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Dependence of Reflectivity Factor–Rainfall Rate Relationship on Polarization

D. S. ZRNIĆ

NOAA/Environmental Research Laboratories, National Severe Storms Laboratory, Norman, Oklahoma

N. BALAKRISHNAN

Indian Institute of Science, Bangalore, India

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ABSTRACT

The reflectivity factor (Z), rainfall rate (R) relationship for weather radars that probe precipitation at low elevation angles is sensitive to polarization. It is shown how to transform a relation that is valid with one polarization (vertical, horizontal or circular) to relations that are applicable to the other two polarizations. We present errors that occur if the transformations are not applied, and an example from literature in which two seemingly different Z, R relations are equivalent, tied by the polarization transformation.

1. Introduction

Although it was early recognized that backscattering cross sections of oblate spheroids depend on the polarization of the incident electric field (Atlas and Wexler 1963), quantitative use of this dependence began only after the pioneering work of Seliga and Bringi (1976). It is therefore not surprising that proposed reflectivity factor (Z), rain rate (R), relationships, until very recently did not contain provisions for adjustments to accommodate changes in polarization. In heavy rain the difference between reflectivity factors at horizontal and vertical polarizations can be over 3 dB. This may affect the empirical Z, R relationships. The purpose of this paper is to examine the effects of polarization on the Z, R relationships.

2. Relationships between reflectivity factors

Consider a rain medium in which drops can be approximated by oblate spheroids that have a minor to major axis ratio a/b and diameter D corresponding to a sphere with volume equal to that of the spheroid. We assume that the diameter is much smaller than the wavelength λ, and neglect the canting angle. Stapor and Pratt (1984) give elements of a matrix that relate the vertically and horizontally polarized electric fields to the incident field. The two elements of concern here are for horizontally (S_{hh}) or vertically (S_{vv}) polarized backscattered fields when the incident field is horizontally or vertically polarized. From these elements and an assumed (or known) drop size distribution $N(D)$ reflectivity factors for the three polarizations are as follows:

$$Z_h = \frac{4\lambda^4}{\pi^4 |K_w|^2} \int_0^{D_m} N(D) |S_{hh}(D, a/b)|^2 dD, \quad (1)$$

$$Z_v = \frac{4\lambda^4}{\pi^4 |K_w|^2} \int_0^{D_m} N(D) |S_{vv}(D, a/b)|^2 dD, \quad (2)$$

$$Z_c = \frac{\lambda^4}{\pi^4 |K_w|^2} \int_0^{D_m} N(D) | S_{hh}(D, a/b) + S_{vv}(D, a/b)|^2 dD, \quad (3)$$

where we have explicitly indicated the dependance of the S parameters on D and a/b, D_m is the equivalent volume diameter of the largest drop, $K_w = (m^2 - 1)/(m^2 + 2)$ and m is the complex refractive index of water. Implicit in our treatment is that D and a/b are uniquely related as shown by Pruppacher and Pitter (1971).

Combining (1) and (2) produces a relationship between the reflectivities at linear polarizations and circular polarization (for more details, see Jameson and Dave 1988)

$$Z_c = (Z_h + Z_v)/4 + \rho_{hv}(Z_h Z_v)^{1/2}/2, \quad (4)$$

where ρ_{hv} is the correlation coefficient, at zero time lag, between vertically and horizontally polarized echoes given by

$$\rho_{hv} = \langle | S_{hh} S_{vv} |^2 \rangle / [\langle | S_{hh} |^2 \rangle \langle | S_{vv} |^2 \rangle]^{1/2}. \quad (5)$$

In rain media the correlation coefficient is very high. Balakrishnan and Zrnić (1990) present average values > 0.98. We thus conclude that Z_c is very close to the

Corresponding author address: Dr. Dusan Zrnić, National Severe Storms Laboratory, 1313 Halley Circle, Norman, OK 73069.

average of the arithmetic and geometric means of Z_h and Z_v.

