Stability of multiple access communication networks

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Abstract. The problem of stability of different multiple access schemes in computer networks is discussed. The results for the main multiple access schemes – slotted ALOHA, controlled ALOHA, CSMA, CSMA/CD, polling systems and collision resolution algorithms are surveyed. The techniques used for the stability of these systems have been emphasized.

Keywords. Multiple access schemes; communication networks; channel access; stability.

1. Introduction

In data communications several resources usually share a common channel of communication. Traditionally, in telephone or radio networks time division multiple access (TDMA) or frequency division multiple access (FDMA) has been used to access the channel. These methods of channel access are efficient if the number of users is not very large and/or the communication traffic is not very bursty. But with the advent of large-scale use of computers it was felt that dedicating a time slot or a frequency band to each user is not very efficient because in such environments both these assumptions are generally violated. This could lead to unusually large delay in the communication and the channel may remain idle for a large fraction of time. This led researchers to look for new methods to share a channel.

The earliest new scheme for sharing channels was ALOHA. An improvement over it was slotted ALOHA. Since then a large number of techniques have been developed. Good reviews are available in Sachs (1988), Bertsekas & Gallager (1987). The purpose of this paper is to explain the main methods of multiple access that have been developed over the years and, more importantly, to present the most useful techniques that have been used to prove the stability of the channel under these methods.

We shall model the multiple access system as a stochastic system where the input traffic generated at the users for transmission is viewed as a stochastic process. In the literature the term stability has been used to imply different things. For us, the stability would usually mean the existence of stationary distributions of some system parameters of interest (e.g. queue lengths, delays) and the convergence to stationary distributions starting from any initial distribution. But we shall also briefly present other forms of stability like existence and convergence of moments of system parameters, structural stability, boundedness of sample paths of system parameters etc.

The paper is planned as follows. Section 2 describes the general multiple access

system and the assumptions made. A unified stability analysis is given in §3 for the slotted ALOHA and TDMA systems. Section 4 discusses the controlled ALOHA system. First the instability of the infinite user model is proved and then various schemes to stabilize it have been mentioned. Again, in §5 for CSMA and CSMA/CD the instability of the infinite user model is shown and then some work concerning its stabilization has been described. Section 6 discusses the polling systems and §7 the collision resolution schemes. Section 8 briefly mentions some hybrid schemes which combine the random access and reservation schemes. Methods used to prove the stability of the various systems have been highlighted, the ergodicity of appropriate Markov chains being the predominant theme.

2. Multiple access system

There are $1 < M \leq \infty$ users which share a common channel to communicate with each other. The messages are assumed packetized and usually having fixed packet lengths. The time axis will at times be slotted, i.e., it will be divided into slots of fixed size sufficient to send a packet if the packets are of the same lengths. In the slotted case the transmission of a packet can start only at the beginning of a slot. A user sends at most one packet at a time if it has something to transmit. The users will be assumed to have finite or infinite buffers to store the packets until successful transmission. When $M = \infty$, each user will be assumed to have a single buffer. If the transmission times of two or more packets overlap then the transmission is unsuccessful and the transmitters will send those packets again. On successful transmission the packets are removed from the buffers. It is assumed that the users can detect a collision or a successful transmission. The exact details of different multiple access methods will be presented at the appropriate places.

The stability of multiple access systems is generally studied either with $M < \infty$ and each user having an infinite buffer or with $M = \infty$. Both of these systems are idealization of the real systems where $M < \infty$ and the buffers at each user are finite. The relations between the stability for $M < \infty$ and $M = \infty$ assumptions have been studied by Paterakis *et al* (1987).

3. Slotted ALOHA and TDMA

In this section we present a technique by Loynes (1962) which has been used to demonstrate the stability of many systems. The advantage of this method is that it is quite intuitive and gives the existence of and convergence to stationary distributions under general stationary assumptions. We first explain the method in general and then apply it to slotted ALOHA and TDMA systems.

Let $\{x_k\}$ be a sequence of stationary random vectors with values in R^n and let $\{z_n\}$ be a sequence with values in R^m defined by (we call $\{x_n\}$ the input sequence and $\{z_n\}$ the state sequence)

$$z_{n+1} = f(z_n, x_n), \quad (1)$$

where f is a measurable function. The following is Loyne's lemma.

Lemma 1. If $f(z, x)$ is a nonnegative monotonically nondecreasing function in the first variable z for each x and is also continuous in z then (1) has a stationary solution.

Proof. Let us denote by z_k^{-n} the state value at time k if the system started at time $-n$ with initial state 0. Then it is easy to see that

$$z_k^{-n-2} \geq z_k^{-n-1} \geq z_k^{-n} \quad \text{a.s.}$$

Hence as $n \rightarrow \infty$, z_k^{-n} converges almost surely (a.s.) to a limit z_k . It can easily be shown that the $\{z_k\}$ sequence so obtained is stationary and satisfies (1) a.s.

