Optimum Difference Mode Excitations for Monopulse Arrays

N. BALAKRISHNAN AND S. RAMAKRISHNA

Abstract—**A technique has been developed to obtain optimum difference mode excitations for monopulse arrays. In addition, a direct interpolation scheme has been devised for situations where near-optimum results would suffice. A parameter designated as u_o is identified with every optimum difference pattern. The choice of this parameter u_o determines the sidelobe level of the optimum pattern. The problem of obtaining the optimum excitations has been shown to be reducible to one of finding out the best approximation that minimizes the maximum deviation (minimax) from the real line ($u = \beta d \cos \varphi$ axis), over a range determined by u_o. This latter problem has been solved using a modified Remez exchange algorithm. An extensive set of design curves has also been presented.**

I. INTRODUCTION

IN ANGLE of arrival measurements employing the monopulse concept, the difference patterns are called optimum if they have the largest normalized slope at the boresight for a given sidelobe level. Price and Hyneman [1] demonstrated that the difference patterns with equal amplitude sidelobes are optimum in the sense that they display both the lowest sidelobe ratio for a given difference pattern beamwidth as well as the largest slope at the boresight.

The design of optimum difference mode excitations is not as straightforward as the design of symmetric Dolph–Chebyshev patterns. However, methods of obtaining approximate equiripple sidelobe structure using the transmutation techniques [1] and the lambda functions [2] have been reported. Techniques of synthesis of arbitrarily prescribed patterns in the minimax sense have been adopted by Ma [3] to synthesize a prescribed difference pattern. Matrix methods, originally developed for the directivity optimization of symmetric patterns, have been employed by Pang and Ma [4] to optimize the difference mode directivity.

In this paper, a numerical method of obtaining optimum difference mode patterns with an exact equiripple sidelobe structure has been developed. Besides, an effort-saving direct interpolation scheme has also been devised to obtain near-optimum patterns.

II. REVIEW OF DOLPH–CHEBYSHEV SYMMETRIC PATTERNS

In this section, the Dolph–Chebyshev method of obtaining symmetric broadside patterns has been briefly reviewed to aid the discussions that follow. The array factor of an N-element equally spaced, symmetrically excited broadside array can be written from [5, p. 188] as

$$f_s(u) = 2 \sum_{k=1}^{N/2} I_k \cos (\overline{2k-1}u), \qquad \text{for } N \text{ even}$$

$$= I_o + 2 \sum_{k=1}^{(N-1)/2} I_k \cos (2ku), \qquad \text{for } N \text{ odd}, \tag{1}$$

where $u = \beta d/2 \cos \varphi$, d is the element spacing, and $I_k = I_{-k}$ is the current in the kth element.

The design of a Dolph–Chebyshev array involves transforming the Chebyshev polynomial using Dolph's transformation

$$x = x_o \cos u \tag{2}$$

and identifying it with the array factor to obtain the optimum excitations.

The largest values of $f_s(\varphi)$, i.e., the main-lobe peak occurs at $\pi/2$ (or $u = 0$) and is equal to R. The value of R can be obtained from

$$T_{N-1}(x_o) = R. \tag{3}$$

Since the magnitude of all the minor lobes is unity, the main-lobe magnitude is R times that of the sidelobes. It is well known that of all the polynomials of degree $(N-1)$ that pass through the point (1, 1), the Chebyshev polynomial $T_{N-1}(x)$ deviates the least from the real line (x-axis) in the range $|x| \leqslant 1$ and hence is the best minimax approximation to the real line in this range [5, p. 187]. Hence, it is apparent that the Dolph's transformation merely transforms this range into $|x| \leqslant x_o$. This, in the u-space corresponds to u_o to $(\pi - u_o)$ where

$$u_o = \cos^{-1}(1/x_o)$$

and

$$f_s(u_o) = 1. \tag{4}$$

It may be noted here that the approximation that minimizes the maximum deviation (minimax) is also known as the Chebyshev approximation [6] or the L_∞ approximation [8].

