

Prediction of the Break-out Noise of the Cylindrical Sandwich Plate Muffler Shells

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ABSTRACT

Break-out noise from the shell often puts a limit on the (net) transmission loss that can be obtained from an engine exhaust muffler. One of the best ways of reducing break-out noise is use of a sandwich plate shell. This short paper presents an analytical model for prediction of transverse transmission loss of sandwich plate shells. Four-pole parameters of the porous layer are used along with an impedance approach for relating radial velocity of the outer (exposed) metallic layer to acoustic pressure of a plane progressive wave inside the muffler shell. This is combined with radiation impedance for outgoing cylindrical waves to predict transverse transmission loss. Thus, a sandwich plate shell is compared with a single layer shell without and with wrapping of another metallic layer by means of a transverse insertion loss. Finally, results of a parametric study are reported to help the muffler designers.

Keywords: Break-out noise, muffler, sandwich plate, noise control.

INTRODUCTION

Noise generated by engines, compressors, fans, etc. is radiated out into the atmosphere at the radiation end of the muffler (or inlet/outlet pipe) and also as break-out noise from the shell of the muffler. An efficient muffler, therefore, should not only have large enough axial transmission loss¹ (TL_a) but also transverse transmission loss (TL_{tp}) so that it has adequate net transmission loss² (TL_n). Transverse transmission loss TL_{tp} can be increased by

means of a thicker shell, a double-wrapped shell, or a sandwich plate shell.³ Prediction of TL_{ip} of a single-layer plate shell has been dealt with in the literature (e.g. see Refs 2, 4 and 5). However, there is little in the published literature on prediction of $TL_{,}$ of a cylindrical sandwich plate shell, or its special case in the form of a double-wrapped shell, that is, without any porous layer between the inner elastic shell and the outer massive plate layer or wrapping. This is what has been attempted in this brief communication.

A cylindrical sandwich plate shell can be modelled as a limiting case of an acoustically lagged pipe.⁶ Thickness of porous layer in typical sandwich-plate muffler shells is about 1–3 mm, whereas that in acoustically lagged pipes is of the order of 50 mm. Inertive impedance of the outer impervious layer is transferred across the porous layer making use of the transfer matrix of the latter,⁶ and then combined with that of the inner metallic layer to evaluate the total wall impedance felt by the plane wave inside the pipe. This is used to calculate the radial wave number for waves in the bulk-reacting porous layer and those radiated out into the atmosphere as break-out noise. Making use of an impedance model, radial velocity of the outer metallic layer is predicted in terms of the acoustic pressure of a plane progressive wave inside the shell. This is then combined with radiation impedance⁵ to predict transverse transmission loss $TL_{,}$ of the shell.² (Second subscript p in $TL_{,}$ is to emphasize that the transverse transmission loss is defined here with respect to acoustic power and not intensity as done by ASHRAE,⁷ so that it can be combined with $TL_{,}$ to predict $TL_{,}$). Finally some parametric studies have been presented.

ANALYTICAL MODEL

Starting with generalized expressions for acoustic pressure p and radial velocity u_r in cylindrical coordinates, it can be shown that the transfer matrix relationship is given by⁶

$$\begin{bmatrix} p_2 \\ u_{r2} \end{bmatrix} = \frac{X}{\det} \begin{bmatrix} J_{1,3}N_{0,2} - J_{0,2}N_{1,3} & \frac{1}{X}(J_{0,3}N_{0,2} - J_{0,2}N_{0,3}) \\ X(J_{1,2}N_{1,3} - J_{1,3}N_{1,2}) & J_{1,2}N_{0,3} - J_{0,3}N_{1,2} \end{bmatrix} \begin{bmatrix} p_3 \\ u_{r3} \end{bmatrix} \quad (1)$$

where

$$X \equiv j \frac{k_{rp}}{Y_p k_p} \quad (2)$$

$$\det \equiv X(J_{1,3}N_{0,3} - J_{0,3}N_{1,3}) \quad (3)$$

$$J_{1,3} \equiv J_1(k_{rp}r_3), \quad N_{1,3} \equiv N_1(k_{rp}r_3), \quad J_{0,2} \equiv J_0(k_{rp}r_2), \text{ etc.} \quad (4)$$