Remark 1. We can also show that if the input sequence $\{x_m\}$ is ergodic then so is the $\{z_n\}$ sequence obtained in the above lemma. Of course, it is possible that the limit sequence $z_k = \infty$ a.s. Therefore, to obtain an a.s. finite sequence we need to have extra conditions which we mention when we consider the particular systems.

Remark 2. It is easy to see that the distribution of z_k^{-n} is the same as that of z_{n+k}^0 . Therefore, since $z_k^{-n} \xrightarrow{W} z_k$ in the above lemma, we also have $z_{n+k}^0 \xrightarrow{W} z_k$ as $n \rightarrow \infty$. Thus if the system starts empty at time 0, then it converges in distribution to z_k . This can be shown for all finite dimensional distributions also.

Remark 3. The above lemma does not guarantee the uniqueness of the stationary distributions. But it can be shown that if more than one stationary distribution exists then the sequence obtained in lemma 1 is stochastically the smallest stationary distribution of (1).

Remark 4. The condition of monotonicity in the above lemma is quite strong and is not satisfied in many computer network systems.

Many practical systems can be modelled more appropriately to have periodic inputs, i.e., the finite dimensional distributions of $\{x_n, x_{n+1}, \dots, x_{n+m}\}$ and $\{x_{n+d}, x_{n+1+d}, \dots, x_{n+m+d}\}$ are the same for some integer $d \geq 1$ and for all n and m . For this periodicity, we show now that the state equation $\{z_n\}$ in (1) has periodic stationary distributions with period d under the assumptions of lemma 1.

Under the assumption of periodicity,

$$z_k^{-n} \stackrel{\text{dist}}{=} z_{k+d}^{-n+d},$$

where $X \stackrel{\text{dist}}{=} Y$ indicates that the random variables X and Y have the same distribution. Since $z_k^{-n} \xrightarrow{W} z_k$ and $z_{k+d}^{-n+d} \xrightarrow{W} z_{k+d}$, z_k and z_{k+d} have the same distribution. The same can be said for all finite dimensional distributions. Hence, we can also show that if the system starts empty at time $n=0$, it converges to these periodic stationary distributions. Similar ideas have been expressed in Rolski (1981) and Bambos & Walrand (1989) also.

Now we apply the above results on a multiple access system which has been analysed in Prathima (1991). This is a new scheme for multiple access that combines the advantages of slotted ALOHA and TDMA. The results for stationary and periodic stationary cases for the slotted ALOHA and TDMA will emerge as special cases.

We assume there are $1 < M < \infty$ users each having infinite buffer to store the packets. The packets are of fixed size and the time axis is slotted. The M users are

divided into L groups $G_1, G_2, \dots, G_L$ (they don't have to be disjoint). Each slot is assigned to one of the groups in a periodic fashion. We take the period as d slots. The question of how to form the groups, how large d should be and how many slots should be assigned to a group in a period is an optimization problem which depends upon the traffic rates of different users. This problem is addressed in Prathima (1991) but presently we assume that the assignment has been made. In the slot assigned to a group, all the users which have packets to transmit at the beginning of that slot transmit a packet with a certain probability. If more than one packet gets sent then there is a collision and those packets will be tried again in the next assigned slot.

We use the following notation:

- k th slot = time interval $(k, k + 1)$;
- $x_k(i)$ = number of new packets generated in the $(k - 1)$ slot at the user i ;
- $t_k(i)$ = 1, if i th user would transmit a packet in the k th slot if it has a packet at time k
= 0, otherwise;
- $z_k(i)$ = queue length at the i th user at time k without including $x_k(i)$;
- 1_A = indicator function of set A ;
- $y_k(i)$ = 1, if the i th user has a successful transmission in the k th slot
= 0, otherwise.

Then the system equation can be written as

$$z_{k+1} = z_k + (x_k - y_k)$$

where $z_k = (z_k(1), z_k(2), \dots, z_k(M))$ and similarly for x_k and y_k . We can write this equation as

$$z_{k+1} = f(z_k, x_k, t_k).$$

The random variables t_k take care of the assignment of the slots to different groups.

We can see that the function f is monotonically nondecreasing in z and is nonnegative. Thus, if $\{(x_k, t_k)\}$ is a periodic stationary process, then there exists a periodic stationary distribution of $\{z_k\}$ and starting at time $t = 0$ with $z_k = 0$, the finite dimensional distributions of $\{z_k\}$ converge to the periodic stationary distributions. For the finiteness of the stationary distributions (proper random variables) we need extra conditions on the rate of the input traffic. It can be shown, as in Sharma (1988), that if $Ex_k < \infty$, $\sum_{k=1}^d E(x_k - y_k) < 0$ and the input traffic $\{(x_k, t_k)\}$ is ergodic, the periodic stationary distributions of $\{z_k\}$ are proper. Also, as in Sharma (1988), it can be shown that if the system starts at $t = 0$ with nonzero finite a.s. initial state then the random variables $\{z_k\}$ remain bounded a.s. Other remarks in Sharma (1988) remain relevant for this system as well.