In the case of unequally spaced arrays, the array factor cannot be represented directly by a polynomial. Hence, the optimum patterns cannot be easily associated with x_o. However, even in such cases u_o can be identified with the optimum pattern. Hence, the general problem of designing Chebyshev arrays reduces to one of finding out the best minimax approximation to the real line in the range of u from u_o to $(\pi - u_o)$, subject to the condition that

$$f_s(u_o) = 1. \tag{5}$$

It should be emphasized that such an approximation can be obtained from the Chebyshev polynomial and the Dolph–Chebyshev transformations only for equally spaced arrays.

The beamwidth of the optimum pattern for a given value of N and sidelobe level can be decreased by increasing the element spacings d of the equally spaced array until as many minor lobes as possible are included in the visible range. From Fig. 1 it is apparent that this optimum element spacing d_{opt} is

Manuscript received April 3, 1980; revised August 6, 1980
The authors are with the Department of Aeronautical Engineering, Indian Institute of Science, Bangalore 560 012, India.

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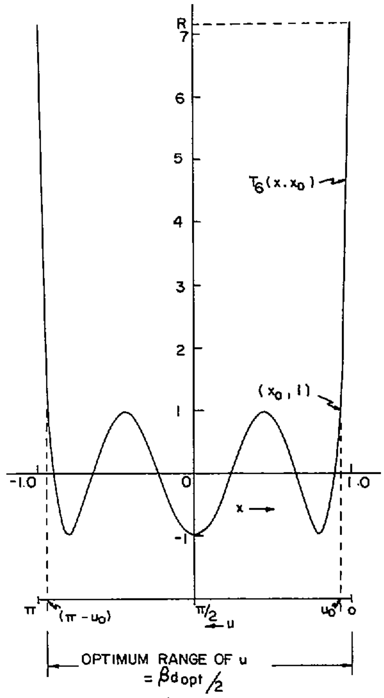

Fig. 1. Transformed Chebyshev polynomial of sixth degree.

obtained from

$$\beta d_{opt}/2 = \pi - u_o$$

or

$$d_{opt} = \frac{\pi - u_o}{\pi} \quad \text{in wavelengths.} \tag{6}$$

The foregoing discussion shows that it is often advantageous to identify the optimum pattern with the parameter u_o. It is worthwhile to note here that increasing u_o decreases the sidelobe level of the optimum pattern and hence can be directly used to control the beamwidth-sidelobe level trade-off.

III. CHARACTERIZATION OF THE OPTIMUM DIFFERENCE PATTERNS

The difference pattern is obtained by exciting the array antisymmetrically. That is

$$I_k = -I_{-k}, \qquad \text{for all } k. \tag{7}$$

The difference pattern of an N-element equally spaced array can then be written as

$$f_d(u) = \sum_{k=1}^{N/2} I_k \sin(\overline{2k-1}u), \qquad \text{for } N \text{ even}$$

$$= \sum_{k=1}^{(N-1)/2} I_k \sin(2ku), \qquad \text{for } N \text{ odd.} \tag{8}$$

Sin (ku) can be expressed as a product of sin u and a $(k-1)$th degree polynomial in cos u. That is,

$$f_d(u) = \sin u \sum_{k=1}^{N/2} a_k \cos^{2(k-1)} u, \qquad \text{for } N \text{ even}$$

$$= \sin u \sum_{k=1}^{(N-1)/2} a_k \cos^{2k-1} u, \qquad \text{for } N \text{ odd.} \tag{9}$$

For the purpose of illustration, an eight-element array can be considered. The array factor can then be written as

$$f_d(u) = \sin u(a_4 \cos^6 u + a_3 \cos^4 u + a_2 \cos^2 u + a_1) \tag{10}$$

where a_i are dependent on the excitations.

Because of the sin u term in (9), the difference pattern is antisymmetric about the boresight ($u = 0$) direction. It can also be seen from (9) that the pattern is symmetric about $u = \pi/2$ when N is even (because of the even powers of cos u) and is antisymmetric when N is odd. The difference pattern has main lobes of equal magnitude on either side of the null at $u = 0$, together with the usual secondary maxima or sidelobes. The pattern is optimum when all the secondary maxima are of equal magnitude [1]. It is apparent from (10) that this optimum pattern for the eight-element array will have only four controllable peaks including the main-lobe peak in the range of u from 0 to $\pi/2$. This range of u is the most significant one for the discussions that follow.

