

Dynamical transitions in network models of collective computation

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The field of neural network modelling has grown up on the premise that the massively parallel distributed processing and connectionist structure observed in the brain is the key behind its superior performance. The conventional network paradigm has mostly centered around a static approach – the dynamics involves gradient descent of the network state to stable fixed-points (or, static attractors) corresponding to some desired output. Neurobiological evidence however points to the dominance of non-equilibrium activity in the brain, which is a highly connected, nonlinear dynamical system. This has led to a growing interest in constructing nonequilibrium models of brain activity – several of which show extremely interesting dynamical transitions. In this paper, we focus on models comprising elements which have exclusively excitatory or inhibitory synapses. These networks are capable of a wide range of dynamical behaviour, including high period oscillations and chaos. Both the intrinsic dynamics of such models and their possible role in information processing are examined.

SINCE the development of the electronic computer in the 1940s, the serial processing computational paradigm has successfully held sway. It has developed to the point where it is now ubiquitous. However, there are many tasks which are yet to be successfully tackled computationally. A case in point is the multifarious activities that the human brain performs regularly, including pattern recognition, associative recall, etc. which are extremely difficult, if not impossible to do using traditional computation.

This problem has led to the development of non-standard techniques to tackle situations at which biological information processing systems excel. One of the more successful of such developments aims at ‘reverse-engineering’ the biological apparatus itself to find out why and how it works. The field of neural network models has grown up on the premise that the massively parallel distributed processing and connectionist structure observed in the brain is the key behind its superior performance. By implementing these features in the design of a new class of architectures and algorithms, it is hoped that machines will approach human-like ability in handling real-world situations.

The complexity of the brain lies partly in the multiplicity of structural levels of organization in the nervous system. The spatial scale of such structures span about ten orders of magnitude – starting from the level of molecules and synapses, going all the way up to the entire central nervous system (Figure 1).

The unique capabilities of the brain to perform cognitive tasks are an outcome of the collective global behaviour of its constituent neurons. This is the motivation for investigating the network dynamics of model neurons. Depending upon one’s purpose, such ‘neurons’ may be either, extremely simple binary threshold-activated elements, or, described by a set of coupled partial differential equations incorporating detailed knowledge of cellular physiology and action potential propagation. However, both simplifying and realistic neural models are based on the theory of nonlinear dynamical systems in high-dimensional spaces¹. The development of nonlinear dynamical systems theory – in particular, the discovery of ‘deterministic chaos’ in extremely simple systems – has furnished the theoretical tools necessary for analysing non-equilibrium network dynamics. Neurobiological studies indicating the presence of chaotic dynamics in the brain and the investigation of its possible role in biological information processing has provided further motivation.

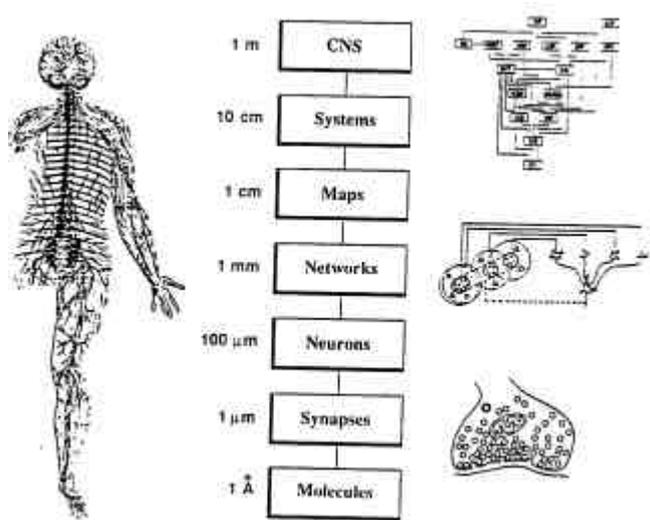


Figure 1. Structural levels of organization of the nervous system (from Churchland and Sejnowski¹).

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Actual networks of neuronal cells in the brain are extremely complex (Figure 2). In fact, even single neurons (Figure 3) are much more complicated than the ‘formal neurons’ usually used in modelling studies, and are capable of performing a large amount of computation^{2,3}. To gain insight into the network properties of the nervous system, researchers have focused on *artificial neural networks*. These usually comprise of binary neurons (i.e. neurons capable of being only in one of two states), $S_i (= \pm 1; i = 1, 2, \dots, N)$, whose temporal evolution is determined by the equation:

$$S_i = F(\sum_j W_{ij} S_j - \mathbf{Q}), \quad (1)$$

where \mathbf{Q} is an internal threshold, W_{ij} is the connection weight from element j to element i , and F is a nonlinear function, most commonly taken as a *sign* or *tanh* (for continuous value S_i) function. Different neural network models are specified by

- network topology, i.e. the pattern of connections between the elements comprising the network;
- characteristics of the processing element, e.g. the explicit form of the nonlinear function F , and the value of the threshold \mathbf{Q} ;
- learning rule, i.e. the rules for computing the connection weights W_{ij} appropriate for a given task, and,
- updating rule, e.g. the states of the processing elements may be updated in parallel (synchronous updating), sequentially or randomly.

