Pramāṇa – J. Phys., Vol. 25, No. 4, October 1985, pp. 343–347. © Printed in India.

Symmetry breaking, phase transition and gravity†

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Abstract. We discuss some recent work in which the non-minimal coupling of gravity with a self-interacting scalar field in the presence of matter can lead to a phase transition when the sign of gravitational interaction changes. It is found that gravity becomes repulsive above a critical temperature which may lie in the range 10^{24} to 10^{32}K which obtains in the very early universe (10^{-35} to 10^{-43} sec) of the standard model. The results are intimately connected with big bang and possible removal of singularity.

Keywords. Symmetry-breaking; phase transition; gravity.

1. Introduction

According to the standard model of cosmology, the universe started with a bang from a superdense and superhot stage at some initial time. In fact at time $t = 0$, we have an initial singularity, when both matter energy density (ρ) and temperature (T) are infinite. Although the observed expansion of the universe (seen through red shift of light coming from distant galaxies) and the presence of uniform 3K microwave radiation give strong support to the model, there have been a few serious difficulties (Guth 1981; Sinha 1983; Linde 1984). In addition to the singularity problem, the other notable ones are: (i) overproduction of magnetic monopoles (mass $\sim 10^{16}$GeV) (ii) the flatness problem wherein the early age of the universe ($t \sim 10^{-43}$sec) the ratio of the energy density ρ to ρ_c (which corresponds to a flat universe) is extremely close to unity. In fact at this stage $|\rho - \rho_c|/\rho_c \approx 10^{-59}$. This is related with the question why the entropy S of the observable universe now $S \sim 10^{87}$. (iii) The horizon or the causality problem concerns the question how two regions of the universe separated by several horizon lengths (ct) are at the same temperature and matter distribution. Why is the universe homogeneous and isotropic at length scale $l \gg ct$? (iv) This concerns the formation of structures in the universe e.g. galaxies, clusters of galaxies and superclusters. Density perturbation $\delta\rho/\rho \sim 10^{-4}$ should have existed in the early universe to account for the observed structure now. The problem is to pinpoint the source of this specific inhomogeneity. There are a host of other problems (see Linde 1984). New models of inflationary universe have been developed in recent years which seem to resolve some of these longstanding problems (Guth 1984; Linde 1982, 1984; Albrecht and Steinhardt 1982). The essential idea in these models is the assumption that at a very early stage ($t \sim 10^{-35}$ sec) the universe was in an unstable vacuum state having large energy

† Based on the talk given at the International Symposium on Theoretical Physics, Bangalore, November 1984.

density $\rho = v(0)$. Under this condition Einstein's equations will permit an exponential expansion of the scale factor, namely, $a(t) \sim a_0 \exp(Ht)$, where H is the Hubble constant at that time

$$H = (8\pi\, v(0)/3M_p^2)^{1/2} \approx 10^{10}\,\text{GeV},$$

$M_p \sim 10^{19}$ GeV being the Planck mass. Starting with the size $a_0 \sim 10^{-20}$ cm, a becomes of the order of 10^{800} cm in the time interval $\Delta t \sim 2 \times 10^{-31}$ sec. During and after this enormous exponential expansion, the concomitant phase transaction to the true vacuum state $V(\phi_0) = 0$, the universe becomes hot and the scale factor now grows as $a(t) \sim t^{1/2}$.

The observable universe ($l \sim 10^{28}$ cm) thus lies inside on bubble of size 10^{800} cm. In one sweep, this scenario and its further improvements seem to explain monopole, flatness, horizon and other problems (see Linde 1984). However, there are serious limitations to these models. These lean heavily on flat-space finite temperature quantum field theory particularly Coleman-Weinberg (C-W) effective potential. The neglect of the space-time curvature and coupling of the scalar field to the background gravitational field is not justified. Concern of this aspect has also been expressed by others (see for example Hu 1984). When one takes into account this coupling, symmetry restoration is inhibited and the potential may not be of C-W type. Thus the conclusions of the new inflationary model are suspect.