Y_p and k_p are respectively complex characteristic impedance and wave number of the porous layer. They depend on structural factor and porosity among other parameters. Structural factor takes into account the effect of the pores and cavities that are perpendicular to the propagation direction of the sound wave. Generally it falls in the range of **1–3**. Most numerical calculations assume a value of unity. Porosity is defined as the ratio of pore volume to total volume. Obviously it is less than unity. For the commercially available mineral wool and glass wool, it lies between 0.95 and 0.99. Thus, assuming porosity and structural factor to be nearly unity,

$$Y_p = \rho_o c_o \left[1 - j \frac{r_p}{\rho_o \omega} \right]^{1/2} \quad (5)$$

$$k_p = k_o \left[1 - j \frac{r_p}{\rho_o \omega} \right]^{1/2} \quad (6)$$

where

R_p is the specific flow resistance of the porous layer,

ρ_o and c_o are density and sound speed of the fluid (air) entrapped in the pores,

$k_o = \omega/c_o$,

$k_{rp} = [k_p^2 - k_z^2]^{1/2}$ is the radial component of k_p , (7)

J and N are Bessel function and Neumann function, respectively,

$$Z_{12} = j\omega I + \frac{1}{j\omega C}, \quad (8)$$

where²

$$\text{inertance } I = \rho_{12} t_{12} \left\{ 1 + \left(\frac{0.025}{r_1} \right)^2 \right\}, \quad (9)$$

$$\text{compliance } C = \frac{r_1}{E_{12}} \left[\frac{r_2^2 + r_1^2}{r_2^2 - r_1^2} + \nu_{12} \right] \cong \frac{r_1^2}{E_{12} t_{12}}, \quad (10)$$

ρ_{12} , t_{12} , E_{12} and ν_{12} are density, thickness, elastic modulus (complex), and Poisson's ratio of the inner pipe, respectively, and

$$t_{12} = r_2 - r_1 \quad (11)$$

Lumped impedance of the jacket (metallic wrapping) over the porous layer (Fig. 1), being wholly inertive, is given by

$$Z_{34} = j\omega \rho_{34} t_{34} \quad (12)$$

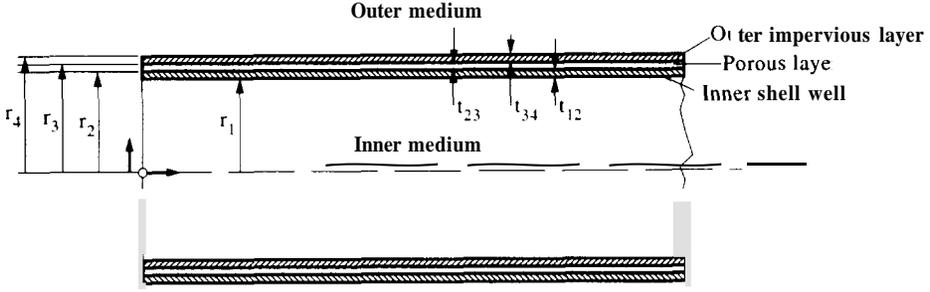


Fig. 1. Schematic cross-section of a sandwich wall shell.

Radiation impedance on the jacket is given by⁵

$$Z_o \equiv Z_{o4} = j \frac{\omega \rho_o}{k_{ro}} \frac{H_o^{(2)}(k_{ro} r_4)}{H_1^{(2)}(k_{ro} r_4)} \quad (13)$$

where $H^{(2)}$ is Hankel function of the second kind, with the subscript indicating the order.