It is easy to see that if $\{t_k\}$ is stationary then the above system becomes slotted ALOHA with periodic input, if $\{(x_k, t_k)\}$ is stationary then it is slotted ALOHA with stationary input, if $\sum_{i=1}^M t_k(i) = 1$ for all k and t_k is deterministic then it is TDMA with periodic input.

Various other results have been obtained on the slotted ALOHA and TDMA systems. Slotted ALOHA with a finite number of users and infinite buffers has been studied by Saadawi & Ephremides (1981) by forming two coupled Markov chains. Stability using Markov chains has also been shown by Tsybakov & Mikhailov (1979). Both these papers assume the input traffic $\{x_n\}$ to be i.i.d. Within this framework most of the ergodicity and nonergodicity results have been summarized in Szpankowski (1986).

For nonhomogeneous, nonperiodic input traffic, analysis has been provided in Sharma (1987). A comparative analysis of TDMA and FDMA is provided in Rubin (1979). The structural stability of slotted ALOHA has been studied in Nelson (1984) and Onozato & Noguchi (1985) via catastrophe theory. That the slotted ALOHA system with $M = \infty$ and unbuffered users is unstable for all input traffic rates has been proved by Rosenkrantz & Towsley (1983) by martingale techniques. We present this result in detail in the next section.

4. Controlled ALOHA

In the last section we studied the slotted ALOHA system in which the retransmission probabilities remain constant. As mentioned there, it has been shown in Rosenkrantz & Towsley (1983 and the earlier references therein) that for an infinite user model the slotted ALOHA system is unstable for any positive total input traffic, i.e., it has been shown that the total number of blocked users goes to infinity with probability 1. This result, which was first noticed in the mid-seventies, has evoked a lot of research to control the retransmission probabilities of the users so that the system becomes stable for some nonzero input traffic (see Hajek 1985, Merakos & Kazakos 1985, Bertsekas & Gallager 1987). The methods used for the study of such systems are quite different from those in the last section. In Merakos & Kazakos (1985) and Hajek (1985) stochastic approximation methods have been used. Theory of Markov chains and large deviations has been used in Lim & Meerkov (1987). Large deviation results have been used in Cottrell *et al* (1983) also. Martingale techniques have been used in Goodman *et al* (1988) to prove the stability of exponential backoff schemes. In §4.1 we show the above mentioned instability of slotted ALOHA for $M = \infty$. Section 4.2 will present a control scheme to stabilize the slotted ALOHA under these circumstances.

4.1 Instability of infinite user slotted ALOHA

We assume that the number of users $M = \infty$, each having a single buffer. Thus if a user has a packet stored then it cannot generate any more packets. Let $\{x_k\}$ be a sequence of i.i.d. random variables where x_k is the number of new packets generated in the system in the $(k-1)$ th slot. Let z_k represent the number of blocked users at time k , i.e., number of users which have packets that have been transmitted unsuccessfully earlier. A new packet generated in slot $k-1$ is transmitted in slot k with probability 1. A blocked user transmits its packet in the next slot with probability p . Then the $\{z_k\}$ sequence is a Markov chain with transition probabilities P_{ij} given as below [in the following $P(x_k = i) = \lambda_i$],

$$\begin{aligned} P_{00} &= \lambda_0 + \lambda_1; & P_{01} &= 0; & P_{0j} &= \lambda_j; \\ P_{i,i-1} &= \lambda_0 i p (1-p)^{i-1}, & i &\geq 1, \\ P_{ii} &= \lambda_1 (1-p)^i + \lambda_0 (1-i p (1-p)^{i-1}), & i &\geq 1, \\ P_{i,i+1} &= \lambda_1 (1-(1-p)^i), & i &\geq 1, \\ P_{ij} &= \lambda_{j-1}, & j &\geq i+2, & i &\geq 1. \end{aligned}$$

The following theorem has been used in Rosenkrantz & Towsley (1983) to show the transience of Markov chain $\{z_k\}$.

Theorem 1. *If an irreducible, aperiodic Markov chain $\{z_k\}$ admits a nonconstant, nonnegative, bounded function f on its state space $\{0, 1, 2, \dots\}$ such that*

$$E[f(z_{k+1})|z_k] \leq f(z_k),$$

then the Markov chain is transient.

When $0 < \lambda_0 < 1$ then $\{z_k\}$ is aperiodic. We further assume that $\lambda_0 + \lambda_1 < 1$, otherwise the chain will be ergodic. Under these conditions a function f has been constructed in Rosenkrantz & Towsley (1983) which satisfies the conditions of theorem 1 and hence $\{z_k\}$ is transient.

4.2 Controlled ALOHA

We have seen in §4.1 that for the infinite user case the slotted ALOHA system is unstable if the retransmission probability p is kept fixed. This happens because if at any time the number of blocked users increases, since p is fixed, the probability that only one user transmits is very low. Therefore, as new packets are generated, the number of blocked packets keeps increasing with a high probability. Hence, attempts have been made to adaptively change p as a function of the number of blocked users z_k so that the probability of successful transmission remains high (Merakos & Kazakos 1985; Hajek 1985; Lim & Meerkov 1987; Tsitsklis 1987).

