q, at the generator terminals and the transmission line impedance, X_e. P, Q and X_e are assumed to vary independently over the following ranges:

$$P: 0.4\text{–}1.0; \quad Q: -0.2\text{–}0.5; \quad X_e: 0.2\text{–}0.7.$$

This encompasses almost all practically occurring operating conditions and very weak to very strong transmission networks.

Fig. 3 shows the open loop poles for this set of plants with P, Q and X_e varied over the specified range in steps of 0.05. As seen, most of the operating conditions in this set do not have adequate damping. It is required to design a stabilizer which places the rotor mode eigenvalue inside the acceptable region shown in Fig. 1.

An attempt to design an H_∞ optimal PSS [3] for this system so as to guarantee robust stability of the closed loop over the entire range of operating conditions mentioned above failed to yield a satisfactory result. The closed loop norm with the H_∞ optimal PSS was found to be greater than 1. Fig. 4 shows the system poles with the H_∞ optimal PSS for the entire range of variations in P, Q and X_e. As seen, this controller does not provide the required damping at all the operating conditions and in fact even fails to stabilize some of the plants within the set.

It is proposed to apply a full state feedback controller using the LMI based approach described in Sections III and IV for achieving the robust D-stability requirement stated in Section II. The implementation details are shown in Fig. 5. Each state is measured, multiplied by the appropriate gain and then summed up before being fed at the reference input of the Automatic Voltage Regulator. In a practical implementation, additional hardware would be required for state measurements. For the 4th order model used here, the states are the deviations in the load angle δ, rotor speed ω, field voltage E_{FD} and the internal voltage E'_q. Of these, δ, ω and E_{FD} can be directly measured using appropriate transducers. The internal voltage E'_q can be computed from the instantaneous values of the stator currents and the equivalent circuit parameters.

A polytopic system is obtained for this set by choosing the A and B matrices corresponding to the external values of P, Q and X_e. All possible combinations of the minimum and maximum values of each of these 3 parameters are taken generating a set of 8 vertex systems corresponding to the 8 corners of a cube in the (P, Q, X_e) space. Of these, the point with minimum Q and maximum P and X_e did not have a steady state load flow solution and was replaced by a nearby point where a solution exists ($P = 1.0$, $Q = -0.2$, $X_e = 0.45$).

An unnecessarily large shift of the system poles into the left half plane should be avoided, since this would be accompanied by large feedback gains. Imposition of additional constraints on the closed loop poles restricting their real parts to be greater than -50 and imaginary parts within $\pm j20$, inhibits such excessive shifting of the system poles due to the feedback. These values

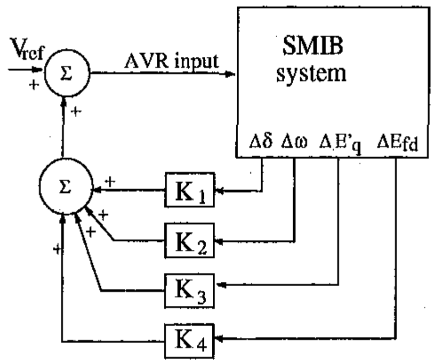

Fig. 5. Proposed controller: implementation details.

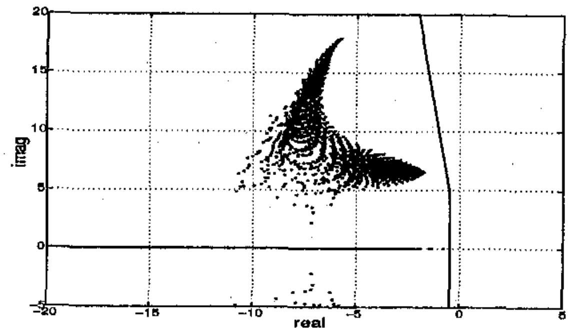

Fig. 6. Closed loop poles with the proposed state feedback controller.

were arrived at by inspecting the extent of the open loop poles for the entire set of plants.