An examination of Fig. 2 in light of earlier discussions on Dolph–Chebyshev patterns, reveals the existence of the control parameter "u_o" for the difference patterns as well. Here again, increasing u_o increases the main lobe to sidelobe amplitude. This will, of necessity, broaden the main beam which, in effect, will reduce the slope of the normalized pattern at the borseight. Thus by systematically varying u_o, one can obtain an optimum trade-off between the sidelobe level and the boresight slope.

Besides, it is apparent from Fig. 2 that the optimum difference pattern satisfies all the requirements of the best minimax approximation to the real line (u-axis) in the range of u, $u_o \leqslant u \leqslant (\pi - u_o)$ [6, pp. 55–62] as for Dolph–Chebyshev arrays. Hence, it can be concluded that of all the difference patterns $f(u)$ which pass through the point $(u_o, 1)$, the optimum difference pattern $f_d(u)$ is the best minimax approximation to the real line (u-axis) in the interval from u_o to $(\pi - u_o)$ Minimiax approximation to the u-axis without the condition

$$f_d(u_o) = 1 \tag{11}$$

is meaningless, for, in such cases the approximation will collapse to the real line itself. The condition imposed by (11) will automatically be met if we seek the minimax approximation with the upper bound of the error prespecified as equal to unity, for, at u_o which is an extemum of the zone of approximation, there will certainly be a peak in the error function [6].

As in the case of Dolph–Chebyshev arrays, the beamwidth of the difference mode pattern can be decreased (an hence the boresight slope increased) by increasing the element spacing until as many sidelobes as possible are included in the visible region. From Fig. 2 it is apparent that the expression for d_{opt} given by (6) is also valid for the difference mode excitations of the monopulse array.

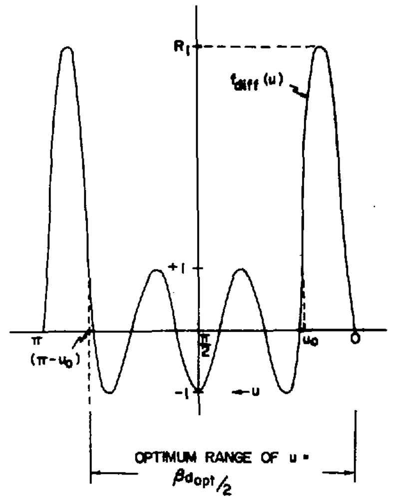

Fig. 2. Optimum difference pattern with equal sidelobes for an eight-element array.

IV. METHOD OF SOLUTION

The characterization of the optimum difference pattern in the previous section has shown that the problem of obtaining the excitation coefficients can be reduced to one of finding out the best minimax approximation. From the discussions on the Dolph-Chebyshev arrays and from Fig. 2 it is clear that this approximation is the transformed polynomial T_{N-2} $(x_o \cos u)$ where

$$x_o = \frac{1}{\cos u_o}. \tag{12}$$

However, the presence of sin u in the array factor of the difference pattern (9) makes it difficult to identify the array factor directly with T_{N-2} $(x_o \cos u)$. This situation is similar to that encountered in conventional unequally spaced arrays. In such cases, closed-form expressions for the excitations are not normally available, and hence recourse can be taken to numerical techniques to obtain the optimum excitations. The Remez exchange algorithm [6, p. 176] has been suitably modified and used in this paper, and the steps involved in such a procedure are detailed below for an eight-element array.

1) Since there are four currents, I_i, $i = 1, 2, 3, 4$ to be determined, the simplest way of accomplishing this is to obtain a set of four simultaneous equations with the currents as unknowns. The first of these equations is obtained from (11), since the optimum pattern has to pass through $(u_o, 1)$.

Then, an initial set of points, $u = u_j^o$, $j = 1, 2, 3$ in the range of u from u_o to $\pi/2$ is chosen and the error between the real line and $f_d(u)$ at these points is equated to $(-1)^j$. Since the error between the real line and $f_d(u)$ is $f_d(u)$ itself, the above condition can be written as

$$f_d(u_j^0) = (-1)^j; \quad j = 1, 2, 3. \tag{13}$$

The initial set of currents I_i^o can be obtained by solving the set of simultaneous equations–(11) and (13). In (11) u_o, the control parameter, is specified. u_1^o, u_2^o, and u_3^o in (13) are chosen arbitrarily in the range u_o to $\pi/2$.

