Application of Chaos Theory to Clutter Classification

[+] P.V.Hareesh[*], N.Balakrishnan[**], K.R.Ramakrishnan[***]
[*]Human Media Group, Matsushita Electric Works Ltd,
1048, Kadoma, Osaka 571-8686, Japan
harish@ai.mew.co.jp
[**]Supercomputer Education and Research Center, Indian Institute of Science,
Bangalore 560 012, India
balki@serc.iisc.ernet.in
[***]Dept. of Electrical Engineering, Indian Institute of Science,
Bangalore 560 012, India
krr@ee.iisc.ernet.in

ABSTRACT

In this paper, the correlation dimension has been shown to be effective in distinguishing between the melting layer and the other regions in a weather phenomenon. Using the correlation integral method of Grassberger and Procaccia, data collected from an airborne radar has been analyzed. It has been shown that the correlation dimension dips to 5.5 in the melting layer where as it is around 7 in other regions. Lyapunov exponent has been found to be positive in these regions emphasizing the fact that the time series is indeed chaotic. The chaos in the time series, it is surmised may have had its origin in the fractal shape of the scatterers in the melting region.

1. INTRODUCTION

Traditionally, time series backscatter data from weather targets, commonly referred to as weather clutter, is modeled as a stochastic process. The first three moments are computed. The power scattered, mean Doppler and the Doppler spectrum width are often used for identifying the nature and dynamics of the scattering targets. Recent introduction of polarimetric capability in radars has given additional information about the shape of the targets. The polarimetric Doppler radars, also referred to as multiparameter radars, have come into use for discrimination of hydrometeor types[1]. Stochastic models are also used in characterization of land and sea surface clutter wherein the experimental time series data is fitted with well known distribution models such as Weibull, lognormal and K-distributions.

The basis for most of the clutter characterization approaches is that of minimizing the number of degrees of freedom required to describe the process. In reality, the clutter echoes have a highly irregular behavior. They provide inadequate information about the sources responsible for the generation of the process. Any clutter classification scheme to be successful must utilize to the full, the distinct nature of the sources of scattering.

Typical weather targets are rain drops, melting ice, hail, needles of stellar crystals etc. and all of them posses distinctly different orientations, fall modes and shape distributions. While the rain drops are oblate spheroid in shape and fall with their minor axis oriented vertically, melting ice, hail stones, needles and crystals tumble while they fall. Further, crystals and needles often possess shapes that exhibit self similarity. Hence, it is only natural to expect the time series data from raindrops, needles and crystals, though appear random to naked eye and to many statistical tests, to have different degrees of freedom. Similar arguments can also be forwarded for the clutter originating from sea, farm land, marshy land and mountainous terrain[2], [22].

In this work, the existence of low dimensional chaotic attractor in backscatter data from the melting layer of a weather phenomenon is established. The possible utility of correlation dimension of this chaotic attractor in distinguishing between different types of weather scatterers is discussed.

2. OUTLINE OF CHAOTIC ANALYSIS

Chaotic (Strange) attractor represents a very universal behavior of dissipative nonlinear dynamical systems. A chaotic attractor can be quantitatively characterized either by its metric properties or by dynamical invariants describing details of the temporal evolution of the considered system[3], [4]. The metric structure of the attractor can be characterized by the dimension of the attractor and the most commonly used invariant in the latter context is lyapunov exponent. The attractor of a dynamical system can easily be obtained if the coupled differential

[+] The work was done while the author was affiliated with Supercomputer Education and Research Center, Indian Institute of Science, Bangalore, India

0-7803-5731-0/99/$10.00 ©1999 IEEE

equation for the relevant variables of the system are known[5]. However, in many experimental situation, neither the relevant variables are available nor even their total number is known, rendering the attractor of the system apriori inaccessible. Fortunately, the attractor can be reconstructed in an artificial phase space, if the time series of one single variable can be measured [3]-[10]. Taken's embedding theorem [6] ensures that the attractor is reliably reconstructed in the limit of sufficiently large dimension D of the artificial phase space. Based on this theorem, different procedures are available in the literature to determine dynamical as well as static invariants of attractors from experimental time series of single variables like a) nearest-neighbor method [11], [12], b) Correlation Integral method [9], [10] and c) Singular-system method [13]. The correlation integral method has been followed in this work and is described below.