Now we present a scheme, given in Tsitsklis (1987) to stabilize the slotted ALOHA. As in §4.1, x_k is the number of new packets generated in slot $k - 1$ and z_k is the number of blocked users at time k (beginning of slot k). Let y_k be the number of attempted transmissions in slot k . At the beginning of each slot every station has an estimate $\hat{z}_k$ of the backlog, which it updates in each slot as below (the users get the feedback from the channel whether a slot was empty, there was successful transmission or a collision).

$$\begin{aligned}\hat{z}_{k+1} &= \max\{1 + \hat{z}_k - 1 + \hat{\lambda}\}, & \text{if } Y_k < 2, \\ \hat{z}_{k+1} &= \hat{z}_k + [1/(e - 2)] + \hat{\lambda}, & \text{if } Y_k \geq 2,\end{aligned}$$

where $\hat{\lambda}$ is the estimate of λ at each user, λ being the expected value of x_k . Actually, $\hat{\lambda}$ can be replaced by e^{-1} and we can still obtain the stability and maximum throughput. The motivation for the updates of z_k is provided in Bertsekas & Gallager (1987) and Rivest (1987).

We define $S_k = (z_k, \hat{z}_k)$. Then under the condition of $\{x_k\}$ being i.i.d., S_k is a Markov chain. Now using the techniques of Hajek (1982) a Lyapunov function can be constructed to prove the following theorem.

Theorem 2. *If $0 < \hat{\lambda} \leq e^{-1}$, $0 < \lambda < e^{-1}$ and $\lambda \leq \hat{\lambda}$, then the Markov process S_k defined above is geometrically ergodic (Tsitsklis 1987).*

In the schemes mentioned in Merakos & Kazakos (1985), Hajek (1985), Tsitsklis (1987) and Rivest (1987) each station has to continuously monitor the channel and update the backlog estimates etc. to follow the algorithm and their estimates should agree with each other for stability. To overcome this disadvantage, other algorithms have been proposed to control the slotted ALOHA (Aldous 1987; Hastad *et al* 1987; Goodman *et al* 1988). In Aldous (1987) it has been shown that for $M = \infty$ the Markov chain of backlogs $\{z_n\}$, when the retransmission probabilities p of the users are

exponentially decreased, is transient. In Goodman *et al* (1988), for number of users $M < \infty$, each with infinite buffer, under the exponential backoff, the Markov chain of queue lengths at each station has been shown to be positive recurrent if the traffic rate is below certain limits. These results have been generalized in Hastad *et al* (1987). Of course for finite user models the uncontrolled slotted ALOHA itself is stable. But the results of Hastad *et al* (1987) show that by controlling the retransmission probabilities we can increase the maximum stable throughput of the system.

The possibility of successful transmission of more than one packet in a slot has been considered in several papers (this is the "capture" effect). For such a slotted ALOHA network, control algorithms have been analysed in Ghez *et al* (1989).

The delay analysis of controlled ALOHA has been considered in Thomopoulos & Sunder (1987) and Clare (1986).

5. CSMA and CSMA/CD

The multiple access scheme CSMA was first proposed and analyzed by Kleinrock & Tobagi (1975). Its stability and control was discussed in Tobagi & Kleinrock (1977). Since then a large number of papers have been devoted to this subject (Bertsekas & Gallager 1987). This scheme has proved very efficient when the propagation delays between different users are much less than the transmission times of the packets. An improvement over CSMA is CSMA/CD (Bertsekas & Gallager 1987; Tobagi & Hunt 1987; Sachs 1988).

In CSMA, $M \leq \infty$ users share a common channel where each user can sense whether the channel is idle or busy. Whenever a user has a packet to send it first senses the channel for sometime (the time depends upon the propagation delays). If channel is free during this time then it sends its packet with probability $0 \leq p \leq 1$. If the channel is busy then it reschedules its sensing of the channel. When it next senses the channel depends upon the protocol assumed. Even after sensing that the channel is free, there is a possibility of a collision. In CSMA the user sends the complete packet even if there is a collision. But in CSMA/CD (CD stands for collision detection) after it senses a collision, the user aborts the transmission and hence frees the channel earlier than in CSMA.

5.1 Instability of CSMA and CSMA/CD

In this section we prove that the infinite user CSMA and CSMA/CD channels are unstable. The analysis follows Medith & Lea (1983) where the Markov chain of blocked users is formed and is shown to be transient by using theorem 1.

There are $M = \infty$ users each with a single buffer. The time axis is partitioned into minislots of lengths which equal the one-way propagation delay of the whole channel. The data packets are of fixed length and require a transmission time of $T > 1$ minislots. In nonpersistent CSMA, if a user has a packet ready for transmission, it senses the channel. If the channel is sensed idle, it transmits the packet in the next slot. If it is found busy, the user schedules its transmission for some future minislot according to a probability distribution. In that slot if it again senses the channel busy it repeats the procedure. A blocked user senses the current slot with probability $0 < f \leq 1$ and thereupon proceeds as above. The nonblocked users are assumed to generate new packets with Poisson distribution and mean rate λ packets/minislot.

