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The dynamics of a mode locked and frequency-doubled laser

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Abstract. The theory for the transient build-up of mode locked pulses in a mode-locked and frequency-doubled laser is presented. The time required for the mode-locked pulsewidth to reach a steady-state value is computed. It is found that steady-state is reached faster in the presence of internal frequency doubling because of the broadening effect of the doubling crystal. The effects of different conversion efficiencies and modulation depths on the mode-locked pulsewidth are investigated and the results are graphically presented.

Keywords. Laser; mode lockimg; frequency doubling; dynamics.

1. Introduction

The CW mode locked internally frequency-doubled Nd:YAG laser is important in satellite optical communication systems, laser radars, underwater communication systems and other similar devices. In the absence of internal frequency conversion the mode-locked Nd:YAG laser emits optical pulses at wavelength 1·06 μm with high repetition rates (~50 MHz), narrow pulsewidth (~100 picosec) and moderate powers (~1 W average). Since most of the optical detectors have poor response at this wavelength (and optimum response in the visible region) this laser can only have a limited application. However, the mode locked Nd:YAG laser can be put to a wider use by converting the output wavelength to 0·53 μm by second harmonic generation. Because the pulse peak-intensity in the output beam is too low to permit efficient second harmonic generation outside the cavity, it is found convenient to employ intracavity second harmonic generation with the harmonic crystal placed inside the laser cavity (Hitz and Osterink, 1971; Rice and Burkhardt 1971). Recently a synchronously mode locked CW dye laser has been operated to produce efficiently continuous trains of picosecond UV pulses by intra-cavity frequency doubling (Yamashita *et al* 1980; Welford *et al* 1980). This laser has been employed to study fast photochemical reactions of molecules (Taylor *et al* 1979).

The pulse characteristics of the mode locked and frequency-doubled laser (MLFD) were considered by many authors. Falk (1975) analysed the MLFD laser in the time domain by following the self-consistent approach of Kuizenga and Siegman (1970) for mode-locked lasers, where the steady-state pulse properties were described in terms of the system parameters. Kennedy (1975) gave a similar analysis in the frequency domain. Prior to the studies of Siegman and Heritier (1980) the analyses of MLFD laser have used approximations such as assuming a gaussian mode locked pulse-shape and then manipulating the parameters of the gaussian pulse to obtain a self-consistent steady-state situation. Siegman and Heritier suggested that the gaussian pulse-shape in a MLFD laser is not an adequate approximation and hence investigated the problem using complex computer techniques. However, we have recently found (Reddy 1982) that the gaussian approximation predicts steady-state pulse-widths comparable to those predicted by the exact computer calculations of Siegman and Heritier. Also, assuming the pulse-shape to be gaussian, Yamashita *et al* (1980) found good agreement between theoretical and experimental results in the case of a synchronously mode locked frequency-doubled laser. Hence in the present analysis we use the approximation that the pulse-shape in the MLFD laser is gaussian.

Earlier studies have mainly confined to steady-state pulse evolution. However, recently the transient evolution of mode locked pulses inside an MLFD laser has been studied (Reddy 1982). In the present paper we investigate the time-dependance of the pulse-width and the pulse-peak intensity in the MLFD laser. We have earlier (Reddy 1982) treated the gain of the laser medium as time-independent and were able to exactly solve the mode locking equation. However, in practice, the laser gain is dependent upon the power in the cavity which in turn is time-dependent. Hence we must re-compute the gain, which depends upon the pulse peak-intensity, after each round-trip of the mode-locked pulse inside the laser cavity. To investigate the problem we follow the approach of Mathieu and Weber (1971) and use Falk's (1975) theory of the MLFD laser.

In § 2 we derive the basic equations for the mode locked pulsewidth, peak intensity and amplitude of the laser medium. An optical pulse having a gaussian shape is assumed to be circulating inside the laser cavity and every time it passes through the laser medium, the active modulator and the second harmonic generation (SHG) crystal, the pulse parameters undergo a small change. Falk (1975) showed that the electric field envelope of the fundamental frequency, emerging from an SHG crystal suffered both decrease in amplitude and broadening in pulse duration; the pulse envelope is however shown to maintain its basic shape. The active medium also contributes to the broadening of the pulse because of the limited laser band-width. A steady state is reached when the broadening effect due to the SHG crystal and the laser active medium is balanced by the pulse-compression effect of the active modulator. The basic equations derived in § 2 represent the changes of pulsewidth, peak intensity and amplitude gain for every round-trip of a pulse inside the cavity. These equations help us to compute the exact number of round-trips the pulse takes to reach the steady-state value.