Besides the three measurable reflectivity factors (1), (2) or (3) there is a reflectivity factor defined as

$$Z = \int D^6 N(D) dD. \quad (6)$$

This reflectivity factor is valid for hypothetical rain in which all drops are spherical. In the past (6) was the accepted definition and often measured drop size distributions were used to calculate Z (for example, see Cunning 1976); it was thought that drop oblateness was secondary compared to several other causes that affect the Z, R relationship. But the measured (1), (2) or (3) and defined (6) reflectivity factors can be significantly different.

3. Dependence of *Z*, *R* relations on polarization

Suppose that a Z, R relation is of the form

$$Z = \alpha R^{\beta} \quad (7)$$

where α and β are constants for a particular rain type, and the reflectivity factor could be for either one of the polarizations (1), (2) or (3). In principle it is fairly simple, albeit laborious, to obtain the Z, R relations for the other polarizations. All that is required is the representative drop size distribution for which the Z, R relation is valid, and some calculations. The representative drop size distribution is inserted in two equations of the triplet (1), (2), (3) to obtain the desired pair of reflectivity factors; these depend on the parameters of the drop size distribution. Explicit dependency $Z = F(R)$ can be formed by calculating rain rate for the given drop size distribution and eliminating the distribution parameters from the equations for Z and R. Because this exact procedure is time consuming and because an alternative exists for which most of the work has already been done we have chosen the less laborious route that is discussed next.

Sachidananda and Zrnić (1987) have obtained, on the basis of extensive simulations and data analysis, a relationship between rain rate and Z_h, Z_{dr}. We choose to express their relationship in terms of Z_h, Z_v as

$$R = 6.84\ 10^{-3} Z_h^{-3.86} Z_v^{4.86}. \quad (8)$$

A similar relationship was proposed by Ulbrich and Atlas (1984) and a slightly modified one by Seliga et al. (1986). These authors express R in terms of Z_h ($mm^6\ m^{-3}$) and Z_{dr} (dB). Because Z_{dr} is in dB units it is not trivial to obtain power relations between R and Z_v or Z_c. The equation proposed by Ulbrich and Atlas is

$$R = 1.93\ 10^{-3} Z_h Z_{dr}^{-1.5}. \quad (9)$$

We have plotted Eqs. (8) and (9) in Fig. 1 where we have also indicated the values for the Marshall–Palmer relation

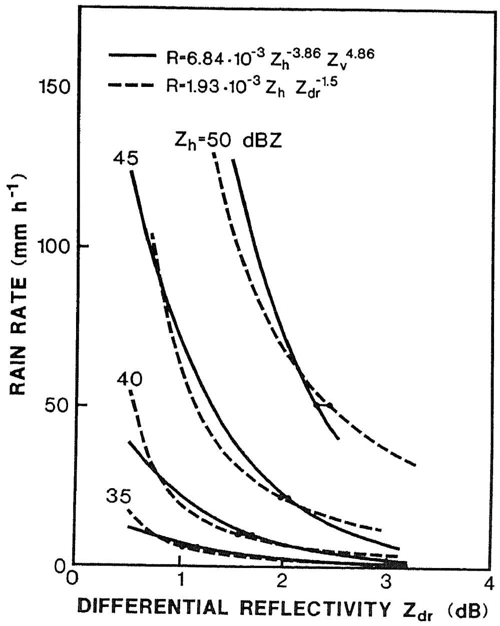

FIG. 1. Two parameter relationships between rain rate, reflectivity factors ($mm^6\ m^{-3}$) and differential reflectivity (dB). Solid curves are from Sachidananda and Zrnić (1987) and dashed from Ulbrich and Atlas (1984).

$$Z_h = 200 R^{1.6}. \quad (10)$$

It should be understood that only some combinations of polarimetric measurands are physically possible in rain and these are represented by the graphs. Because the two formulas agree in the region where measurements are expected to fall, we can use either one with confidence. We choose Eq. (8). For any Z_h, R relationship we can find a corresponding Z_v, R dependence by simply substituting Z_h in Eq. (8). For example consider the Marshall–Palmer Z_h, R relationship (10) and substitute it into (8) to yield

$$Z_v = 188 R^{1.48}. \quad (11)$$

The Z_c, R relationship is obtained by introducing Eqs. (10) and (11) in Eq. (4). We take again $\rho_{hv} = 0.98$ and upon substitution have