The state variables (pressure p and radial velocity u_r) on the inner surface of the pipe are related to those on the outer surface of the jacket as follows:

$$\begin{bmatrix} p_1 \\ u_{r1} \end{bmatrix} = \begin{bmatrix} 1 & Z_{12} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} 1 & Z_{34} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p_4 \\ u_{r4} \end{bmatrix} \quad (14)$$

where

$$p_4/u_{r4} = Z_o, \text{ given by eqn (13),} \quad (15)$$

Z_{12} and Z_{34} are given by eqn (8) and eqn (12), respectively,

T_{ij} are elements of the porous layer transfer matrix given by eqn (1), so that

$$T_{11} = \frac{J_{1,3} N_{0,2} - J_{0,2} N_{1,3}}{J_{1,3} N_{0,3} - J_{0,3} N_{1,3}}, \quad (16)$$

etc.

Equations (14) and (15) yield the total impedance of the wall to plane waves inside the pipe:

$$Z_w \equiv \frac{p_1}{u_{r1}} = Z_{12} + \frac{Z_{34} T_{11} + T_{12}}{Z_{34} T_{21} + T_{22}} \cong Z_{12} \quad (17)$$

This approximation of total wall impedance as impedance of the inner pipe follows obviously from physics of the lagging.

The ratio of the acoustic pressure inside the pipe p_1 to the radial velocity of the outer (radiating) surface u_{r4} may be calculated readily from eqns (14) and (15). Thus,

$$\frac{p_1}{u_{r4}} = Z_0(T_{11} + Z_{12}T_{21}) + T_{11}Z_{34} + T_{12} + Z_{12}(T_{21}Z_{34} + T_{22}), \quad p_1 \equiv p_i \quad (18)$$

Transverse power transmission loss TL, defined as ten times the logarithm to the base 10 of the acoustic power flux associated with an axial plane progressive wave inside the pipe divided by that radiated out from the exposed surface of the outer jacket (Fig. 1) may now be evaluated as follows.

$$TL_{ip} = 10 \lg \frac{W_i}{W_4} = 10 \lg \frac{\frac{|p_i|^2 S_i}{2\rho_i c_i}}{\frac{1}{2} |u_{r4}|^2 \text{Real}(Z_0) S_4}, \quad \text{dB} \quad (19)$$

where

$$S_i = \pi r_1^2, \quad (20)$$

$$S_4 = 2\pi r_4 l, \quad (21)$$

l is length of the lagged pipe,

Z_{04} is given by eqn (13),

p_i/u_{r4} is given by eqn (18) and

T_{ij} may be evaluated from eqn (1), provided k_{rp} is known *a priori*.

k_{rp} , radial component of the wave number in the porous tube, may be calculated as follows.

k_z , the common axial component of the wave numbers in the inner medium, porous tube and the outer medium, is given by²

$$k_z = k_i \left[1 - 2j \frac{\rho_i c_i}{Z_w k_i r_i} \right]^{1/2}, \quad (22)$$

where

subscript i stands for the inner medium,

$Z_w \cong Z_{12}$, as per eqn (17), and

Z_{12} is given by eqns (8)–(10).

$$k_{rp} = \left(k_p^2 - k_z^2 \right)^{1/2} \quad (23)$$

and k_{ro} , the radial component of wave number in the outer medium, required for evaluation of radiation impedance Z_o as per eqn (13), is given by

$$k_{ro} = \left(k_o^2 - k_z^2 \right)^{1/2} \quad (24)$$

Finally, insertion loss due to wrappings may be calculated from the following relationship:

$$IL = (TL_{tp})_{\text{wrapped shell}} - (TL_{tp})_{\text{bare shell}} = TL_{tp3} - TL_{tp1} \quad (25)$$

TL_{tp} for the wrapped shell or sandwich wall shell is given by eqn (19) and TL_{tp} for the bare shell may be calculated from the corresponding relationship:

$$TL_{tp}(\text{bare shell}) = 10 \lg \frac{\frac{p_i}{u_{ri}}}{\frac{1}{2} |u_{ri}|^2 \text{Real}(Z_{02}) S_2}, \text{ dB} \quad (26)$$

where $p_i/u_{ri} = Z_{12}$ given by eqn (8), and $S_2 = 2\pi r_2 l$, $Z_{02} = Z_{04}$ with r_4 replaced by r_2 in eqn (13).