The crucial problem is the singularity itself. It is expected that a solution of this may resolve other problems as well e.g. flatness, horizon, etc.

In what follows, we present the highlights of a model which considers the coupling of the scalar field with curvature and background gravitation field (Sathyaprakash *et al* 1984; Novello 1982; Padmanabhan 1983).

2. Model Lagrangian and some results

We consider the Lagrangian density

$$\mathscr{L} = \mathscr{L}_{\phi G} + \mathscr{L}_{\phi\phi} + \mathscr{L}_G + \mathscr{L}_M, \tag{1}$$

where

$$\mathscr{L}_{\phi G} = \sqrt{-g}\,(\partial_\mu\phi\, \partial^\mu\phi - \tfrac{1}{6} R\phi^2), \tag{2}$$

is the Lagrangian density of the free scalar field (ϕ) with non-minimal ($\frac{1}{6} R\phi^2$) interaction with gravity (R is the curvature scalar).

$$\mathscr{L}_{\phi\phi} = \sqrt{-g}\,\alpha^2 (\phi^2 - \mu^2)^2 \tag{3}$$

is the self-interaction of the scalar field, α^2 and μ^2 being coupling coefficients. This will lead to symmetry-breaking effects.

$$\mathscr{L}_G = \sqrt{-g}\kappa^{-1} R \tag{4}$$

is the usual Einstein Lagrangian density of gravity; k is the gravitational constant.

$\mathscr{L}_M$ is the Lagrangian density of matter and radiation (e.g. fundamental fermions and bosons). It is proposed that the coefficient μ^2 is temperature (T)-dependant. Explicitly, we take (Sathyaprakash *et al* 1984),

$$\mu^2(T) = a^2 T^2, \tag{5}$$

where a is a constant. The field equations for the scalar field of gravity are obtained by the variation of the action $A = \int \mathscr{L}\, d^4x$ with respect to ϕ, $\partial_\mu \phi$ and $g_{\mu\nu}$, etc. For the scalar field, the appropriate equation is (Sathyaprakash *et al* 1984)

$$\Box \phi + \partial V/\partial \phi = 0, \tag{6}$$

with

$$V(\phi) = \alpha^2 (\tfrac{1}{6}\kappa\mu^2 - 1)\left(\frac{\phi^4}{2} - \mu^2\phi^2\right). \tag{7}$$

The vacuum expectation values $\phi = 0, \pm\mu$ are extrema of the effective potential $V(\phi)$ (ref. eq. (7)). It is found that when $\kappa\mu^2 < 6$, V is minimum at $\phi = 0$, and maximum at $\phi = \pm\mu$. On the other hand when $\kappa\mu^2 > 6$, V is maximum at $\phi = 0$, and minimum at $\phi = \pm\mu$. Thus as $\kappa\mu^2$ increases the symmetry $\phi \to -\phi$ is spontaneously broken when $\kappa\mu^2 \geqslant 6$. We will see later that this would lead to a change in sign of the gravitational constant. The equation for gravity at $\phi = \phi_0$ (one of the extrema of V) turns out to be (Satyaprakash *et al* 1984)

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R - \Lambda_{\text{eff}}\, g_{\mu\nu} = -\kappa_{\text{eff}}\, T_{\mu\nu}, \tag{8}$$

where λ_{eff} and κ_{eff} are the effective cosmological and gravitational constants. Let us look at the solutions at $\phi_0 = \pm\mu$ and $\kappa\mu^2 > 6$. We have

$$\kappa_{\text{eff}} = \kappa/(1 - \kappa\,\mu^2/6). \tag{9}$$

In this domain κ_{eff} is negative suggesting repulsive gravity. Translated into a dependence on temperature via the relation $\mu^2 = a^2 T^2$, this becomes $(T > T_c)$