$$Z_c = 50 R^{1.6} + 47 R^{1.48} + 95 R^{1.54}. \quad (12)$$

This is not as handy as the forms (10) or (11) because it is a summation of terms with different exponents on R. But these exponents are very close to each other and therefore could be expanded into a Taylor series and equated to a similar expansion of Eq. (7). Matching of zero-order terms requires that

$$\alpha = 50 + 47 + 95 = 192, \quad (13)$$

and matching of first-order terms produces $\beta = 1.541$. So an excellent approximation (maximum error about 0.1 dB at rain rates less then 150 mm h^{-1}) for Eq. (12) is

$$Z_c = 192R^{1.541}. \quad (14)$$

Even better approximation is obtained if β is computed by equating Eq. (12) to Eq. (7) at a rain rate that is in the middle of the expected range. For example, if the range of rain rates is 0–150 mm h^{-1}, we substitute 75 mm h^{-1} in Eq. (12) and Eq. (7) and find $\beta = 1.5448$. We do not imply that the exponent needs to be carried with a five digit accuracy, such precision is used here to illustrate the quality of these simple fitting procedures. The difference between Eq. (12) and Eq. (7) demonstrates that for practical purposes a three digit accuracy in both the exponent and the multiplying constant are needed to produce errors lower then 0.1 dB.

If the relationship is known for vertical polarization then it should be substituted in Eq. (8) to find Z_h versus R; Z_c versus R is obtained using one of the discussed approximations. For illustration suppose that in (10) $Z = Z_v$ then this procedure produces

$$Z_c = 206R^{1.65} \quad \text{and} \quad Z_h = 217R^{1.755}. \quad (15)$$

When the known relation is Z_c, R the scheme need not be complicated; an iterative procedure that produces the desired result is as follows. Replace Z_c with Z_h (Z_v) and compute the new Z_c, R then replace this new Z_c with Z_v (Z_h) and compute again the Z_c, R relation. This last formula will differ slightly from the formula that we have started with. The difference in coefficients can be used to adjust the Z_v, R and the Z_h, R relations. As an example consider that Z_c, R is given by Eq. (10). Replace this Z_c with Z_h and follow the outlined procedure to obtain Z_c given by Eq. (14). Replace Z_c in Eq. (14) with Z_v and follow the procedure to obtain $Z_h = 206R^{1.68}$. These last two reflectivity factors are used to compute a new $Z_c = 197R^{1.61}$. Now because the difference between the coefficients of the original reflectivity factor and the computed Z_c is 3, we add this number to all the coefficients. A 0.1 adjustment in the exponent may also be applicable.

It is worthy to note that besides providing formulas for the dependence of rainrate on the polarization of the radiated field, our procedure can generate a relationship between differential reflectivity and rain rate. Division of Eq. (10) with Eq. (11) would produce such a formula.

We may also compute the error that would occur if a Z, R relationship for a wrong polarization is used to obtain rain rate. Suppose that the transmitted polarization is vertical but rain rate is computed from formula (10) that is valid for horizontal polarization. The error can be expressed as

$$\text{error} = R - (188/200)^{1/1.6}R^{1.48/1.6} \ [\text{mm h}^{-1}]. \quad (16)$$

This expression is plotted in Fig. 2 where it is apparent that errors > 20% are made if rain rates are larger than

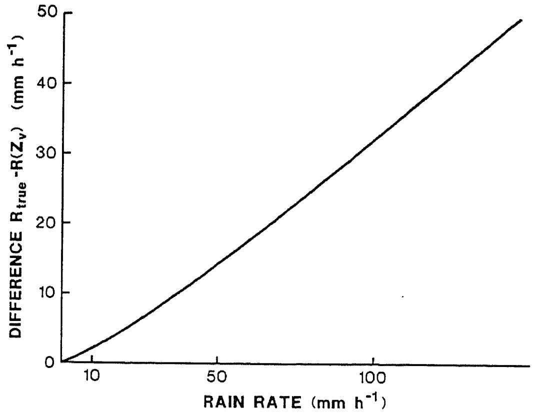

FIG. 2. Error in rain rate when the reflectivity for vertically polarized waves is used in the expression that is valid for horizontally polarized waves.