In § 3 we look at the transient pulse evolution by treating gain as a constant independent of time and pulse intensity and get an exact expression for the pulse-width.

In § 4 we discuss the numerical technique for solving the three basic equations as well as the details of choosing the initial values for starting the numerical computations.

2. Basic equations

In this section the approach followed is that of Washio *et al* (1977). The temporal variation of the pulse shape after N-cavity round-trips is given by

$$I_N(t) = I_{PN} \exp[-2\,\alpha_N t^2], \tag{1}$$

where I_{PN} is the peak-intensity of the pulse and the duration of the pulse is given by

$$\tau_{PN} = (2 \ln 2/\alpha_N)^{1/2}. \tag{2}$$

The laser medium is assumed to be homogeneously broadened. The amplitude gain for laser has a Lorentzian line-shape of the form exp $[g_0/(1 + 2j(\omega - \omega_a)/\Delta\omega)]$ from which we can write the frequency domain transfer functions of the laser as

$$g(\omega) = e^g \exp\left[\frac{-4g}{(\Delta\omega)^2}(\omega - \omega_a)^2\right], \tag{3}$$

where g is the saturated amplitude gain coefficient at the line centre ω_a corresponding to one complete round-trip in the cavity and $\Delta\omega$ is the amplifier linewidth.

The time-domain transfer function of the active AM modulator is assumed to be

$$T(t) = \exp(-\delta^2\,\omega_m^2\,t^2), \tag{4}$$

where δ is the modulation depth and ω_m is the modulation frequency in radians $\sec^{-1}$.

A gaussian pulse $I_N(t) = I_{PN}\exp(-2\alpha_N t^2)$ travelling through a SHG crystal, after having suffered a decrease in amplitude and broadening in pulse-duration, emerges in the form,

$$I_{N_{\text{SHG}}}(t) = I_{PN}\exp(-a)\exp[-2\alpha_N b t^2]; \tag{5}$$

Here a and b are constants defined as

$$a = (\sqrt{2}/2)\alpha_{\text{SHG}}, \quad b = 1 - \sqrt{2}\,\alpha_{\text{SHG}} \tag{6}$$

where α_{SHG} is the second harmonic conversion efficiency.

For every transit of the pulse through the active medium, the AM modulator and the SHG crystal, the pulse parameters I_{PN} and α_N undergo a small change. While the AM modulator compresses the pulse both the gain media and the SHG crystal broaden the pulse duration. Following an earlier analysis (Washio *et al* 1977 and the references cited therein), the pulse-width, peak intensity and the amplitude gain of the laser medium after $(N + 1)$ round trips in the cavity are expressed in terms of their corresponding values after N round-trips:

$$I_{PN+1} = I_{PN}\left(1 + 2g_N - (\alpha_0 + \alpha_1) - \sqrt{2}\,\alpha_{\text{SHG}} - \frac{32 \ln 2}{\Delta\omega^2}\frac{g_N}{\tau_N^2}\right), \tag{7}$$

$$\tau_{N+1} = \tau_N\left(1 + \frac{16(\ln 2)}{\Delta\omega^2\,\tau_N^2}\,g_N + \sqrt{2}\,\alpha_{\text{SHG}} - \frac{\delta^2\,\omega_m^2}{4 \ln 2}\,\tau_N^2\right) \tag{8}$$

$$g_{N+1} = g_N\left(1 - \tfrac{1}{4}(\pi/\ln 2)^{1/2}\frac{\tau_N}{\tau_f} I_{PN}\right) \tag{9}$$

where I_{PN} now is normalized with gain saturation intensity I_s; τ_f is the fluorescence lifetime, α_0 is the linear cavity loss; α_1 is the linear insertion loss of the SHG crystal.