2) The location of the three sidelobe peaks of the patterns $f_d(u)$, denoted as u_j^1, $j = 1, 2, 3$ for the set of currents I_i^o can be determined by setting

$$\frac{\delta f_d(u)}{\delta u} = 0. \tag{14}$$

The above equation gives the locations of the major lobe and the three sidelobes. Confining our attention to the range of u from u_o to $\pi/2$ in (14), the major lobe can be isolated and the sidelobe locations u_1^1, u_2^1, and u_3^1 can be identified. The magnitude of $f_d(u)$ at these extrema are, of course, not equal in general, since the initial set of u_i^o chosen is arbitrary. The next step is to determine the currents I_i^1, $i = 1, 2, \cdots, 4$ such that

$$f_d(u_j^1) = (-1)^j, \qquad j = 1, 2, 3$$

and

$$f_d(u_o) = +1.$$

This iterative process is to be repeated until all the peaks are equal in magnitude within a specified accuracy but with signs alternately plus and minus.

The choice of u_j in the exchange algorithm described above has been confined to the range of u from u_o to $\pi/2$ because the locations of the peaks of the array factor always appear as pairs whose members are equally spaced about the $u = \pi/2$ axis. For any arbitrary choice of u_j, the convergence of the iterations to a unique solution has been described in [7]. If the array elements are fixed at unequal intervals, there is no simple way of ensuring the convergence of the iteration for any arbitrary set of initial spacings. If the element spacings are also to be included as a set of variables, the exchange algorithm will not be applicable. In both these cases of unequally spaced arrays, recourse can be taken to the minimization of the norm using the simplex method [8] since the simplex method does not suffer from the disadvantage of divergence.

V. NUMERICAL RESULTS AND DISCUSSIONS

A computer program has been written in Fortran IV to implement the modified Remez exchange algorithm for equally spaced arrays. The program utilized the values of u_o and N as inputs. Besides the optimum excitation coefficients, the program also computes the relevant optimum parameters like the difference slope at the boresight, element spacing, sidelobe level, and difference mode directivity.

Design curves have been generated from the results obtained for monopulse arrays with the total number of elements N, varying from 8 to 23. The values of u_o have been so chosen as to include arrays with sidelobe levels up to −40 dB below the main beam.

Fig. 3 shows that the sidelobe level decreases for increasing u_o as well as for increasing N. From (6) it is apparent that increasing u_o, to reduce the sidelobe level, decreases d_{opt}. This in effect reduces the array length and broadens the main beam since the main beam width is inversely proportional to the array length. This situation is similar to that encountered in Dolph-Chebyshev arrays.

The excitation coefficients are normalized so as to produce a unit amplitude main-lobe peak. Optimum excitation for only a few representative cases have been presented in Fig. 4. However, a detailed account of the excitation coefficients and the other important radiation characteristics can be found in [7].

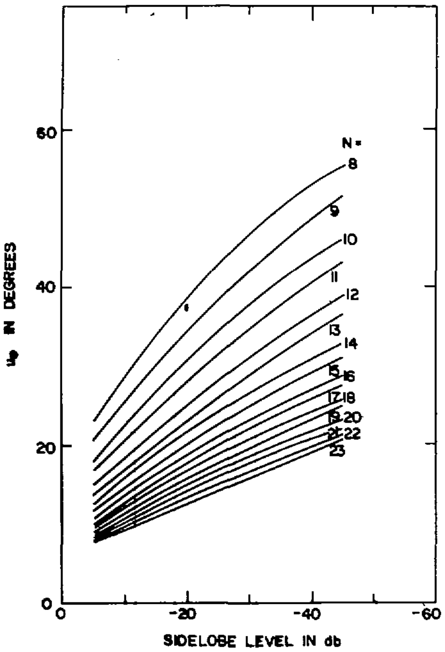

Fig. 3. Variation of sidelobe level with u_o.