One of the limitations of most network models at present



Figure 2. Neuronal network of Purkinje cells in the cerebellum of a hedgehog (image obtained through Golgi staining of neurons). (From <http://weber.u.washington.edu/~chudler/>).

is that they are basically static, i.e. once an equilibrium state is reached, the network remains in that state, until the arrival of new external input⁴. In contrast, real neural networks show a preponderance of dynamical behaviour. Once we recall a memory, our minds are not permanently stuck to it, but can also roll over and recall other associated memories without being prompted by any additional external stimuli. This ability to ‘jump’ from one memory to another in the absence of appropriate stimuli is one of the hallmarks of the brain. It is an ability which one should try to recreate in a network model if it is ever to come close to human-like performance in intellectual tasks. One of the possible ways of simulating such behaviour is through models guided by non-equilibrium dynamics, in particular, chaos. This is because of the much richer dynamical possibilities of such networks, compared to those in systems governed by convergent dynamics⁵.

The focus in this work will be on ‘simple’ network models: ‘simple’ not only in terms of the size of the networks considered when compared to the brain (consisting of $\sim 10^{11}$ neurons and $\sim 10^{15}$ synapses), but ‘simple’ also in terms of the properties of the constituent elements (i.e. the ‘neurons’) themselves, in that, most of the physiological details of real neurons are ignored. The objective is to see and retain what is essential for a particular function performed by the network, treating other details as being of secondary importance for the task at hand. To do that one has to discard as much of the complexity as possible to make the model tractable – while at the same time retaining those features of the system which make it interesting. So, while this kind of modelling is indeed inspired by neuroscience, it is not exclusively concerned with actually mimicking the activity of real neuronal systems.

The Hopfield model

The foundation for computational neural modelling can be traced to the work of McCulloch and Pitts in 1943 on the universal computing capabilities of logic circuits akin to neural nets. However, the interest of physicists was drawn much later, mostly due to the work of Hopfield⁶ who showed the equivalence between the problem of associative

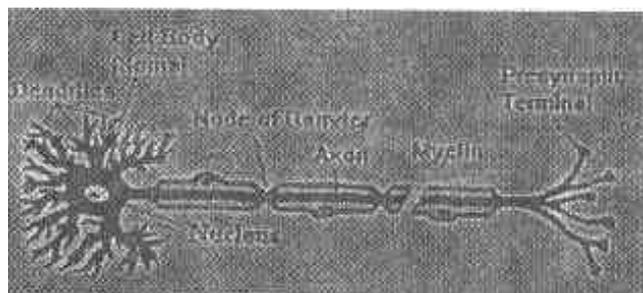


Figure 3. Schematic diagram of a neuron (from <http://www.utexas.edu/research/asrec/neuron.html>).

memory – where, one of many stored patterns has to be retrieved which most closely matches a presented input pattern – and the problem of energy minimization in spin glass systems. In the proposed model, the 2-state neurons, S_i ($i = 1, \dots, N$), resemble Ising spin variables and interact among each other with symmetric coupling strengths, W_{ij} . If the total weighted input to a neuron is above a specified threshold, it is said to be ‘active’, otherwise it is ‘quiescent’. The static properties of the model have been well understood from statistical physics. In particular, the memory loading capacity (i.e. the ratio of the number of patterns, p , stored in the network, to the total number of neurons), $a = p/N$, is found to have a critical value at $a_c \approx 0.138$, where the overlap between the asymptotic state of the network and the stored patterns show a discontinuous transition. In other words, the system goes from having good recall performance ($a < a_c$) to becoming totally useless ($a > a_c$).

It was observed later that dynamically defined networks with asymmetric interactions, W_{ij} , have much better recall performance. In this case, no effective energy function can be defined and the use of statistical physics of spin glass-like systems is not possible. Such networks have, therefore, mostly been studied through extensive numerical simulations. One such model is a Hopfield-like network with a single-step delay dynamics with some tunable weight l :

$$S_i(n+1) = \text{sign} [\sum_j W_{ij}(S_j(n) + l S_j(n-1))]. \quad (2)$$

Here, $S_i(n)$ refers to the state of the i -th spin at the n -th time interval. For $l > 0$, the performance of the model improved enormously over the Hopfield network, both in terms of recall and overlap properties⁷. The time-delayed term seems to be aiding the system in coming out of spurious minimas of the energy landscape of the corresponding Hopfield model. It also seems to have a role in suppressing noise. For $l < 0$, the system shows limit cycle behaviour. These limit-cycle attractors have been used to store and associatively recall patterns⁸. If the network is started off in a state close to one of the stored memories, it goes into a limit cycle in which the overlap of the instantaneous configuration of the network with the particular stored pattern shows large amplitude oscillations with time, while overlap with other memories remains small. The model appears to have a larger storage capacity than the Hopfield model and better recall performance. It also performs well as a pattern classifier if the memory loading level and the degree of corruption present in the input are high.