From the given one dimensional time series, a set of m dimensional vectors can be obtained, whose components are just the time delayed values of the variables. These are the reconstructed phase space vectors be represented by

$$X_i = \left(x_i, x_{i+T}, x_{i+2T}, \ldots, x_{i+(m-1)T}\right), i = 1,2,3\ldots$$

The integer m is called the embedding dimension and τ the delay. Further, if the dimension of the underlying dynamical system is d, then the minimum dimension m_o of a Euclidean space R^m in which we can find a smooth embedding of the attractor is $m_o = 2d+1$ [3]. Thus one can get the spatial information in the m dimensional set from dynamical information in the one dimensional data. A system which has a d-dimensional attractor in its phase space will have a its Taken's vectors lying in a d-dimensional subset of the embedding space R^m.

The correlation dimension of the phase space can be found by using Grassberger and Procaccia algorithm [9]. Correlation integral is given by

$$C(\varepsilon) = \frac{1}{N^2}\sum_{ij=1}^{N}\Theta\left(\varepsilon - \left\|X_i - X_j\right\|\right)$$

where Θ is the Heaviside function. The relationship between the correlation dimension d and the correlation integral is based upon the power law $C(\varepsilon) \approx \varepsilon^d$. The correlation dimension can be found by plotting $C(\varepsilon)$ versus ε on a log log graph. The region in which the power law is obeyed appears as a straight line and the slope computed is an estimate of correlation dimension. Since the dynamical evolution of the system state on a strange attractor is very sensitive to the choice of the initial conditions, the dynamical flow in phase space must possess at least one positive lyapunov exponent. Largest Lyapunov exponent can be estimated using Wolf's algorithm [14] and the Lyapunov Spectrum can be found by Eckmann and Ruelle algorithm [15].

3. RESULTS AND DISCUSSIONS

The data set chosen for analysis is collected from a Ku - band radar flown in an aircraft at 12 KM altitude on 25th May 1992. The data is collected at 512 range gates with a 30 m range resolution and at a PRF of 3.6 kHz. A total of 920 samples at each range gate have been collected for analysis [16].

Assuming the weather phenomenon belongs to the category of dissipative conservative system dynamics, its dynamics can be revealed through the strange attractor structure. Systems with dimensionality > 2 generally have more complicated attracting sets with one or more positive Lyapunov exponents and fractal dimensions.

The correlation dimension of the radar echoes at each altitude have been found using the procedure described in the previous section. The delay τ for the phase space vector has been chosen to be equal to PRT in the present work.

The correlation dimension analysis is not bereft of any problems. Many inquiries have been made in the literature on how much data is required to determine the correct dimension. The arguments of Essex[17], Ruelle[18] and Abarbanal[8] has been followed in this work. In order to establish the correct dimension, analysis of the time series data was carried out at different range gates. The time series of the radar echo at a typical gate located at 4.53 KM from the ground is shown in figure 1. Figure 2 gives the logarithmic plot of correlation integral for different embedding dimensions. It is clear from the figure that there is a region over which the plot is linear. The slopes of the curves for each embedding dimension are computed in this region and the result is plotted in figure 3, which gives an estimate of the fractal dimension at that altitude.

Table 1 Positive Lyapunov exponents estimated at typical Altitudes using 2 different methods.

Altitude(km)	Eckman & Ruelle	Wolf
4.38	1.0636, 1.0341	1.076
4.53	1.8688, 1.0169	1.928

The correlation dimension is calculated at every range gate and the vertical profile is presented in figure 4. Also depicted in figure 4 is the average echo power which when adjusted for range gives the reflectivity factor Z [16]. From the plot of P, it is easy to locate the melting region wherein the largest spread of shapes, size and species concentration are to be found. This will be just below the altitude at which the average power peaks.