The actual time taken in transmission for CSMA is $1 + T$ minislots whether the transmission is a success or a failure (collision). Between two successive transmissions there is at least one idle slot. We consider the backlogged process at the instants of the end of the first idle minislot after a transmission and denote by z_i the number of blocked users at these instants. This is a Markov chain. The transition probability of this Markov chain can be easily calculated under the above assumptions. Then, using theorem 1, the following result can be obtained.

Theorem 3. *The Markov chain $\{z_n\}$ of block users at the embedded points is transient (Medith & Lee 1986).*

A similar result for CSMA/CD has also been obtained in Medith & Lea (1983). As for slotted ALOHA, this instability result has led researchers to obtain control schemes for CSMA and CSMA/CD also. We consider some of these results in the next section.

The delay analysis for CSMA and CSMA/CD has been studied in Beuerman & Coyle (1988) and Tobagi (1982). Matrix geometric methods have been used in Beuerman & Coyle (1988), while Tobagi (1982) uses Z-transform techniques.

5.2 Control of CSMA and CSMA/CD

To stabilise the CSMA and CSMA/CD adaptive algorithms changing the retransmission probabilities etc. have been proposed (Tobagi & Kleinrock 1977; Medith & Lea 1983; Rubin 1983; Coyle & Liu 1985) just as for slotted ALOHA. Medith & Lea (1983) consider only the slotted system, while Coyle & Liu (1985) consider only the unslotted (continuous time) system and both cases are considered by Rubin (1983).

Medith & Lea (1983) control CSMA and CSMA/CD by varying the retransmission probabilities f of the blocked users as $f = \alpha/n$ where n is the backlog. Of course n will not be known to the users and hence will have to be estimated. The estimation schemes which have been used for controlling slotted ALOHA can be useful in this case also (see details later). Under the adaptive scheme of $f = \alpha/n$ Medith & Lea (1983) have shown the ergodicity of the Markov chain $\{z_n\}$ formed in § 5.1. For this they use the following Pake's lemma (Pake 1969).

Lemma 2. *Let $\{x_i\}$ be an irreducible, aperiodic Markov chain whose state space is the set of nonnegative integers. Then, the following conditions are sufficient for the chain to be ergodic:*

$$|E[x_{i+1} - x_i | x_i = n]| < \infty,$$

and

$$\limsup_{n \rightarrow \infty} E[x_{i+1} - x_i | x_i = n] < 0.$$

Using Lemma 2 it has been shown (Medith & Lea 1983) that if

$$\lambda T[1 + (T + 1)(1 - e^{-(\alpha + \lambda)})] < (\alpha + \lambda) T e^{-(\alpha + \lambda)},$$

then Markov chain $\{z_n\}$ for CSMA is ergodic. A similar expression has been obtained for CSMA/CD also. From this condition, α can be appropriately chosen so as to obtain ergodicity for different arrival rates λ .

Coyle & Liu (1985) have chosen the same adaptive scheme $f = \alpha/n$ and provide conditions for ergodicity, null-recurrence and transience of the Markov chain for the

unslotted CSMA/CD. The system has been studied for exponential as well as phase type distributions and explicit closed form expressions obtained using the matrix-geometric techniques of Neuts (1981).

More recently, control schemes have been proposed which do not require the stations to know the total system backlog (Thomopoulos 1988; Cunningham & Medith 1988). In (Thomopoulos 1988) minimum mean square error predictor has been used for the number of blocked users z_n as below (the updating is done at the end of the idle period, the successful transmission period and the collision period).

$$\begin{aligned}\hat{Z}_{n+1} &= \hat{Z}_n - \alpha/(\alpha + \lambda_{\text{eff}}), & \text{if successful transmission,} \\ &= 0, & \text{if idle period,} \\ &= \lambda_{\text{eff}}/[1 - (\alpha + \lambda_{\text{eff}})/(\exp(\alpha + \lambda_{\text{eff}}) - 1)], & \text{if collision,}\end{aligned}$$

where $\lambda_{\text{eff}} = \lambda$ (length of the period).

Then the retransmission probability f is updated as

$$f_{n+1} = \min(\beta, \alpha/\hat{Z}_{n+1}).$$

If the traffic intensity λ is not known then an estimate of λ is also provided in Thomopoulos (1988). Under these estimates the following result for stability has been obtained. The packet length is taken as 1 slot.

COROLLARY.

The maximum stable throughput obtained by 1-persistent CSMA, under the above adaptive scheme, is the solution of

$$(1 - x)e^{-x} = 1/(1 + \tau),$$

where τ is the time required by the stations to sense the channel being idle. For $\tau \ll 1$, $\lambda_{\text{max}} \approx 1 - \sqrt{2\tau}$. For CSMA/CD the maximum stable throughput is obtained by solving the equation

$$(1 - x)e^{-x} = 1/(1 + \tau/\tau_c),$$

where τ_c is the fraction of a slot used in collision detection.