The travelling salesman problem

To see how collective computation can be more effective than conventional approaches, we can look at an example from the area of combinatorial optimization: the Travelling Salesman Problem (TSP). Stated simply, TSP involves

finding the shortest tour through N cities starting from an initial city, visiting each city once, and returning at the end to the initial city. The non-triviality of the problem lies in the fact that the number of possible solutions of the problem grows as $(N-1)!/2$ with N , the number of cities. For $N = 10$, the number of possible paths is 181,440 – thus, making it impossible to find out the optimal path through exhaustive search (brute-force method) even for a modest value of N . A ‘cost function’ (or, analogously, an energy function) can be defined for each of the possible paths. This function is a measure of the optimality of a path, being lowest for the shortest path. Any attempt to search for the global solution through the method of ‘steepest descent’ (i.e. along a trajectory in the space of all possible paths that minimizes the cost function by the largest amount) is bound to get stuck at some local minima long before reaching the global minima. The TSP has also been formulated and studied on a randomly dilute lattice⁹. If all the lattice sites are occupied, the desired optimal path is easy to find; it is just a Hamilton walk through the vertices. If however, the concentration p of the occupied lattice sites (‘cities’) is less than unity, the search for a Hamilton walk through *only* the randomly occupied sites becomes quite nontrivial. In the limit $p \rightarrow 0$, the lattice problem reduces to the original TSP (in continuum).

A neural network approach to solving the TSP was first suggested by Hopfield and Tank¹⁰. A more effective solution is through the use of Boltzmann machines¹¹, which are recurrent neural networks implementing the technique of ‘simulated annealing’¹². Just as in actual annealing, a material is heated and then made to cool gradually, here, the system dynamics is initially made noisy. This means, that the system has initially some probability of taking up higher energy configurations. So, if the system state is a local optima, because of fluctuations, it can escape a sufficiently small energy barrier and resume its search for the global optima. As the noise is gradually decreased, this probability becomes less and less, finally becoming zero. If the noise is decreased at a sufficiently slow rate, convergence to the global optima is guaranteed. This method has been applied to solve various optimization problems with some measure of success. A typical application of the algorithm to obtain an optimal TSP route through 100 specific European cities is shown in Figure 4 (ref. 13).

Nonequilibrium dynamics and excitatory–inhibitory networks

The Hopfield network is extremely appealing owing to its simplicity, which makes it amenable to theoretical analysis. However, these very simplifications make it a neurobiologically implausible model. For these reasons, several networks have been designed incorporating known biological facts – such as, the Dale’s principle, which states that a neuron has either exclusively excitatory or exclusively inhibitory synapses. In other words, if the i -th neuron is

excitatory (inhibitory), then $W_{ji} > 0$ (< 0) for all j . It is observed that, even connecting only an excitatory and an inhibitory neuron with each other leads to a rich variety of behaviour, including high period oscillations and chaos¹⁴⁻¹⁶. The continuous-time dynamics of pairwise connected excitatory-inhibitory neural populations have been studied before¹⁷. However, an autonomous two-dimensional system (i.e. one containing no explicitly time-dependent term), evolving continuously in time, cannot exhibit chaotic phenomena, by the Poincaré-Bendixson theorem (see e.g. Strogatz¹⁸). Network models updated in discrete time, but having binary-state excitatory and inhibitory neurons, also cannot show chaoticity, although they have been used to model various neural phenomena, e.g. kindling, where epilepsy is generated by means of repeated electrical stimulation of the brain¹⁹. In the present case, the resultant system is updated in discrete-time intervals and the continuous-state (as distinct from a binary or discrete-state) neuron dynamics is governed by a nonlinear activation function, F . This makes chaotic behaviour possible in the model, which is discussed in detail below.

If X and Y be the mean firing rates of the excitatory and inhibitory neurons, respectively, then their time evolution is given by the coupled difference equations:

$$\begin{aligned} X_{n+1} &= F_a(W_{xx} X_n - W_{xy} Y_n), \\ Y_{n+1} &= F_b(W_{yx} X_n - W_{yy} Y_n). \end{aligned} \tag{3}$$

The network connections are shown in Figure 5. The W_{xy} and W_{yx} terms represent the synaptic weights of coupling between the excitatory and inhibitory elements, while W_{xx} and W_{yy} represent self-feedback connection weights. Although a neuron coupling to itself is biologically implausible, such connections are commonly used in neural network models to compensate for the omission of explicit terms for synaptic and dendritic cable delays. Without loss of generality, the connection weightages W_{xx} and W_{yx} can be