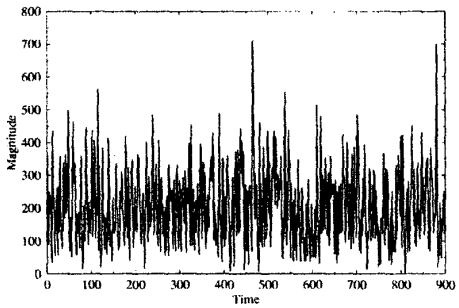

FIG.1. Time evolution of typical Radar echo at an altitude of 4.53 Km

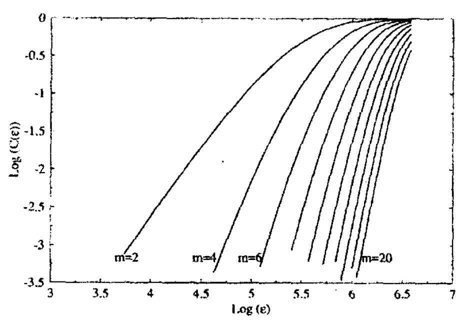

FIG.2. Logarithmic plot of Correlation Integral

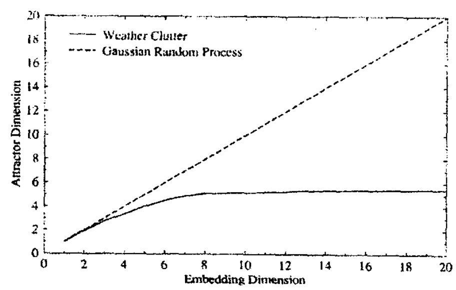

FIG.3. Dimension analysis of a typical Radar echo at an altitude of 4.53 Km

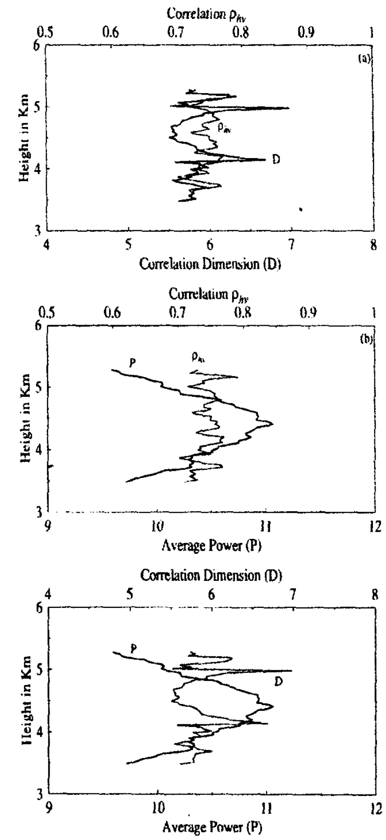

FIG.4 Vertical Profiles of Correlation Dimension, Average Power.

Lyapunov exponent is also estimated using the available methods. Table 1 shows the results for typical altitudes 4.38 km and 4.53 km. It indicates that that the phenomenon is indeed chaotic.

Above the melting layer, most of the precipitation habitates are frozen particles and do not contribute to intense scattering because of the low dielectric constant of ice. Below the melting layer, it is a pure rain medium, in this case with oriented oblate spheroidal shapes of narrow size and shape distribution. For a detailed analysis of the meteorological phenomenon readers are referenced to [19]. It is to be noted that in a Gaussian random process, the embedding dimension and the attractor dimension would be equal and in the case of low dimensional chaotic attractor, the attractor dimension would be lowered. The attractor dimension can be used thus for distinguishing between the different regions of the weather phenomenon. The correlation dimension fell from its large value of around 7 above the freezing level to 5.5 at the melting layer and then increased again to be above 7 in the rain region. This indeed act as a parameter to distinguish between oriented rain medium and a highly active melting region. Further, the evidence of chaos in the time series also establishes that the weather clutter can be viewed as a manifestation of a deterministic dynamic phenomenon involving a limited number of key variables. The attractor dimension of around 7 also indicates that no more than 7 variables are needed to describe the process completely.

The chaotic nature of the scattered echoes from the region of the melting layer may have been caused by the scatterer shapes which are often fractal themselves [19]. However, because of the low intensity of scattering above the melting layer, even if particles with fractal shapes are present above the melting layer, the effect of the shapes of the scatter is unlikely to be seen to cause chaotic behavior in the time series.