10 mm h^{-1}. In this particular example the rain rate would be underestimated because for the same rain rate, Z_v is smaller than Z_h but it is used in the equation for Z_h.

The outlined procedure can be used to generate a Z, R relation for any one of the three polarizations providing that such a relation for one polarization is known. But as we shall see in the next section, that information, until very recently, was seldom available in the literature.

4. Polarization of the electric field for some Z, R relations

We have examined all the preprints from conferences on radar meteorology and found very scant information concerning polarization of the transmitted electric field. Most authors use Eq. (10) irrespective of polarization. Barge (1970) used it with circular polarization. But the majority of papers relied on data from the WSR-57 radar, which has horizontal polarization, and therefore we may assume that Eq. (10) would be better suited for such polarization. The very few cases that specifically identify polarization state of the transmitted wave are discussed next.

Crane (1975) cites that $270R^{1.3}$ worked well; in his experiments the polarization was vertical. For same rain rate this equation produces lower Z then Eq. (10) as expected for vertical polarization. On the other hand Joss and Waldvogel (1970) claim that $300R^{1.5}$ is a relation that offers overall best performance, although for specific rain types there are better formulas. Their radar was observing at vertical incidence hence these results should be almost valid for horizontal polarization. It can be shown that this last formula equals to Eq. (10) at a rain rate of 58 mm h^{-1}, but otherwise is very close to it. This again suggests that Eq. (10) is good for horizontal polarization.

At the latest radar meteorology conference Crozier

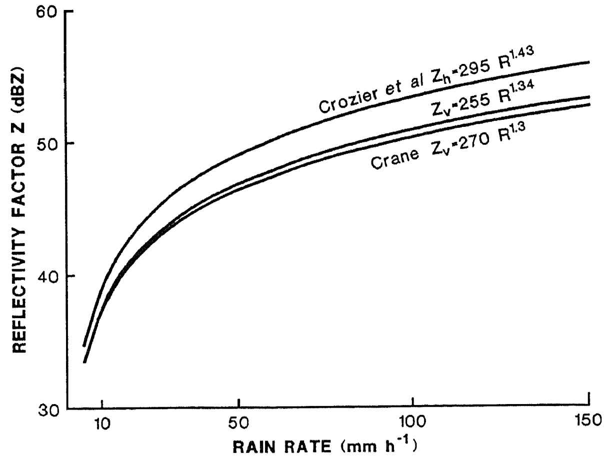

FIG. 3. Z_h, R relationship from Crozier et al. (1989), Z_v, R relationship from Crane (1975) and a Z_v, R relationship obtained from the Crozier et al. relationship using Eqn. (8).

et. al (1989) claimed that the relation $295R^{1.43}$ fits best data in the southern Ontario region. The polarization of the Canadian radar is horizontal. We do not expect large variation between rainfall types in Ontario and Virginia where Crane took his measurements. Thus we decided to calculate the Z_v, R relation from the Canadian relation and compare it to the Crane (1975) relation. Our calculation produces $Z_v = 255R^{1.34}$. This is plotted in Fig. 3 together with the other relations. Note the very good agreement between it and the Crane relation. At least in this case the conclusion is clear, the two relations (Canadian and Crane) are equivalent, tied by the polarization transformation.

5. Conclusion

Except in papers that deal directly with polarimetric measurements of rain, thus far there have been no attempts to impose polarization sensitive adjustments to the Z, R relations. We have shown how to derive a formula that relates a reflectivity factor for orthogonal linear or circular polarization with a rain rate, provided that one such relationship and the polarization are known. The derivation and evidence from literature suggest that the familiar formula $Z = 200R^{1.6}$ is correct for horizontal polarization. Our results are simple and can be obtained by substitutions requiring minimal calculations. They imply that bias errors larger than 20% occur if a wrong polarization is used in the Z, R relation. Although there are several causes that contribute to the variability in rain rates that are estimated from single parameter Z, R relations polarization should not be one of these because it is well understood and can be quantified.

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