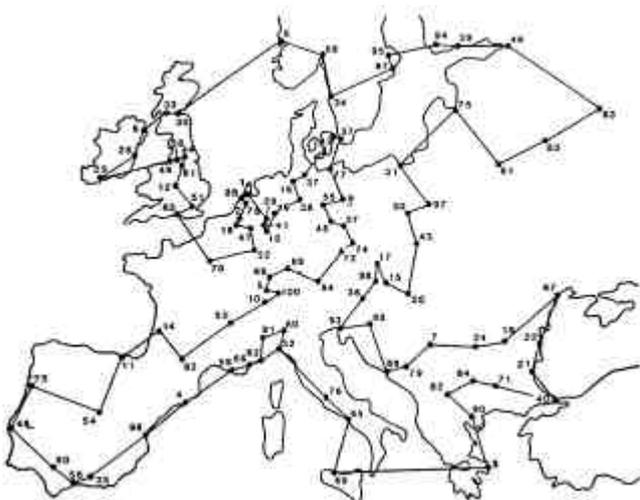


Figure 4. An optimal solution for a 100-city TSP (from Aarts *et al.*¹³).

absorbed into the gain parameters a and b and the correspondingly rescaled remaining connection weightages, W_{xy} and W_{yy} , are labelled k and k' , respectively. For convenience, a transformed set of variables, $z_n = X_n - kY_n$ and $z'_n = X_n - k'Y_n$, is used. Now, if we impose the restriction $k = k'$, then the two-dimensional dynamics is reduced effectively to that of an one-dimensional difference equation (i.e. a 'map'),

$$z_{n+1} = F(z_n) = F_a(z_n) - kF_b(z_n), \tag{4}$$

simplifying the analysis. The dynamics of such a map has been investigated for both piecewise linear and smooth, as well as asymmetric and anti-symmetric, activation functions. The transition from fixed point behaviour to a dynamic one (asymptotically having periodic or chaotic trajectory) has been found to be generic across the different forms of F . Features specific to each class of functions have also been observed. For example, in the case of piecewise linear functions, border-collision bifurcations and multifractal fragmentation of the phase space occur for a range of parameter values¹⁶. Anti-symmetric activation functions show a transition from symmetry-broken chaos (with multiple coexisting but disconnected attractors) to symmetric chaos (when only a single chaotic attractor exists). This feature has been used to show noise-free 'stochastic resonance' in such neural models²⁰, as discussed in the following section.

Stochastic resonance in neuronal assemblies

Stochastic resonance (SR) is a recently observed cooperative phenomena in nonlinear systems, where the ambient noise helps in amplifying a sub threshold signal (which would have been otherwise undetected) when the signal frequency is close to a critical value²¹ (see Gammitoni *et al.*²² for a recent review). A simple scenario for observing such a phenomenon is a heavily damped bistable dynamical system (e.g. a potential well with two minima) subjected to an external periodic signal. As a result, each of the minima is alternately raised and lowered in the course of one complete cycle. If the amplitude of the forcing is less than the barrier height between the wells, the system

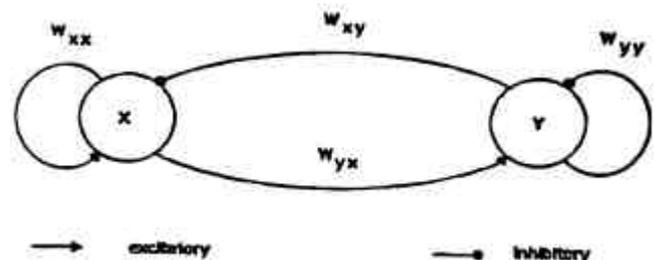


Figure 5. The pair of excitatory (x) and inhibitory (y) neurons. The arrows and circles represent excitatory and inhibitory synapses, respectively.

cannot switch between the two states. However, the introduction of noise can give rise to such switching. This is because of a resonance-like phenomenon due to matching of the external forcing period and the noise-induced (average) hopping time across the finite barrier between the wells, and as such, it is not a very sharp resonance. As the noise level is gradually increased, the stochastic switchings will approach a degree of synchronization with the periodic signal until the noise is so high that the bistable structure is destroyed, thereby overwhelming the signal. So, SR can be said to occur because of noise-induced hopping between multiple stable states of a system, locking on to an externally imposed periodic signal.

