A DELAYLESS IFIR ADAPTIVE FILTER STRUCTURE WITH ADAPTED FILTERBANK

K.Rajgopal J.Dinesh Babu
Department of Electrical Engineering,
Indian Institute of Science,
Bangalore, India
kasi@ee.iisc.ernet.in

Abstract—In many applications such as acoustic echo cancellation and wideband active noise control, the Least Mean Square (LMS) based adaptive filters with hundreds of taps are used, resulting in lower convergence and high computational complexity. In recent years researchers have developed adaptive filters based on subband techiques to improve the convergence rate and reduce the computational complexity. One such approach is IFIR structure based adaptive filters also called Filter Bank Adaptive Filters(FBAF). Though the technique improves the convergence rate, it introduces a delay in the signal path. FBAFs usually use fixed paraunitary Perfect Reconstruction (PR) filterbanks for preprocessing the data. An arbitrary FIR transfer function can be modelled by a delayless IFIR structure with no conditions on the interpolating filter coefficients. The interpolators can be designed using optimization techniques with knowledge of input signal statistics to improve the convergence rate. In this paper, we propose an algorithm to adapt the interpolators themselves and the model filters simultaneously to reduce the mean square error and hence require no offline optimization procedure to design the interpolators.

1. INTRODUCTION

Adaptive filtering techniques are used in many applications [1] particularly, the adaptive FIR filters in view of their stability and unimodal performance properties. The convergence rate of the fullband Least Mean Squares (LMS) algorithm becomes poor as the number of taps increases and the input becomes correlated.

To address this problem, subband structures have been proposed for the adaptive filters [2], [3]. Convergence rate is improved because the spectral dynamic range is significantly reduced in each subband. Most of the subband adaptive structures employ subsampling and a few do not for eg: IFIR structure. A critically subsampled Subband Adaptive Digital Filter (SADF) employing a filterbank with Perfect Reconstruction (PR) property cannot model an arbitrary FIR transfer function exactly due to aliasing [4]. Solution to this problem is oversampling of the filterbanks or introduction of adaptive crossterms between the subbands which increases the computational complexity. These structures introduce another problem: a delay into the signal path by the subfilters of the filterbank. For applications like active noise control, delay seriously limits the bandwidth over which good cancellation can be achieved. Several delayless subband adaptive filter structures have been proposed [5], [6] recently.

In a system identification application shown in Fig.1, an IFIR structure as shown in Fig.2 is capable of modelling any FIR system for appropriately chosen sparse filters. Usually the interpolators are chosen to be synthesis filters of a paraunitary PRFB ($M = D$). Recently, Sridharan [7] proposed a delayless IFIR structure to model an arbitrary FIR transfer function relaxing the paraunitary PRFB constraints on the interpolating filter coefficients ($M = D + 1$). The interpolating filters were designed using optimization procedures to improve the convergence rate. The optimization procedure asssumes a priori knowledge of input statistics. Section 3 describes this model. A Normalised LMS (NLMS) type algorithm was used to adapt the sparse filters in a system identification scenario as described in Section 4A.

In this paper, we propose a delayless FBAF structure based on the model in [7] which does not require any offline optimization procedure to design the interpolators [8]. The adaptation algorithm updates the coefficients of the interpolating filters and the model filters simultaneously as described in Section 4A and 4B. In Section 5 we show the computer simulation in order to verify the effectiveness of the proposed scheme.

2. IFIR STRUCTURE IN SYSTEM IDENTIFICATION SETUP

In a system identification framework shown in Fig.1, let $S(z)$ be the FIR system transfer function to be identified. $\hat{S}(z)$ denote the IFIR model which approximates the system $S(z)$.

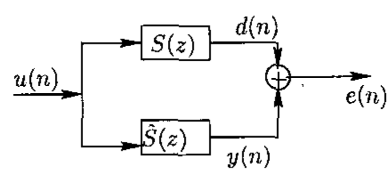

Figure 1. System Identification setup

The M channel IFIR structure consisting of the interpolator followed by the sparse filter is shown in Fig.2.

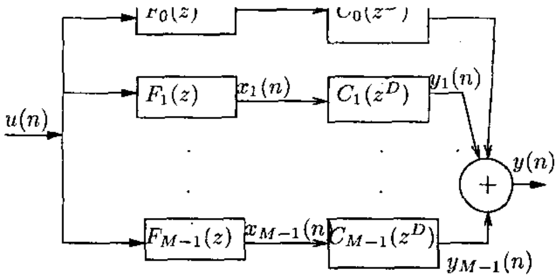

Figure 2. IFIR Structure

$\hat{S}(z)$ is given by

$$\hat{S}(z) = \Sigma_{k=0}^{M-1} F_k(z) C_k(z^D) \quad (1)$$

where D is the sparsity factor of the sparse filter.