The feasibility problem was solved for (X, L) and the required state feedback matrix was obtained. Fig. 6 shows the closed loop poles of the set of plants with the stipulated variations in P, Q and X_e. As seen, the rotor mode has been shifted into the desired region of the complex plane for the entire set of plants.

The system response to a 5% step disturbance input at the AVR voltage reference was simulated with and without the proposed controller at various operating conditions using a nonlinear model. The state measurements are passed through a high pass filter with a time constant of 2 seconds to inhibit stabilizer output in steady state. The system response at a typical operating condition is shown in Fig. 7. The natural response is very poorly damped. There is a considerable improvement in the system response with the proposed controller. This closed loop performance can be compared with the system response with a conventional and an H_∞ optimal controller shown in Fig. 8. At this operating condition, the conventional as well as the H_∞ optimal controllers perform well.

The behavior of the SMIB system now operating at a large P and leading power factor in the same transmission network is shown in Fig. 9. The system is unstable without a controller. The system is also unstable with the conventional, as well as, H_∞, optimal controllers as seen in Fig. 10. The system is very well damped with the proposed controller. Further, the stabilizer output hits a peak value of around 0.05 p.u. which complies

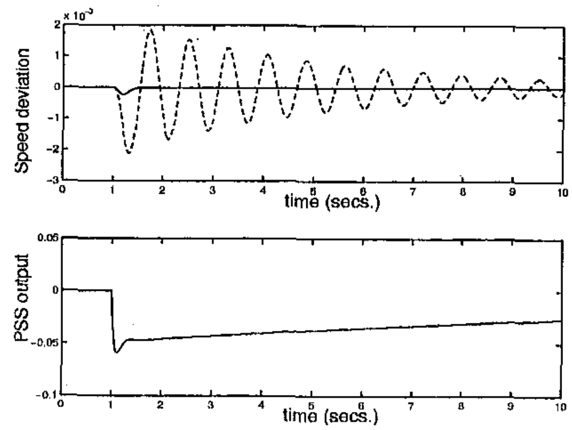

Fig. 7. Step response of the SMIB system, case I. $P = 1.0$, $Q = 0.4$, $X_e = 0.4$. - - - without controller, —with proposed controller.

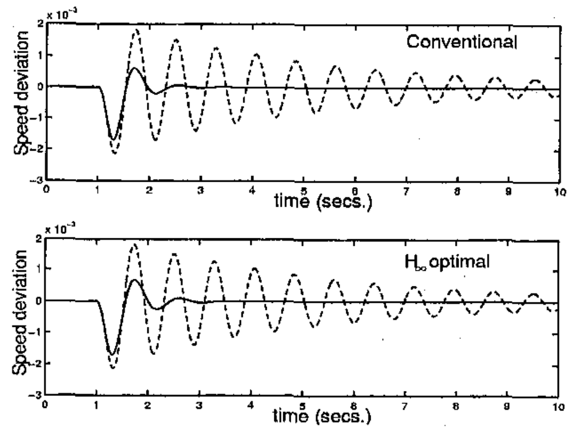

Fig. 8. Step response with the conventional and H_∞ optimal controllers, case I. $P = 1.0$, $Q = 0.4$, $X_e = 0.4$. - - - without controller, —with controller.

well with the typical limits of 0.05–0.1 p.u. imposed on the PSS output in practical implementations.

Power system stabilizers are primarily used to damp out the low frequency small oscillations. However, use of high controller gains and the resulting controller output saturation for large deviations of system variables can have a detrimental affect on the large disturbance response and transient stability of the system. Hence, the system response to a large disturbance in the presence of the proposed stabilizer was also evaluated. Fig. 11 shows the system response to a 3 phase fault at the generator terminals with the generator operating at full load and a power factor of 0.92 with a transmission line impedance of 0.4 p.u.. The fault was cleared after 100 milliseconds. Hard limits of ± 0.1 p.u. were imposed on the stabilizer output. It is seen that the proposed stabilizer does not adversely affect the system transient stability and damps out the oscillations following the fault clearing.