These results assume significance in light of the observation of SR in the biological world. It has been proposed that the sensory apparatus of several creatures use SR to enhance their sensitivity to weak external stimulus, e.g. the approach of a predator. Experimental studies involving crayfish mechanoreceptor cells²³ and even, mammalian brain slice preparations²⁴, have provided evidence of SR in the presence of external noise and periodic stimuli. Similar processes have been claimed to occur for the human brain also, based on the results of certain psychophysical studies²⁵. However, in neuronal systems, a non-zero signal-to-noise ratio is found even when the external noise is set to zero²⁶. This is believed to be due to the existence of 'internal noise'. This phenomenon has been examined through neural network modelling, e.g. in Wang and Wang²⁷, where the main source of such 'noise' is the effect of activities of adjacent neurons. The total synaptic input to a neuron, due to its excitatory and inhibitory interactions with other neurons, turns out to be aperiodic and noise-like. The evidence of chaotic activity in neural processes of the crayfish²⁸ suggests that nonlinear resonance due to inherent chaos might be playing an active role in such systems. Such noise-free SR due to chaos has been studied before in a non-neural setting²⁹. As chaotic behaviour is extremely common in a recurrent network of excitatory and inhibitory neurons, such a scenario is not entirely unlikely to have occurred in the biological world. There is also a possible connection of such 'resonance' to the occurrence of epilepsy, whose principal feature is the synchronization of activity among neurons.

The simplest neural model²⁰ which can use its inherent chaotic dynamics to show SR-like behaviour is a pair of excitatory-inhibitory neurons with anti-symmetric piecewise linear activation function, viz. $F_a(z) = -1$, if $z < -1/a$, $F_a(z) = az$, if $-1/a \leq z \leq 1/a$, and $F_a(z) = 1$, if $z > 1/a$. From eq. (4), the discrete time evolution of the effective neural potential is given by the map,

$$z_{n+1} = F(z_n + I_n) = F_a(z_n + I_n) - kF_b(z_n + I_n),$$

where I is an external input. The design of the network ensures that the phase space $[-1 + (kb/a), 1 - (kb/a)]$ is divided into two well-defined and segregated sub-intervals L: $[-1 + (kb/a), 0]$ and R: $[0, 1 - (kb/a)]$. For $a < 4$, there is no dynamical connection between the two sub-intervals and the trajectory, while chaotically wandering over one of the sub-intervals, cannot enter the other sub-interval. For $a > 4$, in a certain range of (b, k) values, the system shows both symmetry-broken and symmetric chaos, when the trajectory visits both sub-intervals in turn. The chaotic switching between the two sub-intervals occurs at random. However, the average time spent in any of the sub-intervals before a switching event, can be exactly calculated for the present model as

$$\langle n \rangle = \frac{1}{bk \left(1 - \frac{bk}{a}\right) - 1}. \quad (5)$$

As a complete cycle would involve the system switching from one sub-interval to the other and then switching back, the 'characteristic frequency' of the chaotic process is $\nu = 1/(2\langle n \rangle)$. For example, for the system to have a characteristic frequency of $\nu = 1/400$ (say), the above relation provides the value of $k \approx 1.3811$ for $a = 6$, $b = 3.42$. If the input to the system is a sinusoidal signal of amplitude A and frequency $\sim \nu$, we can expect the response to the signal to be enhanced, as is borne out by numerical simulations. The effect of a periodic input, $I_n = A \sin(2\pi \nu n)$, is to translate the map describing the dynamics of the neural pair, to the left and right, periodically. The presence of resonance is verified by looking at the peaks of the residence time distribution³⁰, where the strength of the j -th peak is given by

$$P_j = \int_{n_j - a n_0}^{n_j + a n_0} N(n) dn \quad (0 < a < 0.25). \quad (6)$$

For maximum sensitivity, a is set as 0.25. As seen in Figure 6, the dependence of P_j ($j = 1, 2, 3$) on external signal frequency, ν exhibits a characteristic non-monotonic profile, indicating the occurrence of resonance at $\nu \approx 1/(2\langle n \rangle)$. For the system parameters used in the simulation, $\langle n \rangle = 200$. The results clearly establish that the switching between states is dominated by the sub-threshold periodic signal close to the resonant frequency. This signal enhancement through intrinsic dynamics is an example of how neural systems might use noise-free SR for information processing.

Formation of neural assemblies via activity synchronization

Dynamical transitions leading to coherence in brain activity, in the presence of an external stimulus, have received considerable attention recently. Most investigations of these phenomena have focussed on the phase synchronization of oscillatory activity in neural assemblies. An example is the detection of synchronization of '40 Hz' oscillations within and between visual areas and between cerebral hemispheres of cats³¹ and other animals. Assemblies of neurons have been observed to form and separate depending on the stimulus. This has led to the speculation that, phase synchronization of oscillatory neural activity is the mechanism for 'visual binding'. This is the process by which local stimulus features of an object (e.g. colour, motion, and shape), after being processed in parallel by different (spatially separate) regions of the cortex, are correctly integrated in higher brain areas, forming a coherent representation ('gestalt').