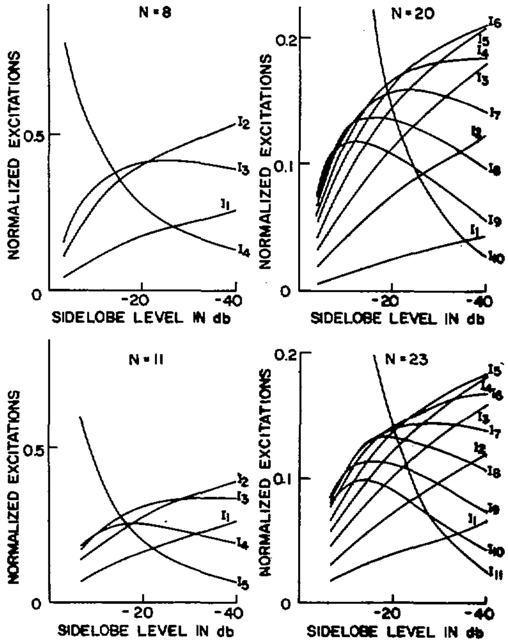

Fig. 4. Optimum difference mode excitations.

It can be seen from Fig. 4 that the current in the innermost element increases with decreasing sidelobe level while that in the outermost element decreases. As the sidelobe level is decreased, the envelope of the excitation amplitudes in the elements increases from a nearly linear odd variation to a sinusoidal type of variation. This is similar to the increasing current taper in the Dolph–Chebyshev arrays where the excitation envelope changes from a nearly uniform distribution to a cosinusoidal type of variation.

The slope of the optimum difference pattern at the boresight can be obtained from (8) as

$$\left.\frac{\delta f_d(u)}{\delta\varphi}\right|_{\substack{\varphi=\pi/2\\ u=0}} = \sum_{k=1}^{M} I_k \beta x_k$$

where

$$x_k = kd_{\text{opt}}, \qquad \text{for } N \text{ odd}$$

$$= (k-0.5)\, d_{\text{opt}}, \qquad \text{for } N \text{ even}$$

and I_k are the normalized optimum excitations.

The trade-off between the slope at the boresight and the sidelobe level is given in Fig. 5 which clearly shows that the boresight slope decreases with 1) decreasing sidelobe level and 2) decreasing N.

For a given N, the difference mode directivity increases initially and then decreases with decreasing sidelobe level and is shown in Fig. 6. The level of the sidelobe around which this change takes place increases with increasing N. This form of variation in directivity with sidelobe level and N is also encountered in the optimum Chebyshev arrays [3].

A. Design Procedure to Obtain Specific Difference Pattern Characteristics

When both the lower bound of the difference slope at the boresight and the upper bound of the sidelobe level are specified, the designer's job is one of choosing the minimum num-

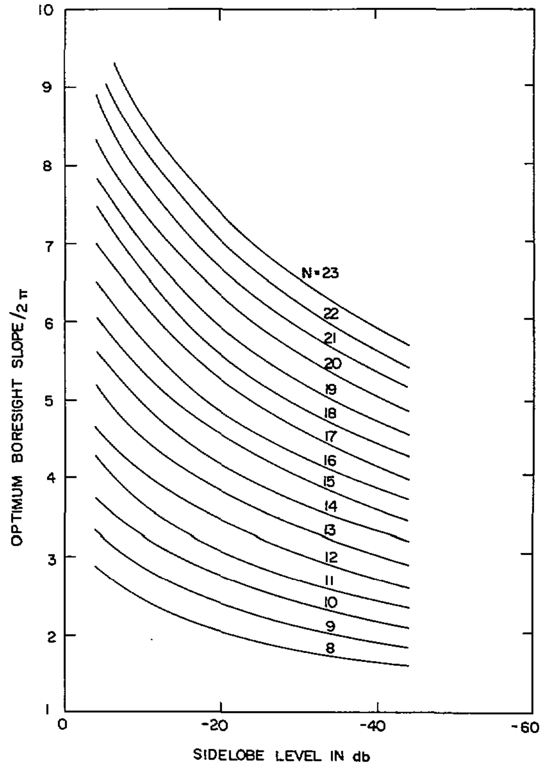

Fig. 5. Variation of the optimum boresight slope with sidelobe level and N.