Thomopoulos (1988) defines the stability in the following way: the average rate of successful transmission should be maintained and the expected delay should remain bounded as the number of users sharing the channel approaches infinity.

The control scheme used in Cunningham & Medith (1988) applies stochastic approximation techniques, following Merakos & Kazakos (1985) and Hajek (1982, 1985).

6. Polling schemes

In this scheme, the different users sharing the channel are polled in a certain order for using the channel. The polling in a physical system can be done in various ways. One common method is by sending a token to different stations whenever the channel is free. When a station/user is polled, it may or may not be allowed to send more than one packet/message at a time. Even TDMA can be considered a polling system.

But in general, in a polling system the disadvantage of TDMA, in having the channel idle for a long time when there are a large number of users, is not there. Therefore, these schemes can be very efficient in local area networks (Sachs 1988; Bertsekas & Gallager 1987). Hence, polling systems have been extensively studied. Since rather exhaustive surveys are available for these systems (see Takagi 1986, 1990) we shall limit ourselves to only the references which are directly relevant for the stability of the polling systems.

The polling systems have been studied under various assumptions. The number of stations $M < \infty$. The stations can have finite or infinite buffers. The time axis can be slotted or unslotted. The number of messages allowed for transmission at a time from a user can be one, $k \geq 1$, exhaustive (i.e. all packets queued at the station) or gated (with various restrictions). The walk time, i.e., the time to poll from one user to another or the time a token takes to reach from one user to another can be zero, deterministic or random. If the walk time is zero then the stability of polling systems can be studied in the generality of §3. If walk time is nonzero then the stationary distributions in such generality may not exist. Therefore, in this section we consider the stability of polling systems where each user has an infinite buffer (stability problem does not arise for the finite buffer case), the walk times are nonzero and the message generation sequences are Poisson processes (for the unslotted case) or i.i.d. sequences (for the slotted case).

The main references for stability of the polling systems are Eisenberg (1972), Hashida (1972), Konheim & Meister (1974), Zhdanov & Saksonov (1979) and Swartz (1980). A nice survey emphasising the performance and technological aspects of token rings is given in Bux (1989). In Konheim & Meister (1974) and Swartz (1980) the time axis is considered slotted with i.i.d. input sequences while in Eisenberg (1972), Hashida (1972) and Zhdanov & Saksonov (1979) the system is unslotted. The arrival processes at each user are Poisson in Hashida (1972) and Eisenberg (1972), while more general renewal arrival process at each user is considered in Zhdanov & Saksonov (1979). But the criterion of stability, which is ergodicity of appropriate Markov chains in Hashida (1972) and Eisenberg (1972), is the existence of finite limit of expected values of certain system parameters (for Poisson input the results in Zhdanov & Saksonov (1979) reduce to that of others).

We present, in the following, the results of Zhdanov & Saksonov (1979). Eisenberg (1972), Hashida (1972), Konheim & Meister (1974) and Swartz (1980) have used Laplace–Stieltjes transform techniques, which involve many tedious manipulations. Also, as mentioned above, Zhdanov & Saksonov (1979) do convey something in the case of general renewal input process also.

There are $1 < M < \infty$ users each with an infinite buffer. The input messages generated at the users are independent renewal processes. The interarrival time at user i has mean $0 < \alpha_i < \infty$ and the service time has general distribution with mean $0 < \beta_i$. Walk time from user i to user $i + 1$ is random having a general distribution with finite mean $0 < \gamma_i < \infty$. All random variables are independent of each other. We consider in the following only the gated system although the exhaustive service system has also been treated by Zhdanov & Saksonov (1979).

For the system described above, it is easy to show that

$$t_{ik} = \left[\sum_{j=1}^M \gamma_j \right] / \alpha_i + \left[\sum_{j=1}^{i-1} t_{jk} \beta_j + \sum_{j=i+1}^M t_{jk-1} \beta_j \right] / \alpha_i,$$

where t_{ik} is the expected value of the total time the server spends at user i in the k th cycle (i.e., the k th time it comes to user i). This can be rewritten as

$$t_k = (I - A)^{-1} B t_{k-1} + (I - A)^{-1} \gamma,$$

where,

$$t_k = (t_{1k}, t_{2k}, \dots, t_{Mk})^T,$$

$$\gamma = \left(\sum_{i=1}^M \gamma_i \right)^{-1} (\beta_1/\alpha_1, \beta_2/\alpha_2, \dots, \beta_M/\alpha_M)^T,$$

$$A = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \rho_2 & 0 & \dots & 0 \\ \rho_3 & \rho_3 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ \rho_M & \rho_M & \dots & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} \rho_1 & \rho_1 & \rho_1 & \dots & \rho_1 \\ 0 & \rho_2 & \rho_2 & \dots & \rho_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \rho_N \end{bmatrix},$$

and

$$\rho_i = \beta_i/\alpha_i.$$

Now, it is well-known that this recursive equation has a finite limiting value if and only if $(I - A)^{-1}B$ has all its eigenvalues in a unit circle in the complex plane. It has been shown in Zhdanov & Saksonov (1979) that a sufficient condition for this is

$$\sum_{i=1}^M (\beta_i/\alpha_i) < 1. \quad (2)$$

The finiteness of the limiting value of t_{ik} implies the finiteness of the limiting value of expected valued η_{ik} (as $k \rightarrow \infty$) of the number of customers served in k th cycle at user i .