Recent neurobiological studies³² have shown that many cortical neurons respond to behavioural events with rapid modulations of discharge correlation. Epochs with a particular correlation may last from $\sim 10^{-2}$ to 10 secs. The observed modulation of correlations may be associated with changes in the individual neuron's firing rates. This supports the notion that a single neuron can intermittently participate in different computations by rapidly changing its coupling to other neurons, without associated changes in firing rate. The mechanisms of such dynamic correlations are unknown. The correlation could probably arise from changes in the pattern of activity of a large number of neurons, interacting with the sampled neurons in a correlated manner. This modification of correlations between two neurons in relation to stimulation and behaviour most probably reflects changes in the organi-

zation of spike activity in larger groups of neurons. This immediately suggests the utilization of synchronization by neural assemblies for rapidly forming a correlated spatial cluster. There are indeed indications that such binding between neurons occurs and the resultant assemblies are labelled by synchronized firing of the individual elements with millisecond precision, often associated with oscillations in the so-called gamma-frequency range, centered around 40 Hz.

Mostly due to its neurobiological relevance as described above, the synchronization of activity has also been investigated in network models. In the case of the excitatory-inhibitory neural pair described before, even $N = 2$ or 3 such pairs coupled together give rise to novel kinds of collective behaviour¹⁵. For $N = 2$, synchronization occurs for both unidirectional and bidirectional coupling, when the magnitude of the coupling parameter is above a certain critical threshold. An interesting feature observed is the intermittent occurrence of desynchronization (in 'bursts') from a synchronized situation, for a range of coupling values. This intermittent synchronization is a plausible mechanism for the fast creation and destruction of neural assemblies through temporal synchronization of activity. For $N = 3$, two coupling arrangements are possible for both unidirectional and bidirectional coupling: *local coupling*, where nearest neighbours are coupled to each other, and *global coupling*, where the elements are coupled in an all-to-all fashion. In the case of bidirectional, local coupling, we observe a new phenomenon, referred to as *mediated synchronization*. The equations governing the dynamics of the coupled system are given by:

$$z_1^{n+1} = (z_1^n + l z_2^n),$$

$$z_2^{n+1} = (z_2^n + l [z_1^n + z_3^n]),$$

$$z_3^{n+1} = F(z_3^n + l z_2^n).$$

For the set of activation parameters $a = 100$, $b = 25$ (where F is of anti-symmetric, sigmoidal nature), we observe the following feature over a range of values of the coupling parameter, l : the neural pairs, z_1 and z_3 which have no direct connection between themselves synchronize, although z_2 synchronizes with neither. So, the system z_2 appears to be 'mediating' the synchronization interaction, although not taking part in it by itself. This is an indication of how long-range synchronization might occur in the nervous system without long-range connections.

For a global, bidirectional coupling arrangement, the phenomenon of 'frustrated synchronization' is observed. The phase space of the entire coupled system is shown in Figure 7. None of the component systems is seen to be synchronized. This is because the three systems, each trying to synchronize the other, frustrate all attempts at collective synchronization. Thus, the introduction of structural disorder in chaotic systems can lead to a kind of

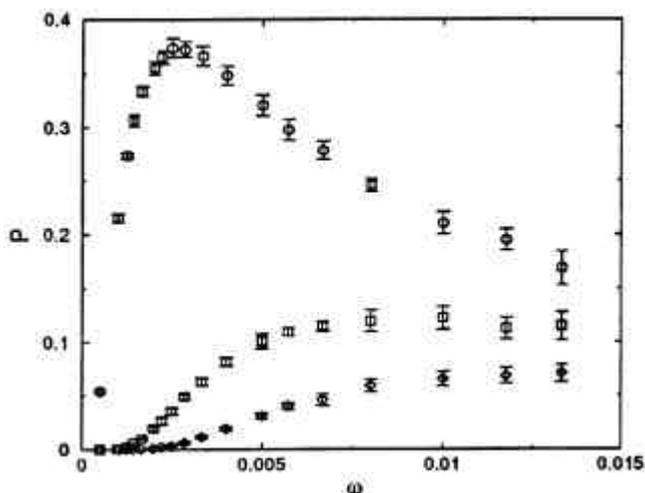


Figure 6. Peak strengths of the normalized residence time distribution, P_1 (circles), P_2 (squares) and P_3 (diamonds), for periodic stimulation of the excitatory-inhibitory neural pair ($a = 6$, $b = 3.42$ and $k = 1.3811$). Peak amplitude of the periodic signal is $d = 0.0005$. P_1 shows a maximum at a signal frequency $\omega \approx 1/400$. Averaging is done over 18 different initial conditions, the error bars indicating the standard deviation.

'frustration'³³, similar to that seen in the case of spin glasses. These features were of course studied for very small systems ($N=2$ or 3), where all the possible coupling arrangements could be checked. For larger N values, the set of such combinations quickly becomes a large one, and was not checked systematically. We believe, however, that the qualitative behaviour remains unchanged.