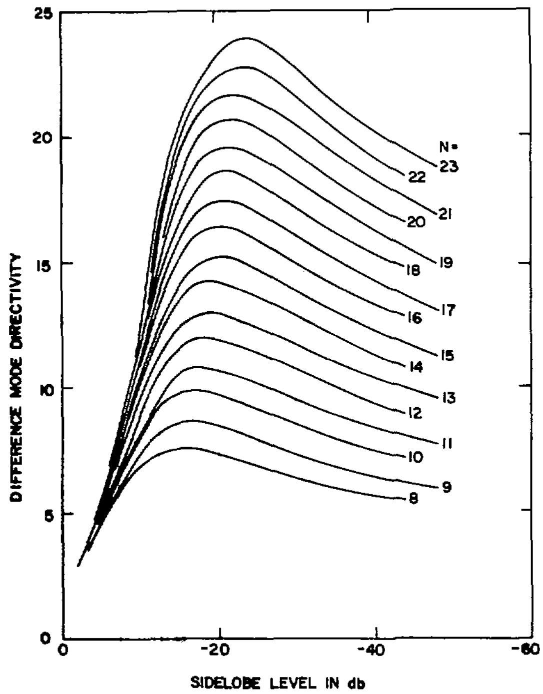

Fig. 6. Directivity variations with sidelobe level of the optimum difference mode patterns.

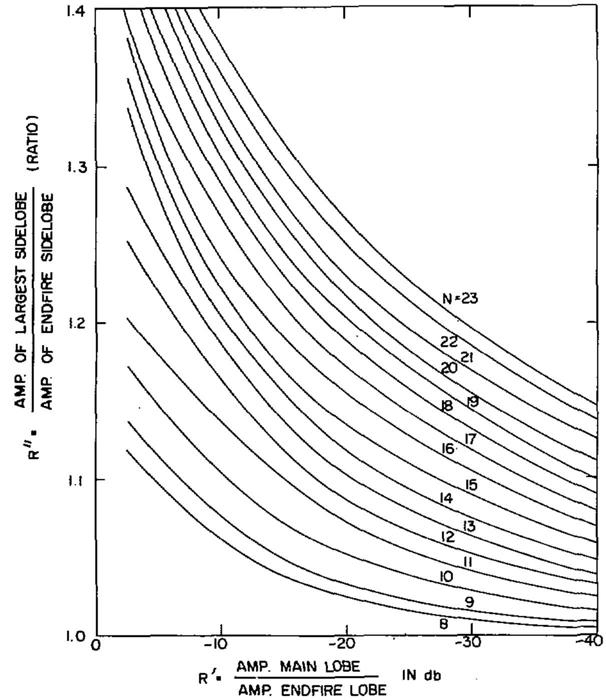

Fig. 7. Sidelobe level variation in the near-optimum design using direct interpolation.

ber of elements and the optimum excitation coefficients. The minimum number of elements can be directly obtained from Fig. 5. The optimum excitations can be read from the design curves corresponding to the value of N from Fig. 4. For values of N for which excitations are not provided, the value of u_o can be read from Fig. 3. With this value of u_o the modified exchange algorithm can be used to get the optimum excitation coefficients.

VI. NEAR-OPTIMUM DIFFERENCE MODE EXCITATIONS

In the previous sections, the problem of obtaining the optimum excitation coefficients of an N-element array has been reduced to one of finding out the best minimax approximation to the real line in the range $(u_o, \pi - u_o)$ of u. It has also been shown that the transformed polynomial $T_{N-2}\ (xx_o)$ is indeed the required approximation. However, since the difference pattern function cannot be identified directly with $T_{N-2}\ (xx_o)$, the exchange algorithm in which the pattern function is systematically interpolated or equated to $+1$ or -1 alternately at a set of points has been used. In situations where near-optimum results would suffice, it would be worthwhile to evolve techniques that will reduce the total computational effort involved. In this section, a collocation of points has been identified where the pattern function is directly equated to the transformed Chebyshev polynomial of proper order to obtain near-optimum excitations.

In fact, the near-optimum pattern of an N-element array can be viewed as some kind of an approximation to the transformed Chebyshev polynomial T_{N-2} (cos u/cos u_o) in the range $u_o \leqslant u \leqslant (\pi - u_o)$. The near-optimum excitations can now be obtained by equating $f_d(u)$ and T_{N-2} (cos u/cos u_o) at a set of $(N-1)$ points. Since T_{N-2} (cos u/cos u_o) is the best minimax approximation required, the closeness of $f_d(u)$ to the optimum will depend on the choice of the set of points where they are equated or interpolated.