B. Three Machine System

The proposed design method was also applied for designing a decentralized controller for a 3 machine, 9 bus power system. In

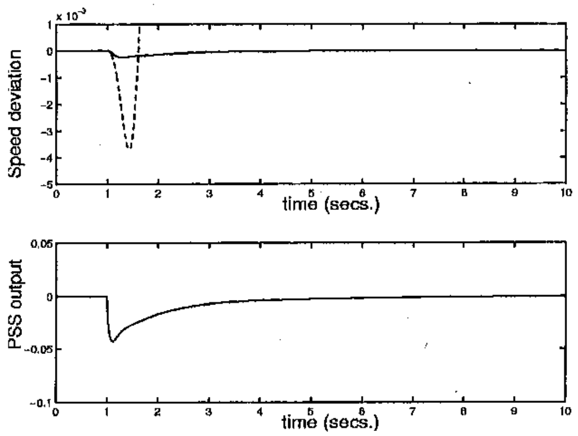

Fig. 9. Step response of the SMIB system, case II. $P = 1.0$, $Q = -0.2$, $X_e = 0.4$. - - - without controller, — with proposed controller.

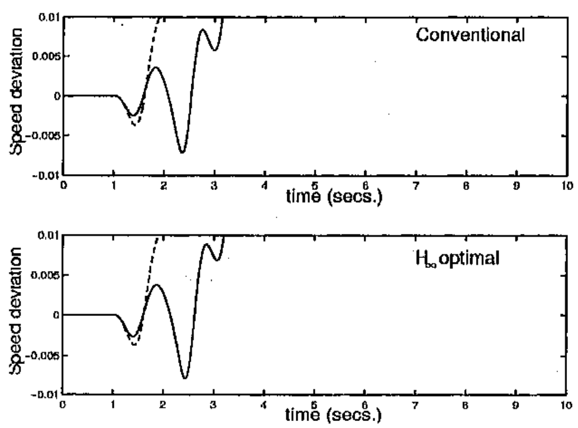

Fig. 10. Step response with the conventional and H_∞ optimal controllers, case II. $P = 1.0$, $Q = -0.2$, $X_e = 0.4$. - - - without controller, —with controller.

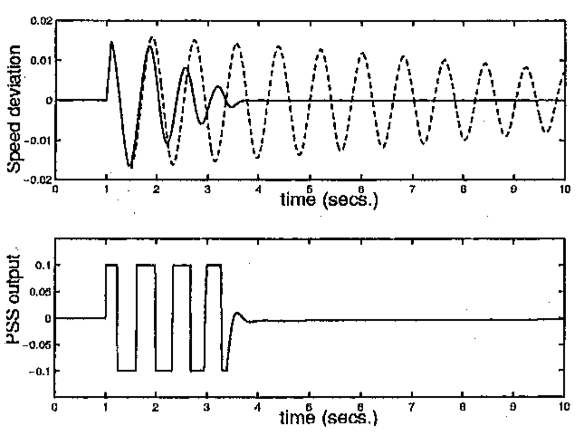

Fig. 11. System response to a 100 ms, 3 phase fault at generator terminals.

this scheme, each of the generators is fitted with a partial state feedback controller so that only locally available states are fed back at each generator. This implies that the state feedback matrix K of the overall system is block diagonal. This is schematically shown in Fig. 12, where the submatrices K_{11}, K_{22}, and

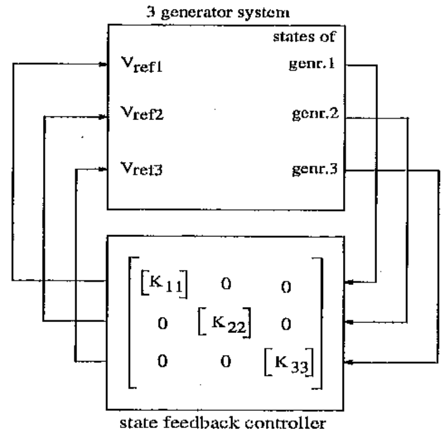

Fig. 12. Schematic of the decentralized state feedback controller.