In (8) the second and third terms in the brackets show the pulse-broadening effect of the gain medium and the SHG crystal respectively. The fourth term shows the pulse-narrowing effect of the active AM modulator. In (7) the fourth term in brackets shows the loss due to second harmonic conversion by the crystal and the fifth term shows the power-dependent-gain decrease attributed to a spectrally-broadened short pulse. The gradual decrease in the laser medium gain during the pulse build-up is shown in (9).

3. Transient pulse evolution

Since the net change in τ per round-trip inside the laser cavity is very small, we can rewrite (8) in the form

$$\frac{\mathrm{d}\tau}{\mathrm{d}t} = \left(\frac{16\,(\ln 2)}{\Delta\omega^2\,\tau^2}g + \sqrt{2}\,\alpha_{\mathrm{SHG}} - \frac{\delta^2\,\omega_m^2}{4\ln 2}\,\tau^2\right)\frac{c}{2L}, \tag{10}$$

where c is the velocity of light in vacuum and L is the length of the laser cavity.

In the above equation we have dropped the subscript N and now this equation represents the rate of change of mode locked pulse-width as a continuous function of time. Another important point to be noted is that the gain of the laser medium is treated as a time-independent function in (10).

The transient solution of (10) is

$$\tau = (2\ln 2)^{1/2}\left(\frac{32g}{\Delta\omega^2}\right)^{1/2}\left(\frac{2}{M_0}\tanh(M/M_0) - \sqrt{2}\,\alpha_{\mathrm{SHG}}\right)^{-1/2}, \tag{11}$$

where $$M_0 = \left(\frac{16g}{\Delta\omega^2}\delta^2\,\omega_m^2 + \alpha_{\mathrm{SHG}}^2/2\right)^{1/2}, \tag{12}$$

and $M\,(= t/c/2L)$ is the number of round-trips inside the laser cavity.

We can show that at steady-state (*i.e.* as $t \to \infty$) (10) has a solution identical to that given by Falk (1975). For the particular case, of $\alpha_{\mathrm{SHG}} = 0$, *i.e.* in the absence of intracavity second harmonic generation, (11) reduces to

$$\tau = \tau_0\,[\tanh(M/M_0)]^{-1/2}, \tag{13}$$

and (12) reduces to

$$M_0 = 1/4\delta\sqrt{g}\ \ (\Delta f/f_m), \tag{14}$$

where $$\tau_0 = (2\ln 2/\pi)^{1/2}\ \ (\sqrt{g}/\delta)^{1/2}\ \ (1/f_m\,\Delta f)^{1/2}, \tag{15}$$

which is the same as the steady-state pulsewidth for AM mode locking derived by Kuizenga and Siegman (1970).

Equation (13) was earlier derived by Kuizenga *et al* (1973) to study the problem of simultaneous Q-switching and mode locking in a cw Nd:YAG laser. They found that under normal operating conditions such a simultaneously Q-switched mode locking process does not allow sufficient time for mode locked pulses to build-up. Hence mode locking is incomplete and pulses are very broad. This problem is overcome by allowing the system to prelase before the Q-switch is opened.

The mode-locked pulsewidth obtained from (11) and (13) are plotted in figure 1 as a function of number of roundtrips inside the cavity (M). The system parameters used are, $\delta^2 = 0{\cdot}04, f_m = 50$ MHz, and $g = 0{\cdot}0525$. The pulsewidth in the presence of intracavity SHG crystal is calculated for two different values of α_{SHG}. From figure 1 it is clear that the mode locked pulsewidth approached a steady-state value relatively faster in the presence of internal harmonic conversion and builds up to steady state condition in a shorter time. Therefore prelasing may not be necessary for complete mode locking for a simultaneously Q-switched mode locked laser. It is also clear from figure 1 that the steady-state pulse from an MLFD laser is broader in duration than that from a simply mode-locked laser. This is because of the pulse-broadening effect of the SHG crystal which increases with the α_{SHG} value.

4. Numerical solutions

In the previous section it was assumed that the laser gain is a constant and we concentrated on getting an exact expression for the pulsewidth as a function of the number of round-trips. In this section we solve the basic equations (7), (8) and (9) treating gain as dependent on pulse peak intensity and pulsewidth which in turn are time-dependent. We update the gain after every round-trip by using the computed values of pulse peak intensity and pulsewidth. This updated value is used to calculate the new peak intensity and pulsewidth corresponding to the next round-trip.