In the classical method [9], one constructs $f_d(u)$ by collocation with T_{N-2} (cos u/cos u_o) at the $(N-1)$ points which are the zeros of the polynomial T_{N-1} (cos u/cos u_o). However with this collocation the condition imposed by (11) will not be satisfied and hence there will be a difference in the value of the u_o specified and the one obtained. Since the u_o obtained is approximate, d_{opt} will also be approximate and this often leads to a very large endfire sidelobe.

In order to ensure that (11) is always satisfied, collocation at

$$u_i = \cos^{-1}\left[\cos u_o \cos \frac{i\pi}{N-2}\right], \quad i = 0, 1, \cdots, N-2$$

has been employed and has been found to be successful. The points u_i are the peaks of T_{N-2} (cos u/cos u_o).

It should be pointed out here that when the optimum spacings are maintained, the endfire lobe, in view of (11), is always unity.

Since most of the practical arrays are designed with sidelobe levels of -20 dB or less, it can be seen from Fig. 7, that the largest sidelobe level obtained is always less than 1.26 times the level of the endfire lobe. Secondly, this level decreases as R' (the main lobe to endfire lobe level) is decreased and the near optimum results become closer to the optimum results.

In Fig. 7 it is clear that a decrease in R' decreases the level of the largest sidelobe. This is in contrast to that obtained by employing the technique of transmutation in [1], where decreasing R' increases the level of the largest sidelobe. However, the level of the largest sidelobe, obtained using the techniques of transmutation and the direct interpolation presented above, increases with increasing N. Though the upper bound of R'' is nearly the same (around 1.5), in both techniques, the value of R'' in the useful range of R' is considerably smaller in the direct interpolation technique presented here. Besides, the transmutation technique in [1] is not applicable to arrays with an odd number of elements, whereas the method given in this paper is applicable to both odd and even element arrays. Hence, it is clear that the method described above is of more practical significance than that described in [1].

It may be mentioned here that the collocation of points used here can also be used as a starting point in the Remez exchange algorithm and that this will effect a faster convergence (in two or three interations).

VII. CONCLUSION

Two methods have been described in this paper to obtain excitation coefficients for monopulse arrays. The first method aims at obtaining an exact equiripple sidelobe structure. The characterization of the optimum difference mode patterns, as made out herein, has reduced the problem of obtaining optimum excitation coefficients to one of finding out the best minimax approximation to the real line in a particular zone. A numerical algorithm has been used to generate an extensive set of design data. A scrutiny of these data reveals that the optimum parameters of the difference pattern behave in the same qualitative manner as those of the optimum Dolph–Chebyshev patterns.

The second method generates a set of excitations which provide a near-optimum sidelobe structure. This method has also been shown to be of more practical significance than those described in [1].

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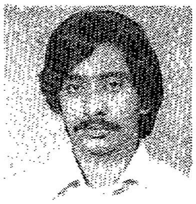

N. Balakrishnan received the B.E. degree in electronics and communication from the University of Madras, India in 1972 and the Ph.D. degree in antennas from the Indian Institute of Science, Bangalore, India, in 1979.

In 1973 he joined the Department of Aeronautical Engineering, Indian Institute of Science, where he is currently employed as an Assistant Professor. His fields of interest are digital electronics, solid-state microwaves, computers, and antenna array theory.

S. Ramakrishna was born in Visakhapatnam, Andhra Pradesh, India, on September 17, 1938. He received the B.Sc. degree from Osmania University in 1957, the M.S. degree in nuclear physics, and the Ph.D. degree in radio astronomy from the Indian Institute of Science, Bangalore, India.

From 1962 to 1968 he was a Senior Scientific Officer with the Defense Electronics Research Laboratory, Hyderabad, India, where he was Head of the Space Electronic Division. In 1969 he worked for L.R.D.E. Bangalore in the Radar Division. He joined the Department of Aeronautical Engineering at the Indian Institute of Science in 1969 where he is a Professor. He was also the Chairman of the Central Instruments and Services Laboratory of the Indian Institute of Science. His research interest are missile guidance and instrumentation, radar systems, avionics, atmospheric physics, and antenna theory.