If the arrival processes at the users are independent Poisson processes then the above result gives the ergodicity of the Markov chain of queue length processes formed at the instants of arrival of the server at i th user. Analysis in the following differs somewhat from that in Zhdanov & Saksonov (1979) and provides a little stronger result which is equivalent to that obtained in Hashida (1972) and Eisenberg (1972). We denote $(z_n(1), z_n(2), \dots, z_n(M))$ by z_n , where $z_n(j)$ is the queue length at the j th user when the server arrives at the i th queue (fixed in the following) in the n th cycle. Then $\{z_n\}$ is an aperiodic, irreducible Markov chain. We show in the following that the above result on finiteness of the limit of the expected values implies that $\{z_n\}$ is ergodic.

It can be easily seen that $z_n(j)$, $j \neq i$ is a.s. dominated by the queue length at user j when the server visits j either in n th cycle or in $(n+1)$ th cycle. Thus $\lim_{n \rightarrow \infty} \sup E[z_n(j)] < \infty$. Now we define a random sequence $\{s_n\}$ by

$$s_n = \sum_{j=1}^M z_n(j).$$

Then $Es_n = \sum_{j=1}^M E[z_n(j)]$ and $\lim_{n \rightarrow \infty} \sup E(s_n) < \infty$. Since $\{z_n\}$ is irreducible, if $\{z_n\}$ is not an ergodic sequence then $P\{z_n = a\} \rightarrow 0$ for any vector $a = (a_1, \dots, a_M)$. This implies that $P\{S_n = k\} \rightarrow 0$ for all $k \geq 0$. Therefore,

$$\liminf_{n \rightarrow \infty} Es_n = \liminf_{n \rightarrow \infty} \sum_{k=0}^{\infty} P\{S_n \geq k\} \geq \sum_{k=0}^{\infty} \lim_{n \rightarrow \infty} P\{S_n \geq k\} = \infty.$$

Hence $\{z_n\}$ is an ergodic Markov chain.

We now mention a few more papers on the stability of the polling systems. The stability in these papers may not imply the ergodicity of any underlying Markov chain. An influential paper is by Kuehn (1979). Analysis of this kind has been followed up in Servi (1985) and Ibe & Chang (1989). See Takagi (1990) for many more references.

7. Collision resolution algorithms

We have seen in the earlier sections that for infinite user models, slotted ALOHA, CSMA and CSMA/CD are all unstable for any nonzero input traffic. Collision resolution algorithms (CRA) introduced in Capetanakis (1979) and Tsybakov & Mikhailov (1978) were the first algorithms among the random multiple access methods which provide nonzero stable throughput. The currently available CRA have throughputs upto 0.487 which exceed the throughput of $1/e$ available from controlled ALOHA. The IEEE special issue (*IEEE Trans. Inf. Theory* 1985) contains the history, most of the references to earlier literature and the state of the art till 1984 on these algorithms. We shall mention some more recent work at the end of this section.

The number of users $M = \infty$ and each user has a single buffer. The system is slotted and the packet lengths are fixed. Total system packet generation process is Poisson. A nonblocked binary stack algorithm is described as follows (Tsybakov 1985).

- (i) If a station has a packet to transmit, then in the time interval from the packet's generation to its successful transmission, it assigns to this packet an integer L_n in slot n . L_n is called its packet level at time n . The packet is transmitted over the channel in slot n if and only if $L_n = 0$.
- (ii) Every new packet is assigned level 0.
- (iii) If $L_n = 0$ for a packet and there is collision in slot n then $L_{n+1} = 1$ with probability $1/2$, and $L_{n+1} = 0$ otherwise.
- (iv) If $L_n \geq 1$ for a packet and there is a collision in slot n then $L_{n+1} = L_n + 1$.
- (v) If $L_n \geq 1$ and there is no collision in slot n then $L_{n+1} = L_n - 1$.