In time domain, let $s(n)$ and $\hat{s}(n)$ denote the impulse response of the system and the IFIR model respectively. $\hat{s}(n)$ is a linear combination of a double indexed set of functions $\{\eta_{k,m}(n), 0 \leq k \leq M-1, m = 0, 1, 2, \ldots\}$ where $\eta_{k,m}(n) = f_k(n - mD)$. $f_k(n)$ is the impulse response of the k^{th} interpolator. The index m indicates the time and k corresponds to the channel index. When $\{\eta_{k,m}(n)\}$ is a linearly independent set then it is called filterbank basis functions.

Therefore $\hat{s}_k(n)$, the impulse response of the k^{th} channel, is given by

$$\hat{s}_k(n) = \Sigma_m c_k(m) f_k(n - mD) \quad (2)$$

The overall system is modelled by combining contributions from all the M channels as

$$\begin{aligned} \hat{s}(n) &= \Sigma_{k,m} c_k \eta_{k,m}(n) \\ &= \Sigma_{k=0}^{M-1} \Sigma_m c_k(m) f_k(n - mD) \\ &= \Sigma_{k=0}^{M-1} [c_k(n)] \uparrow D \odot x_k(n) \end{aligned} \quad (3)$$

The output of the system to an input $u(n)$ is given by

$$\begin{aligned} y(n) &= \Sigma_p u(p) \hat{s}(n-p) \\ &= \Sigma_p u(p) \Sigma_{k=0}^{M-1} \Sigma_m c_k(m) f_k(n - mD - p) \\ &= \Sigma_{k=0}^{M-1} \Sigma_m c_k(m) x_k(n - mD) \\ &= \Sigma_{k=0}^{M-1} [c_k(n)] \uparrow D \odot x_k(n) \end{aligned} \quad (4)$$

where $x_k(n) = \Sigma_p u(p) f_k(n-p)$

The time-domain equations of the IFIR model described above can be written in matrix form as follows. Let the length of each model filter be L_c and the length of each interpolator be L_f. Usually $L_f > D$.

the IFIR structure can model. Then $L_r = D(L_c - 1) + L_f$. $\mathbf{c_k}$ and $\hat{\mathbf{s}}_k$ represent vectors of dimension L_c and L_s respectively given by

$$\begin{aligned} \mathbf{c_k} &= [c_k(0), c_k(1), \ldots, c_k(L_c - 1)]^T \\ \hat{\mathbf{s}}_k &= [\hat{s}_k(0), \hat{s}_k(1), \ldots, \hat{s}_k(L_r - 1)]^T \end{aligned} \quad (5)$$

Equation 2 can be written in matrix notation as $\hat{\mathbf{s}}_l = \mathbf{F}_l \mathbf{c}_l$

$$\text{where } \mathbf{F}_k = \begin{bmatrix} f_k(0) & 0 & \ldots & 0 \\ f_k(1) & 0 & \ldots & 0 \\ \vdots & \vdots & & \\ f_k(D) & f_k(0) & \ldots & 0 \\ \vdots & \vdots & & \end{bmatrix}$$

The overall model $\hat{\mathbf{s}}$ is a sum of contributions from all the channels $\hat{\mathbf{s}}_k$

$$\begin{aligned} \hat{\mathbf{s}} &= \hat{\mathbf{s}}_0 + \hat{\mathbf{s}}_1 + \ldots + \hat{\mathbf{s}}_{M-1} \\ &= [\mathbf{F}_0 \mathbf{F}_1 \ldots \mathbf{F}_{M-1}][\mathbf{c}_0^T \mathbf{c}_1^T \ldots \mathbf{c}_{M-1}^T]^T \\ &= \mathbf{Fc} \end{aligned} \quad (6)$$

The dimension of vector $\mathbf{c}$ is L_t, where $L_t = ML_c$. The IFIR structure can model a system s=$[s(0), s(1), \ldots, s(L_s - 1)]^T$ of length L_s if it satisfies the following condition

Condition A: $rank(F) \geq L_s$

Condition A can be translated into the following conditions on L_r and L_t as follows.