Image segmentation in an excitatory–inhibitory network

Sensory segmentation, the ability to pick out certain objects by segregating them from their surroundings, is a prime example of 'binding'. The problem of segmentation of sensory input is of primary importance in several fields. In the case of visual perception, 'object-background' discrimination is the most obvious form of such sensory segmentation: the object to be attended to, is segregated from the surrounding objects in the visual field. This process is demonstrated by dynamical transitions in a model comprising excitatory and inhibitory neurons, coupled to each other over a local neighbourhood. The basic module of the proposed network is a pair of excitatory and inhibitory neurons coupled to each other. As before, imposing restrictions on the connection weights, the dynamics can be simplified to that of the following one-dimensional map:

$$z_{n+1} = F_a(z_n + I_n) - kF_b(z_n + I'_n), \quad (7)$$

where the activation function F is of asymmetric, sigmoidal nature:

$$F_a(z) = 1 - e^{-az}, \text{ if } z > 0, \\ = 0, \text{ otherwise.}$$

Without loss of generality, we can take $k = 1$. In the

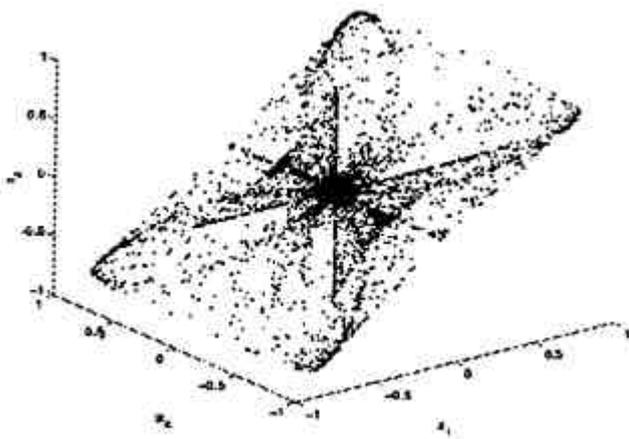


Figure 7. Frustrated synchronization: Phase space for three bidirectional, globally coupled neural pairs (z_1, z_2, z_3) with coupling magnitude $l = 0.5$ ($a = 100, b = 5$ for all the pairs).

following, only time-invariant external stimuli will be considered, so that:

$$I_n = I'_n = I.$$

The autonomous behaviour (i.e. $I, I' = 0$) of the isolated pair of excitatory–inhibitory neurons show a transition from fixed point to periodic behaviour and chaos with the variation of the parameters a, b , following the 'period-doubling' route, universal to all smooth, one-dimensional maps. The introduction of an external stimulus of magnitude I has the effect of horizontally displacing the map to the left by I , giving rise to a reverse period-doubling transition from chaos to periodic cycles to finally, fixed-point behaviour. The critical magnitude of the external stimulus which leads to a transition from a period-2 cycle to fixed point behaviour is given as³⁴.

$$I_c = \frac{1 - \frac{2}{m}}{(m)^{1/m} - (a/m)} + \frac{1}{m} [\ln(m) - 1]. \quad (8)$$

To make the network segment regions of different intensities ($I_1 < I_2$, say), one can fix m and choose a suitable a , such that $I_1 < I_c < I_2$. So elements, which receive input of intensity I_1 , will undergo oscillatory behaviour, while elements receiving input of intensity I_2 , will go to a fixed-point solution.

The response behaviour of the excitatory–inhibitory neural pair, with local couplings, has been utilized in segmenting images and the results are shown in Figure 8. The initial state of the network is taken to be totally random. The image to be segmented is presented as external input to the network, which undergoes 200–300 iterations. Keeping a fixed, a suitable value of m is chosen from a consideration of the histogram of the intensity distribution of the image. This allows the choice of a value for the critical intensity (I_c), such that, the neurons corresponding to the 'object' converge to fixed-point behaviour, while those belonging to the 'background' undergo period-2 cycles. In practice, after the termination of the specified number of iterations, the neurons which remain unchanged over successive iterations (within a tolerance value) are labelled as the 'object', the remaining being labelled the 'background'.

The image chosen is that of a square of intensity I_2 (the object) against a background of intensity I_1 ($I_1 < I_2$). Uniform noise of intensity ϵ is added to this image. The signal-to-noise ratio is defined as the ratio of the range of grey levels in the original image to the range of noise added (given by Θ). Figure 8 shows the results of segmentation for unit signal-to-noise ratio. Figure 8a shows the original image while segmentation performance of the uncoupled network is presented in Figure 8b. As is clear from the figure, the isolated neurons perform poorly in identifying the

‘background’ in the presence of noise. The segmentation performance improves remarkably when spatial interactions are included in the model. We have considered discrete approximations of circular neighbourhoods of excitatory and inhibitory neurons with radii r_{ex} and $r_{in}(r = 1, 2)$, respectively, in our simulations.