A generalization of the above algorithm is nonblocked J -ary stack algorithm in which the steps (iii) and (iv) in the above algorithm are changed to:

- (iii)' If $L_n = 0$ for a packet and there is a collision in slot n then L_{n+1} takes value j with probability P_j , where $j \in \{0, 1, \dots, J-1\}$.
- (iv)' If $L_n \geq 1$ and there is a collision in slot n then

$$L_{n+1} = L_n + J - 1.$$

The above algorithm has been modified to eliminate certain predictable collisions. The details are in Massey (1981, pp. 73-137). In another algorithm called blocked stack algorithm (Tsybakov & Mikhailov 1978), the newly generated packets are not

allowed to transmit immediately. If there has been a collision in the recent past, all the packets involved in that collision are first allowed to transmit. Only after their successful transmission, packets generated during that time transmit and then if there is a collision, these packets get into what is called a collision resolution period. The disadvantage of the blocked algorithm as compared to the nonblocked one, explained above, is that in the blocked case all the stations have to keep monitoring the channel throughout whether they have a packet to transmit or not. A further variation along the theme of collision resolution is the part and try algorithm (Tsybakov & Mihailov 1980).

In the following we provide the stability results for the blocked stack algorithm. The analysis for the nonblocked is along the same lines. Assume that x_k is the number of new packets generated in the system in the k th slot. The sequence $\{x_k\}$ is assumed i.i.d. Also, let

$$P\{x_k = m\} = \lambda^m e^{-\lambda}/m!, \quad m = 0, 1, \dots$$

The analysis can easily be extended to the general distributions. Let us denote by τ_k the length of a collision resolution period given that the collision multiplicity (number of packets involved in the initial collision) is k . Then, the following recursive equation holds.

$$E[\tau_k] = P_0(1 + E(\tau_k)) + \sum_{n=1}^k P_n(2 + E(\tau_{k-n}) + E(\tau_n)),$$

where $P_l = 2^{-k} \binom{k}{l}$ and the initial conditions are $E[\tau_0] = E[\tau_1] = 0$. This provides $E[\tau_2] = 3.5$, $E[\tau_3] = 6$ etc. Also, it can be shown (Tsybakov & Mikhailov 1978) that

$$[(8/3) - (1/168)]k - 2 \leq E[\tau_k] \leq (8/3)k - 2. \quad (3)$$

We denote by ξ_n the multiplicity of the n th conflict. Under the assumption of $\{x_n\}$ being i.i.d., it can be shown that $\{\xi_n\}$, and $\{\tau_n\}$ are homogeneous Markov chains. Using the results on the ergodicity of Markov chains similar to theorem 1 and lemma 2, the following theorem can be obtained.

Theorem 4. *Under the above assumptions the following hold (Tsybakov & Mikhailov 1978)*

- (i) *If $\lambda < \liminf_{k \rightarrow \infty} (k/E\tau_k)$ then $\{\xi_n\}$ is an ergodic Markov chain.*
- (ii) *If $\lambda \leq \liminf_{k \rightarrow \infty} k/E\tau_k$ then $\{\xi_n\}$ is a recurrent Markov chain.*
- (iii) *If $\lambda > \limsup_{k \rightarrow \infty} k/E\tau_k$ then $\{\xi_n\}$ is a transient Markov chain.*

From the estimates given in (3), theorem 4 states that if $\lambda < 3/8$ then $\{\xi_n\}$ is ergodic and if $\lambda \geq 3/8$ then it is recurrent. It has been shown (Tsybakov & Mikhailov 1978) that if $\lambda < 3/8$ then $\{\tau_n\}$ is also an ergodic Markov chain, that $\lim E\tau_n < \infty$ and that $\lim E\xi_n < \infty$.

Recently sharper asymptotic estimates have been obtained for the $E[\tau_k^m]$, for $m \geq 1$ (Jacquet & Szpankowski 1989). The performance of the collision resolution algorithms can be improved somewhat if the channel feedback is used to estimate the collision multiplicity also (Cidon & Sidi 1986). This estimate is then used to choose dynamically the J -parameter in a J -ary stack algorithm.

In a part and try algorithm (Tsybakov & Mihailov 1980; Gallagher 1978) the

collision resolution is done by allowing the colliding packets to transmit in order of their time of generation. This is done by splitting the time axis and allowing packets to be transmitted if they arrived in a certain subset of the time axis. In Mosely & Humblet (1985) and Khasminskii (1989) the problem of splitting the time axis optimally has been addressed by forming a Markov decision problem.

The collision resolution algorithms discussed above have been in the framework when the channel propagation time is large. In local area networks (LAN), collisions and carriers can be sensed in a short time and, exploiting this feature, collision resolution algorithms for LAN have been studied in Georgiopoulos *et al.* (1986, pp. 185–214).

8. Hybrid techniques

As described in §3, random access schemes perform well when there are a large number of bursty users with light traffic and reservation based schemes like TDMA provide good performance when the number of users is small and traffic intensity is high. This has motivated many researchers to develop hybrid schemes combining random access and reservation techniques. Our scheme in §3 is motivated by the same idea. Several references to the hybrid schemes are provided in Sachs (1988). Also, several hybrid schemes are mentioned in Mehravari (1984) and it has been shown that under heavy load the TDMA is better than any random access scheme. Multiple access schemes with priority have been reviewed in de Moraes (1990).

An error in the first draft of this paper was pointed out by the reviewer.

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