The fact that Correlation dimension can be used to identify the melting layer has applications in precise estimation of the rain rate from satellite borne sensors where it is important to know the height of the rain column. This work also supports the finding of Haykin who has used the correlation dimension and a neural network for classification of ground and sea clutter [2], [20], [21], [22].

4. ACKNOWLEDGEMENT

The authors wish to thank Dr.D.S.Zrnic and Dr.Durden for their help.

5.REFERENCES

[1] R. J. Doviak and D. S. Zrnic, *Doppler Radar and Weather Observation* : Academic press, Inc., 1992.

[2] Simon Haykin and Henry Leung, "Chaotic Modeling of radar backscatter from an ocean surface," *Proc. Oceans'91*, Hawaii, October 1991.

[3] P. Grassberger, and I. Procaccia, "Measuring the strangeness of strange attractor," *Physica 9D*, 1983, pp. 189-208.

[4] James Theiler, "Estimating fractal dimension," *J.Opt.Soc. Am*, Vol.7, No.6, June 1990.

[5] Moon. Francis C, *Chaotic Vibrations* : Wiley, New York, 1987.

[6] F. Takens, "Detecting strange attractor in turbulence," *Lect. Notes Math*, No.898, 1981, pp. 366-381.

[7] J. Theiler, "Efficient algorithm for estimating the correlation dimension from a set of discrete points," *Physical Review A*, Vol.36, No.9, Nov. 1987.

[8] H. D. I. Abarbanel, R. Brown, J. J. Sidorowich, and L.S. Tsimring, "The analysis of observed chaotic data in physical systems," *Rev. Mod. Phys.*, Vol.65, 1993, pp. 1331-1392.

[9] P. Grassberger, and I. Procaccia, *Phys. Rev. Lett.*, Vol.50, 1983, pp. 346.

[10] P. Grassberger, and I. Procaccia, *Phy. Rev. A*, Vol.28, 1983, pp. 2591.

[11] Termonia and Alexandrowitch, *Phy. Rev. Lett.* Vol. 45, 1983, pp. 1265.

[12] R. Badii, and A. Politi, *J. Stat. Phys*, Vol.40, 1985, pp. 725.

[13] D. S. Broomhead and G. P. King, *Physica 20D*, 1986, pp. 725.

[14] A. Wolf, J. B. Switt, H. L. swinney, and J. A. Vastaro, *Physica D*, Vol.16, 1985, pp. 285.

[15] J. P. Eckmann and D. Ruelle, *Rev. Mod. Phys*, Vol.57, 1985, pp. 617

[16] D. S. Zrnic, N. Balakrishnan, A. V. Ryzhkov, S. L. Durden, "Use of Copolar Correlation Coefficient at Nearly Vertical Incidence," *IEEE Trans. on Geos.. And Remote sensing*, Vol.32, No.4, July. 1994.

[17] C. Essex, and M. A. H. Nerenberg, "Comments on Deterministic Chaos: The science and the fiction by Ruelle," *Proc. R. Soc. London*, Ser. A, Vol. 435, 1991, pp. 287-292.

[18] D. Ruelle, "Deterministic Chaos: the science and the fiction," *Proc. R. Soc. London*, Ser. A, Vol. 427, 1990, pp. 241-248.

[19] N. Balakrishnan and D. S. Zrnic, "Use of Polarization to characterize precipitation and discriminate large hail," *J. Atmos. Sci*, Vol. 47, No.13, Jul. 1990, pp. 1525-1540.

[20] H. Leung and S. Haykin, "Is there a radar clutter attractor," *Appl. Phys. Lett*, Vol. 56, No.6, Feb. 1990

[21] S. Haykin and H. Leung, " Neural network modeling of radar backscatter from an ocean surface using chaos theory," *Proc. CNNNOE'91*, Washington, DC, Aug. 1991.

[22] S. Haykin and P. Sadasivan, " Chaotic dynamics of sea clutter," *chaos*, Vol. 7, No.4, Dec. 1997, pp. 777-802.