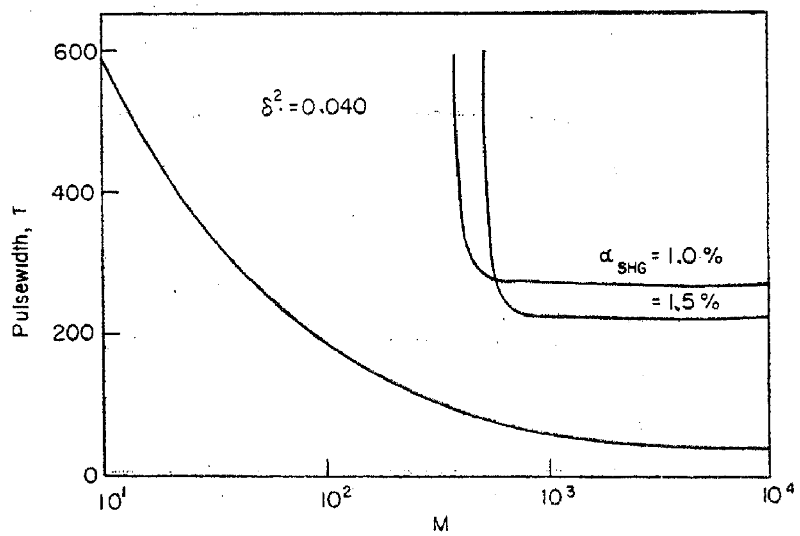

Figure 1. Time dependence (M = number of cavity roundtrips) of the pulsewidth τ for different values of α_{SHG}.

Before we proceed to solve the basic equations we re-arrange equations (7), (8) and (9) in the following form (Murray 1981):

$$\frac{\mathrm{d}\,I_P}{\mathrm{d}\,M} = I_P\left(G - \alpha_0 - \alpha_1 - \sqrt{2}\,\alpha_{\mathrm{SHG}} - 2\,\frac{G}{T^2}\right)\frac{c}{2L}, \tag{16}$$

$$\frac{\mathrm{d}\,T}{\mathrm{d}\,M} = T\left(\frac{G}{T^2} - 2\,\delta^2\,(f_m/\Delta f)^2\,T^2 + \sqrt{2}\,\alpha_{\mathrm{SHG}}\right)\frac{c}{2L}, \tag{17}$$

$$\frac{\mathrm{d}\,G}{\mathrm{d}\,M} = -\,\mathrm{SGT}\,I_P\,\frac{c}{2L}, \tag{18}$$

where we have defined,

$$G = 2g:\quad T = \tau\,.\,\Delta\,\omega/(8\ln 2)^{1/2},\quad S = \tfrac{1}{2}\,(\pi/2)^{1/2}\,\frac{1}{\tau_f\,\Delta\omega}. \tag{19}$$

In the above equations we have dropped the subscript N, as they represent the changes in the pulse parameters per cavity round-trip.

Since our interest is in the pulse-broadening effect of the SHG crystal inside the cavity we assumed that, to start with, there is a mode-locked pulse of ultra-short duration given by (13) with $M = 10{,}000$. The initial peak intensity is also assumed to be small. Starting with such a pulse equations (16) to (18) are solved numerically and the results are summarized in figures 2 to 7. The initial values chosen are: $I_P = 10^{-3}$ and $G = 0{\cdot}105$. The initial pulse-width is computed from (13).