TL_{tp1} denotes TL_{tp} of a base shell of thickness, t_{12} . TL_{tp2} denotes TL_{tp} of a shell of thickness t_{12} wrapped with another layer thickness t_{34} , and TL_{tp3} denotes TL_{tp} of a sandwich plate shell. Thus,

$$IL = TL_{tp3} - TL_{tp2} \quad (27)$$

would represent the effect of the intermediate porous layer.

PARAMETRIC STUDIES AND CONCLUSIONS

The following values were selected for the different geometrical parameters, typical of engine exhaust mufflers (see Fig. 1):

shell radius $r_1 = r_{sh} = 0.05, 0.1, 0.15$ m

thickness $t_{34} = t_{12} = 0.0005, 0.001, 0.002$ m

thickness $t_{23} = 0.001, 0.002, 0.003$ m

specific flow resistance $SFR = 10000, 20000, 30000$ Pa s/m²

Italic values denote default values. Length of the shell is taken to be 0.75 m.

Figure 2 shows plots of the transverse transmission loss of a bare shell (single layer) (TL_{tp1}), double wrapped shell (TL_{tp2}), sandwich wall shell (TL_{tp3}) and the insertion loss IL defined by eqn (25) for the default configuration of a 0.75 m long shell. It may be noticed that the TL_{tp2} curve more or less coincides with the TL_{tp1} , indicating that outer wrapping without a porous layer inbetween is of little use. The intermediate layer of course makes a significant difference. Incidentally, the low frequency disadvantage (marginally negative values of IL) is of no practical significance because TL_{tp1} is anyway very high at low frequencies.

The effect of variation in shell radius on the transverse IL of a sandwich plate shell has been observed to be practically negligible. Therefore, it has

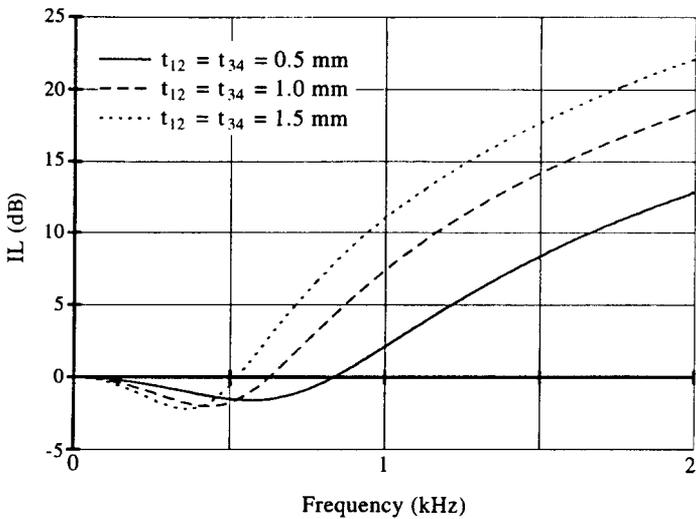
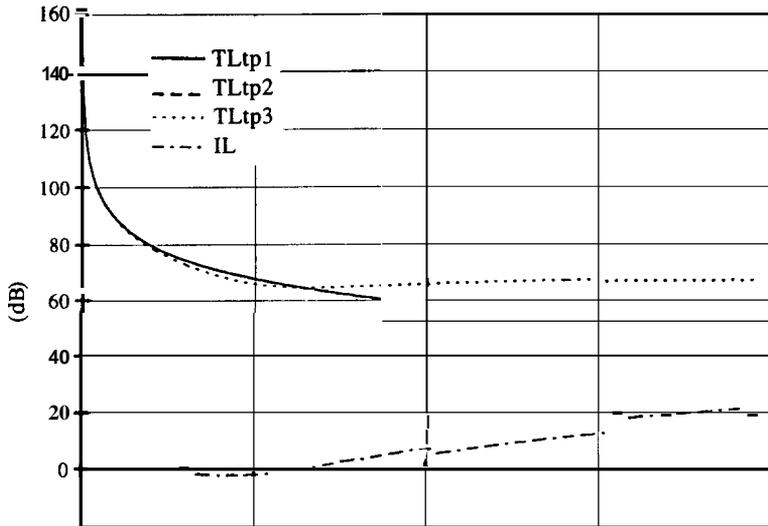


Fig. 3. Effect of thickness of the shell wall and outer wrapping on the transverse insertion loss of a sandwich plate shell.

not been shown here. Incidentally, it means that although TL_{tp1} is a strong function of shell radius,² yet $IL = TL_{tp3} - TL_{tp2}$ is not.