Results for $r_{ex} = 1$, $r_{in} = 2$ and $r_{ex} = r_{in} = 2$ are shown in Figure 8 *c, d* respectively. The two architectures show very similar segmentation results, at least up to the iterations considered here, although the latter is unstable. Excepting for the boundary of the ‘object’, which is somewhat broken, the rest of the image has been assigned to the two different classes quite accurately. More naturalistic images have also been considered, such as a 5-bit ‘Lincoln’ image, and satisfactory results have been obtained³⁴. Note that, a single value of a (and hence I_c) has been used for the entire image. This is akin to ‘global thresholding’. By implementing local thresholding and choosing a on the basis of local neighbourhood information, the performance of the network can be improved.

Outlook

We have pointed out some of the possible uses of dynamical transitions in a class of network models of computation, namely excitatory–inhibitory neural networks updated at discrete time-intervals. Dynamics however plays an important role in a much broader class of systems implementing collective computation – cellular automata³⁵, lattices of coupled chaotic maps³⁶, ant-colony models³⁷, etc. Other examples may be obtained from the ‘Artificial Life’³⁸ genre of models. However, even in the restricted region that we have focused on, several important issues are yet to be addressed.

One important point not addressed here is the issue of

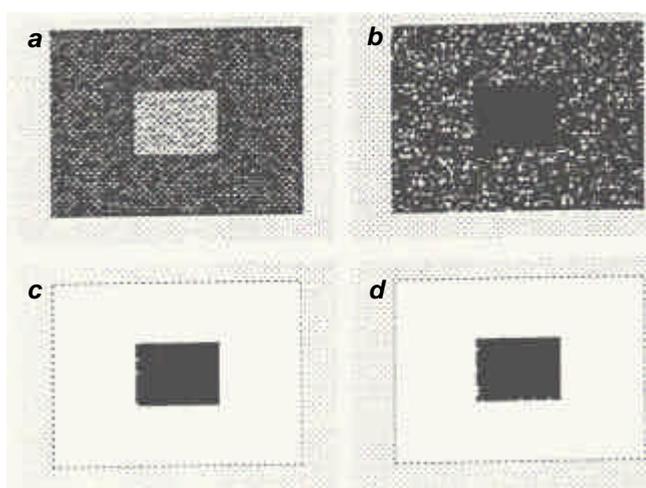


Figure 8. Results of implementing the proposed segmentation method on noisy synthetic image: *a*, original image; *b*, output of the uncoupled network; *c*, output of the coupled network ($r_{ex} = 1$, $r_{in} = 2$); and *d*, output of the coupled network ($r_{ex} = r_{in} = 2$), after 200 iterations ($a = 20$, $b/a = 0.25$ and tolerance = 0.02).

learning. The connection weights $\{W_{ij}\}$ have been assumed constant, as they change at a much slower time scale compared to that of the neural activation states. However, modification of the weights due to learning will also cause changes in the dynamics. Such bifurcation behaviour, induced by weight changes, will have to be taken into account when devising learning rules for specific purposes. The interaction of chaotic activation dynamics at a fast time scale and learning dynamics on a slower time scale might yield richer behaviour than that seen in the present models. The first step towards such a programme would be to incorporate time-varying connection weights in the model. Such time-dependence of a system parameter has been shown to give rise to interesting dynamical behaviours, e.g. transition between periodic oscillations and chaos. This suggests that varying the environment can facilitate memory retrieval if dynamic states are used for storing information in a neural network. The introduction of temporal variation in the connection weights, independent of the neural state dynamics, should allow us to develop an understanding of how the dynamics at two time-scales interact with each other.

Parallel to this, one has also to look at the *learning dynamics* itself. Freeman³⁹, among others, has suggested an important role of chaos in the Hebbian model of learning⁴⁰. This is one of the most popular learning models in the neural network community and is based on the following principle postulated by Hebb⁴⁰ in 1949:

When an axon of cell A is near enough to excite cell B and repeatedly or consistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A’s efficiency, as one of the cells firing B, is increased.

According to the principle known as *synaptic plasticity*, the synapse between neurons A and B increase its ‘weight’, if the neurons are simultaneously active. By invoking an ‘adiabatic approximation’, we can separate the time scale of updating the connection weights from that of neural state updating. This will allow us to study the dynamics of the connection weights in isolation.

The final step will be to remove the ‘adiabatic approximation’, so that the neural states will evolve, guided by the connection weights, while the connection weights themselves will also evolve, depending on the activation states of the neurons, as:

$$W_{ij}(n + 1) = F_e(W_{ij}(n), X_i(n), X_j(n)),$$

where $X(n)$ and $W(n)$ denote the neuron state and connection weight at the n -th instant, F is a nonlinear function that specifies the learning rule, and \ominus is related to the time-scale of the synaptic dynamics. The cross-level effects of such synaptic dynamics interacting with the chaotic network dynamics might lead to significant departure from the overall behaviour of the networks

described here. However, it is our belief that, network models with non-equilibrium dynamics are not only more realistic⁴¹ in the neurobiological sense, as compared to the models with fixed-point attractors (such as, the Hopfield network⁶), but also hold much more promise in capturing many of the subtle computing features of the brain.

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