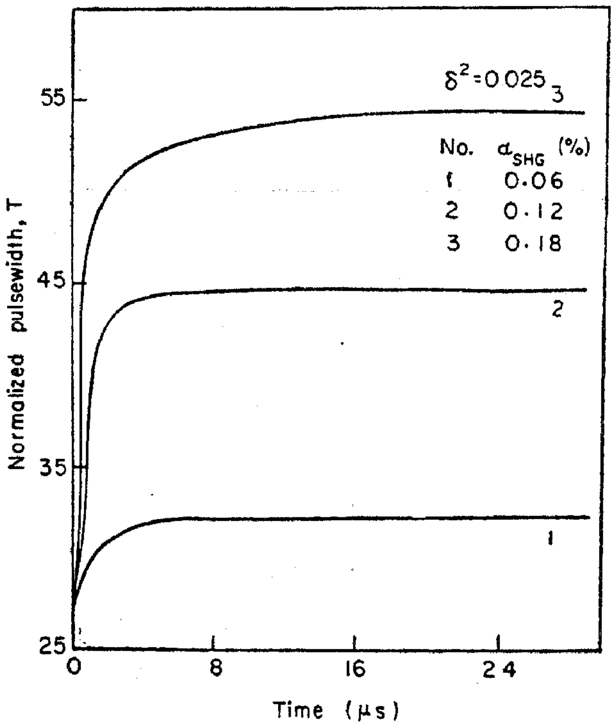

Figure 2. Time dependence of the normalized pulsewidth $T\ [=\tau\ \Delta\omega/(8\ln 2)^{1/2}]$ for three different values of α_{SHG} and for a constant modulation depth.

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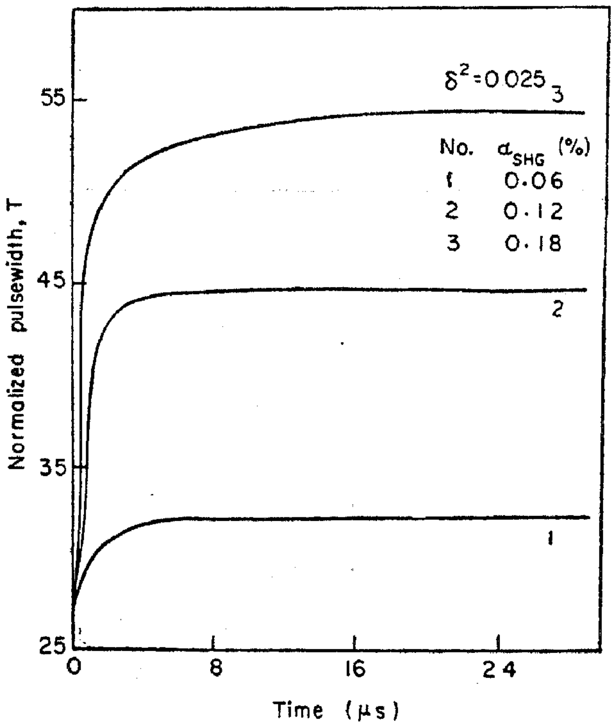

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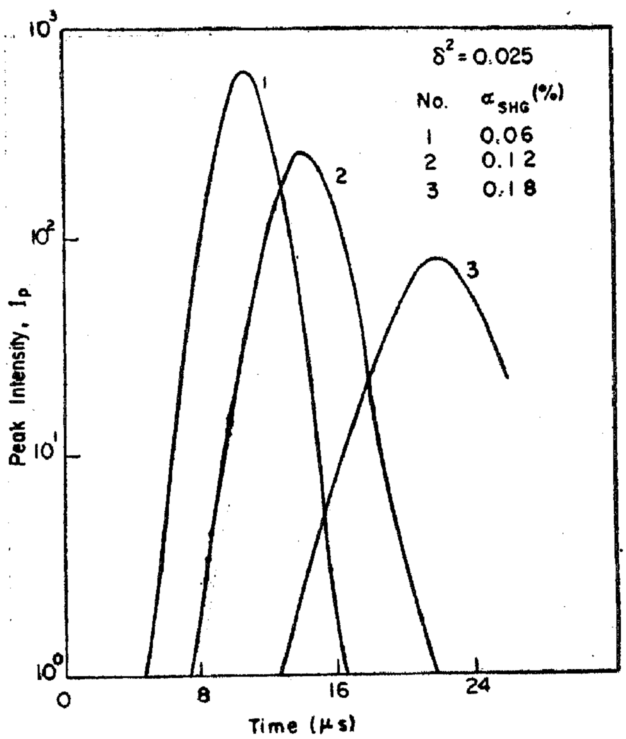

Figure 3. Time dependence of normalized pulse peak-intensity, I_p, for three different SH conversion efficiencies and for a constant modulation depth.