K_{33} are the feedback gains of each of the three generators. The locally measured states—$\Delta\delta$, $\Delta\omega$, $\Delta E_q'$ and ΔE_{FD} are fed back at the AVR reference input of each machine after multiplication by suitable feedback gains.

As explained in Section III, the state feedback matrix is obtained as $K = LX^{-1}$ where X is a symmetric, positive definite matrix and L is the matrix introduced to obtain linearity. If the matrices L and X used in the LMI formulation are restricted to be block diagonal then the product LX^{-1} will also have a block diagonal structure. Such a requirement on the L and X matrices merely means restricting certain off diagonal elements to be zero which is an easily implemented additional constraint in the optimization problem. Such a restriction does not destroy the convexity of the matrix inequality (2). Hence, the same numerical method can still be used for obtaining a feasible solution.

Fig. 13 shows the single line diagram of the test system used. The system loads were assumed to vary from 35–250% of the base case generating a range of operating conditions. The nominal, minimum and maximum loading conditions are given in Table I. Fig. 14 shows the open loop poles of the system for the range of load variation in steps of 30%. The two low frequency oscillatory modes can be seen to be poorly damped.

The state matrices corresponding to the minimum, nominal and maximum loading conditions were chosen as the vertices of the polytope used in the design. The LMI problem was constructed by writing LMI (2) at each of these 3 vertices with the additional constraints on closed loop poles found necessary in the single machine case. A feasible solution (X, L) was obtained. Fig. 15 shows the system poles with the proposed feedback controller at each machine. As seen, the low frequency modes have been shifted into the acceptable region. Again, a simulation of the system response to small disturbances showed the PSS outputs at each of the 3 machines to be within the practical limits of ± 0.1 p.u.

System response at the nominal operating condition, to a small impulse disturbance at the voltage reference input of machine 1 is shown in Fig. 16. The system oscillations in terms of the rotor speed deviations from steady state of all the 3 machines are seen to be very well damped with the controller.

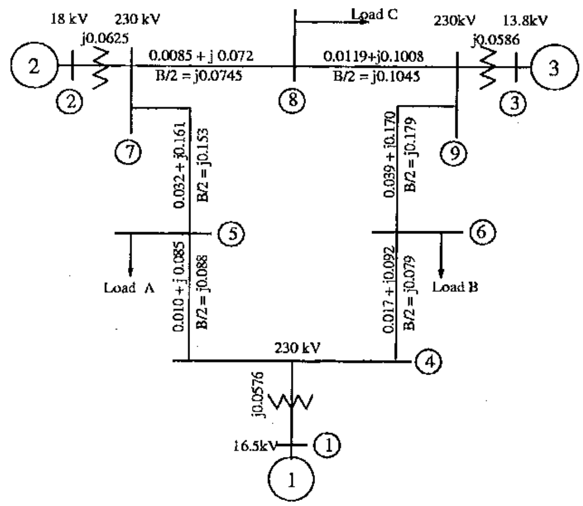

Fig. 13. Line diagram: 9 bus, 3 machine test system.

TABLE I
LOADING CONDITIONS FOR THE 9 BUS, 3 MACHINE SYSTEM

Bus no.	Power injection (pu)					
	Minimum		Nominal		Maximum	
	P	Q	P	Q	P	Q
1*	-	-	-	-	-	-
2+	0.57	-	1.63	-	3.99	-
3+	0.30	-	0.85	-	2.08	-
4	0.00	0.00	0.00	0.00	0.00	0.00
5	-0.44	-0.18	-1.25	-0.50	3.06	1.23
6	-0.32	-0.11	-0.90	-0.30	-2.21	-0.74
7	0.00	0.00	0.00	0.00	0.00	0.00
8	-0.35	-0.12	-1.00	-0.35	-2.45	0.86
9	0.00	0.00	0.00	0.00	0.00	0.00

* slack bus, + PV bus.