Figure 3 shows that an increase in thicknesses t_{12} and t_{34} results in a marked increase in transverse IL at medium and high frequencies.

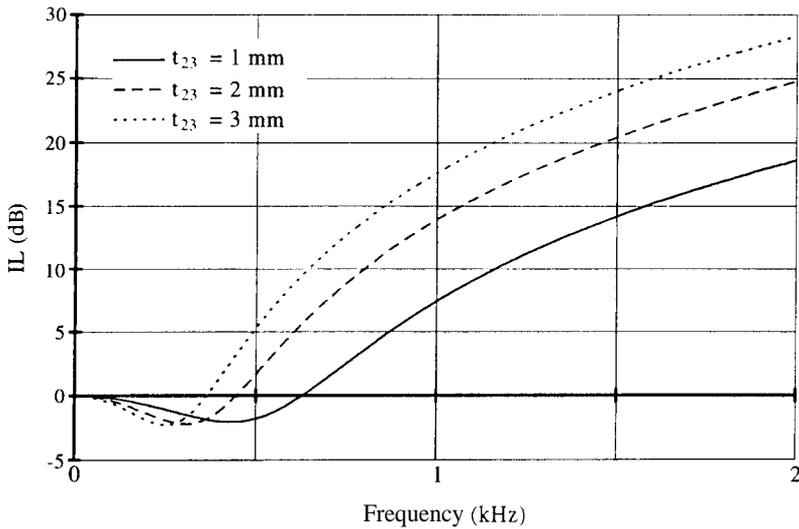
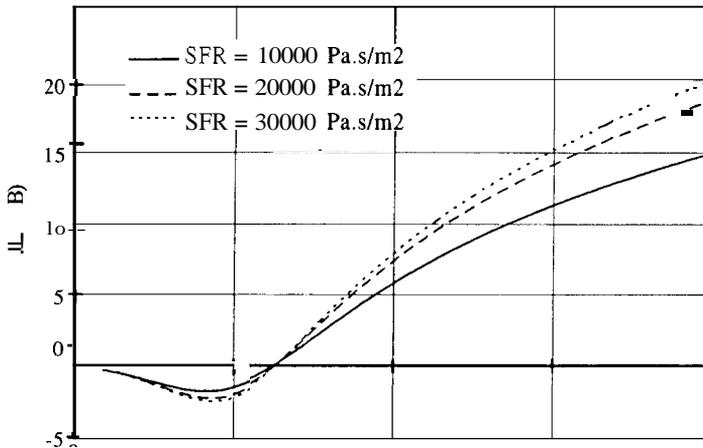


Fig. 4. Effect of the porous layer thickness on the transverse insertion loss of a sandwich plate shell.



It can be noticed from Fig. 4 that an increase in the thickness of the porous layer results in a significant increase in the transverse IL of a sandwich plate shell at medium as well as high frequencies.

Effect of the specific flow resistance on IL is also significant, although not as marked as that of thicknesses, as can be seen from Fig. 5.

In conclusion, break-out noise can be reduced by replacing a single-layer shell by a sandwich plate shell. Further reduction in break-out noise may be achieved by increasing t_{12} , t_{23} , t_{34} and the specific flow resistance of the porous layer. Optimum design would call for transverse IL to be greater than or equal to axial IL so that net IL is within 3dB of the axial IL for which the muffler has been designed.² However, it may be kept in mind that the method presented here holds strictly for low frequencies or very long shells because the finite length effects⁴ have been neglected and only the breathing mode has been considered. The intermediate porous layer would also help in minimizing the flexural vibration amplitudes and hence break-out noise arising out of resonances due to finite length of the shell.

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