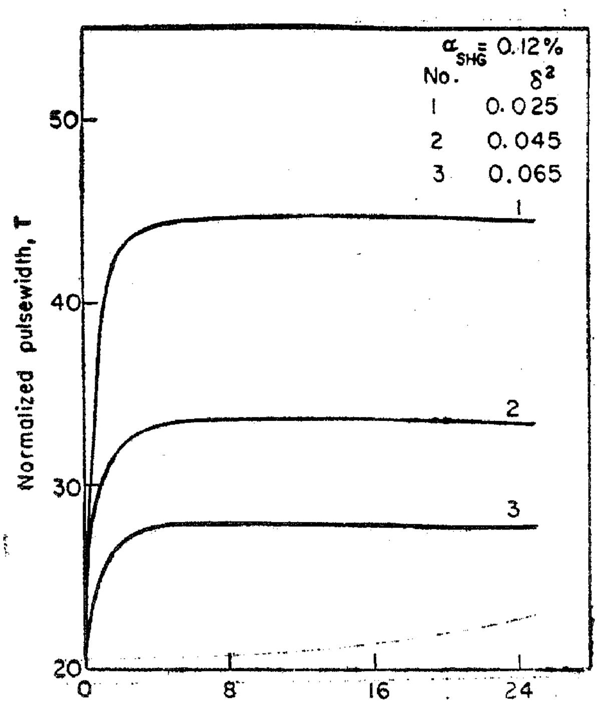

Figure 4. Time dependence of normalized pulsewidth for three different values of modulation depth at a constant SH conversion efficiency.

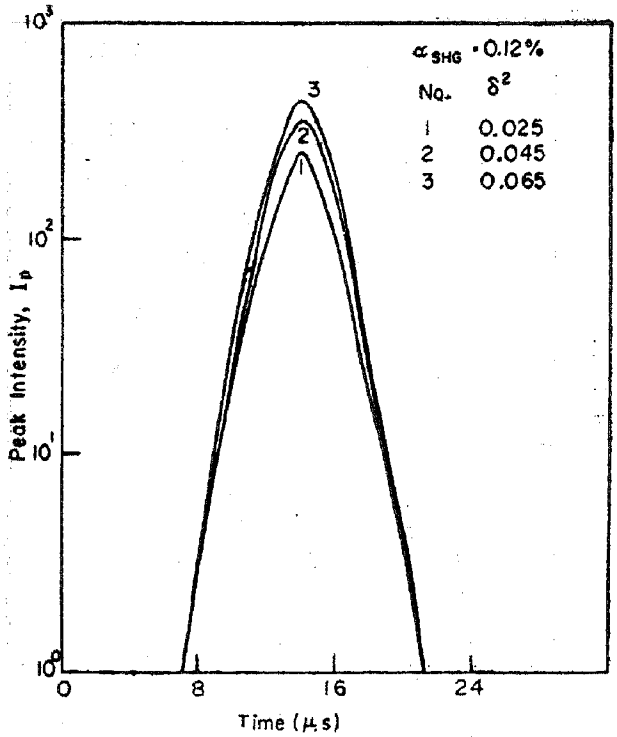

Figure 5. Time dependence of pulse peak-intensity for three different values of modulation depth at a constant SH conversion efficiency.

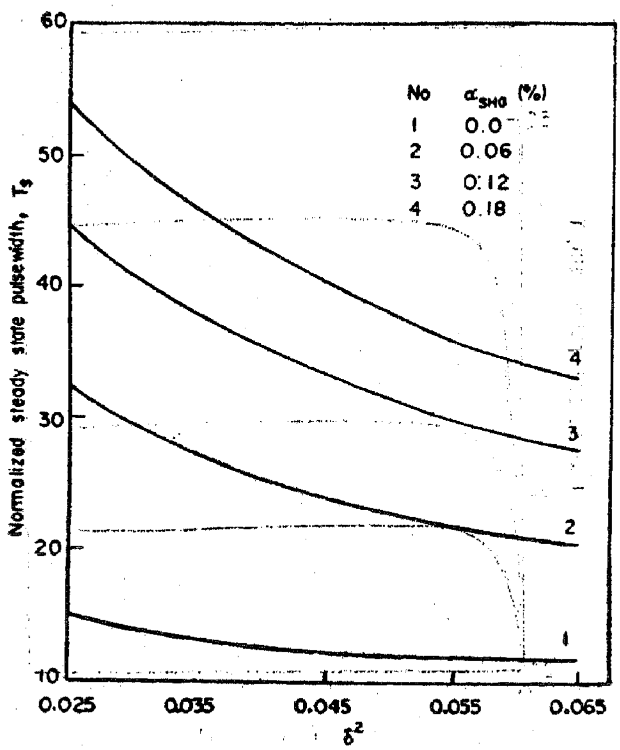

Figure 6. Normalized pulsewidth T_s at steady-state as a function of modulation depth for different SH conversion efficiencies.