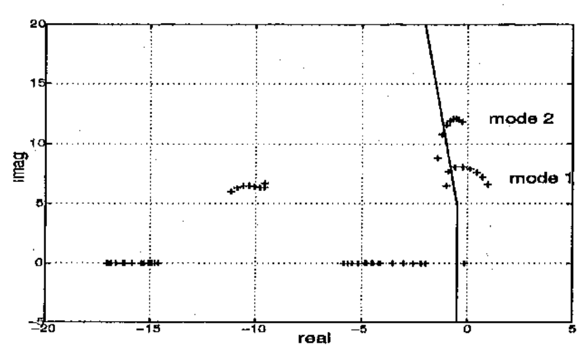

Fig. 14. Open loop poles (9 bus, 3 machine system).

VI. CONCLUSIONS

The proposed technique for the design of robust fixed parameter power system stabilizers is seen to provide the desired closed loop performance over the prespecified range of operating conditions. The performance evaluation of the proposed stabilizer on single and multimachine systems shows that this increased robustness could be achieved with reasonable feedback gain magnitudes. The four states required to be fed back are easily measurable. Further, in the multi machine case, the control is decentralized and only locally measured variables are fed back at each generator. The design procedure is simple and bears much potential for practical implementations.

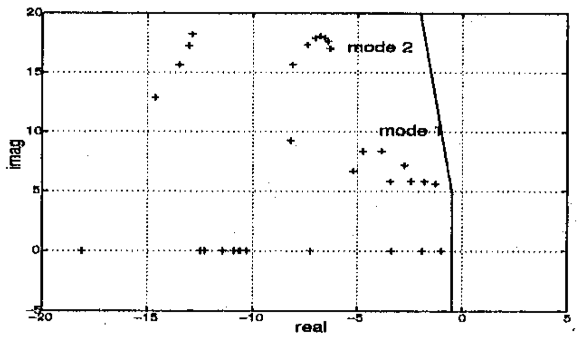

Fig. 15. Closed loop poles (9 bus, 3 machine system).

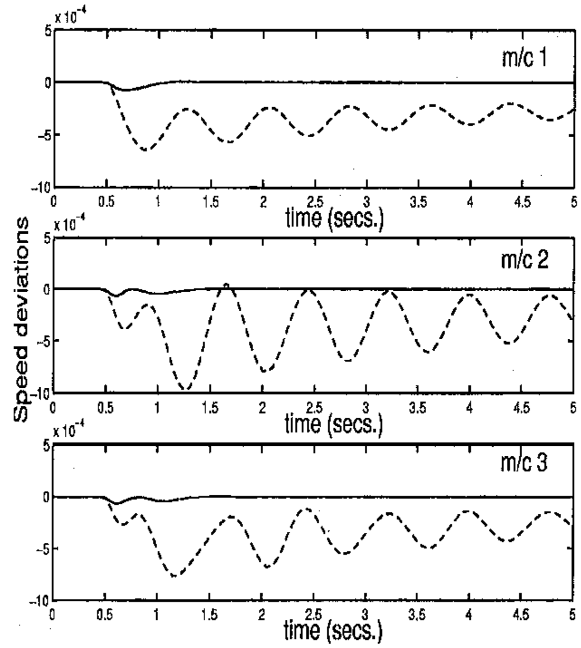

Fig. 16. Small disturbance response of the 3 machine system. - - - without controller, —with controller.

APPENDIX
SYSTEM DATA

Single Machine System:

Generator:

$$x_d = 2.0, \quad x_d' = 0.244, \quad x_q = 1.91,$$
$$T_{do}' = 4.18, \quad D = 0.0, \quad H = 3.25,$$
$$\omega_b = 314.15, \quad V_{inf} = 1.0.$$

Generator AVR:

$$K_a = 50.0,$$

$$T_a = 0.05.$$

The multi-machine system data was taken from reference [8].

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P. Shrikant Rao received his B.E. degree in electrical engineering from V.R.C.E., Nagpur, in 1993 and is currently a Research Student at the Indian Institute of Science, Bangalore. His research interests include power system stability and robust control.

I. Sen received his Ph.D. degree from IISc, Bangalore, in 1981. He is currently an Associate Professor in the Department of Electrical Engineering at the Indian Institute of Science, Bangalore. His research interests include power system stability, adaptive control and energy management systems.