$$\kappa_{\text{eff}} = -\kappa \Bigg/ \left(\frac{T^2}{T_c^2} - 1\right), \tag{10}$$

where $T_c^2 = 6/a^2\kappa$ is the critical temperature for the phase transition from repulsive to attractive gravity. For $a^2 = 1/12$, the critical temperature is of the order of the Planck temperature ($\sim 10^{32}$ K). In standard cosmological model both temperature and density increase as we go backward in time. Thus the above analysis would predict that above a critical temperature (or density), gravity will become repulsive. One can show that below a critical radius also repulsive gravity will set in. This mechanism may be the cause of big bang whose effects we are seeing now. With the expansion and rapid fall in temperature when we are in the region $\kappa\mu^2 < 6$, the gravitational constant $\kappa_{\text{eff}} = \kappa$, becomes a true constant and equals its present positive value and gravitational interaction is attractive.

The occurrence of repulsive gravitational interaction above a critical temperature T_c appears to be a fairly general feature. This seems possible for different choices of self-interaction for the scalar field. For example if we take for $\mathscr{L}_{\phi\phi}$ the form (Sathyaprakash *et al* 1984)

$$\mathscr{L}_{\phi\phi} = \sqrt{-g}\,(\Lambda\kappa^{-1} - \sigma\phi^4), \tag{11}$$

where σ is a coupling constant Λ the cosmological constant, then the corresponding potential

$$V(\phi) = \tfrac{1}{2}\sigma\phi^4 - \frac{\Lambda}{6}\phi^2 \tag{12}$$

has minimum at

$$\phi_0 = \pm\left(\frac{\Lambda}{6\sigma}\right)^{1/2} \tag{13}$$

assuming, of course, that $\Lambda/\sigma > 0$. The potential is maximum for $\phi = 0$. The symmetry $\phi \to -\phi$ is again broken for (13). For this case we have

$$\Lambda_{\text{eff}} = \Lambda, \tag{14}$$

$$\kappa_{\text{eff}} = \kappa \Big/ \left(1 - \frac{\Lambda\kappa}{36\sigma}\right). \tag{15}$$

A temperature dependent Λ which starts from a very large value and falls to negligible value as the universe expands and cools will stradle the phase transition from repulsive to attractive gravity.

To give a rough idea, the temperature and time dependence can be taken as (Sinha *et al* 1976)

$$\Lambda \sim \frac{1}{t^2} \text{ i.e. } \Lambda(T) = bT^4,$$

where b is a constant having the appropriate dimension. The present value of Λ at $T = 3$ K, is 10^{-56} cm^{-2}. This would fix $b \sim 10^{-58}$. Now (15) can be recast as

$$\kappa_{\text{eff}} = \kappa \Big/ \left(1 - \frac{T^4}{T_c^4}\right), \tag{16}$$

where $T_c = (36\sigma/\kappa b)^{1/4}$. Now $\kappa \sim 10^{-49}$ (in appropriate units) and σ can range from 10^{-2} to 10^{-12} (Linde 1984). Thus the critical temperature above which κ_{eff} can be negative will range from 10^{24} to 10^{27} K.

The rough estimate for the two choices shows that the critical temperature for transition from repulsive to attractive gravity lies in the range 10^{32}K to 10^{24}K, and this change would have occurred in the very early universe at time $t = 10^{-43}$ sec to 10^{-35} sec.

3. Concluding remarks

In the foregoing, we have discussed the highlights of our recent work (Sathyaprakash *et al* 1984) where gravity can become repulsive under some extreme conditions. This arises from a non-minimal coupling of a self-interacting scalar field with curvature. It is found that above a critical temperature the effective gravitational interaction constant can become negative. This temperature may lie in the range 10^{24} to 10^{32}K which obtains in the very early universe (10^{-35} to 10^{-43} sec) of the standard model. The appearance of repulsive gravity may provide the cause of the big bang.

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