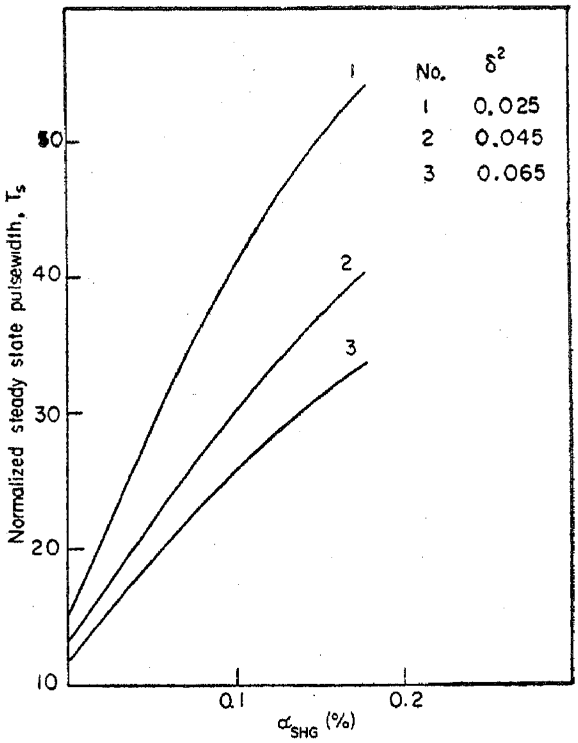

Figure 7. Normalized pulsewidth at steady-state as a function of SH conversion efficiency for different modulation depths.

Figures 2 and 3 show how the normalized pulsewidth and pulse-peak intensity vary with the internal frequency conversion efficiency of the SHG crystal for a constant modulation depth. Figure 2 shows that the pulsewidth increases to a certain value and then remains constant (at steady-state value). Pulse broadening is very fast. As the conversion efficiency is increased broadening also increases. The constant steady-state value to which the pulse-width settles down increases with increase in SHG conversion efficiency. Figure 3 shows the variation of pulse peak intensity with the SHG conversion efficiency, for a constant modulation depth. The highest value of the pulse peak intensity decreases with the increase of α_{SHG}. Figure 4 shows how the pulse-width depends on modulation depth for a fixed value of α_{SHG}. It is clear that the pulsewidth settles down to a constant value in a short duration and this pulsewidth decreases with increase of the modulation depth. Figure 5 shows that the highest value of the pulse peak intensity increases with increase of the modulation depth. The steady-state mode locked pulsewidth is plotted as a function of modulation depth for different conversion efficiencies in figure 6 and as a function of conversion efficiency for different modulation depths in figure 7. It is seen that shorter pulse width is obtained by increasing the modulation depth and decreasing the SH conversion efficiency.

5. Conclusions

Here a mode locked and internally frequency-doubled laser has been investigated. The transient build-up of mode locked pulse inside the laser cavity is studied in detail. The doubler crystal is found to increase the pulse duration. The number of roundtrps inside the laser cavity for the mode locked pulse to reach steady-state (*i.e.* when the pulse broadening effect of the laser amplifier and the doubler crystal is balanced by the pulse compression effect of the AM modulator) is computed. Comparing with a mode locked laser without frequency doubling, it is found that the MLFD laser approaches steady-state much faster. To start with, a weak and narrow pulse is assumed to be present in the cavity. Starting with this initial pulse the basic MLFD equations are solved and the results presented in the form of graphs demonstrate the broadening effect of the internal frequency conversion crystal on the mode locked pulse. It is shown that small SH conversion efficiencies and large modulation depths result in shorter pulsewidths.

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