Condition B: $L_r, L_t \geq L_s$

A special case of the IFIR structure results when the interpolators are the synthesis filters of a paraunitary PRFB. In this case M=D and $\{\eta_{k,m}(n)\}$ is an orthogonal set of basis functions. There exists a unique solution $c_{*k}(m)$ which models an unknown system exactly with a delay.

The constant $c_{*k}(m)$ is the inner product of the system impulse response and the basis functions given by

$$\begin{aligned} c_{*k}(m) &= \Sigma_p s(p) f_k(p - mM) \\ &= \Sigma_p s(p) h_k(mM - p) \end{aligned} \quad (7)$$

The synthesis filter response is the time-reversed one of analysis filter response, $h_k(n) = f_k(-n)$. The features of $s(n)$ corresponding to the time index m in the k^{th} frequency bin is captured by the basis function $\eta_{k,m}(n)$ weighted by the constant $c_{*k}(m)$. If the length of analysis and synthesis filters is L_f, then the reconstruction delay associated with the PRFB is $K = L_f - 1$ and the modelled system is delayed version of the actual system.

$$\hat{S}_*(z) = z^{-L_f+1} S(z) \quad (8)$$

Therefore this special case could be called Delaying IFIR structure.

$$\mathbf{c}_{*k} = \mathbf{H}_k \mathbf{s}$$

where $\mathbf{H}_k = \begin{bmatrix} h_k(0) & 0 & \ddots & 0 & \dots \\ h_k(M) & h_k(M-1) & \dots & h_k(0) & \dots \\ h_k(2M) & h_k(2M-1) & \dots & . & \dots \\ \vdots & \ddots & & & \\ 0 & . & \dots & . & \dots \end{bmatrix}$

$$\begin{aligned} \mathbf{c}_* &= [\mathbf{c}_{*0}^T \mathbf{c}_{*1}^T \dots \mathbf{c}_{*M-1}{}^T]^T \\ &= [\mathbf{s}^T[\mathbf{H}_0^T \mathbf{H}_1^T \dots \mathbf{H}_{M-1}^T]]^T \\ &= \mathbf{H}\mathbf{s} \end{aligned} \quad (9)$$

where $\mathbf{H} = [\mathbf{H}_0^T \mathbf{H}_1^T \dots \mathbf{H}_{M-1}^T]^T$
Using equation 6 and 9 we get

$$\hat{\mathbf{s}}_* = \mathbf{F}\mathbf{H}\mathbf{s} \quad (10)$$

In the case of Delaying IFIR structure, L_c is chosen to be $\lceil \frac{L_s+L_f-1}{M} \rceil$. The dimension of $\mathbf{F}$ and $\mathbf{H}$ is $L_r x L_t$ and $L_t x L_s$ respectively and $L_r > L_t > L_s$. $\mathbf{FH} = [\mathbf{0I0}]^\mathbf{T}$, where $\mathbf{I}$ is the identity matrix of size L_s.

Remark:
In general, the length of the interpolators can be arbitrary and large. In the delayless IFIR structure, a smaller interpolator length (order 4, 8, or 16) is sufficient and computationally advantageous.
Therefore, the following condition holds good for the delayless IFIR structure discussed in the next section.
Condition C: $L_c > L_f > D$

3. DELAYLESS IFIR STRUCTURE

In the delaying IFIR structure, $\mathbf{FH} = [\mathbf{0I0}]^\mathbf{T}$ where $\mathbf{I}$ is the identity matrix of size L_s. The matrices $\mathbf{H}$ and $\mathbf{F}$ have a special sparse structure so that they can be implemented as an analysis and synthesis filter bank. In a IFIR structure, the decomposition is not performed explicitly and the model filters derivation involves the decomposition operation. Hence the sparse structure of $\mathbf{H}$ can be sacrifised to make it delayless. In a delayless IFIR structure, matrix $\mathbf{H}$ is such that $\mathbf{FH} = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix of size L_s. This implies $L_r = L_s$, condition for delayless filter structure. The delayless condition along with conditions B, C implies that $M > D$ and $L_t \geq (L_r = L_s)$.
The computational compexity of the IFIR structure is proportional to $\frac{M}{D}$. Therefore the case $M = D + 1$ is chosen and is referred to as delayless IFIR structure. Now $\mathbf{H}$ is no longer sparse and is the right inverse of $\mathbf{F}$.

Design choice for delayless IFIR Structure

For a given system of length L_s, the sparsity factor D and interpolator length L_f can be chosen independently however the number of channels M and model filter length L_c are chosen as follows.

$L_c = \lceil \frac{L_s-L_f}{D} \rceil + 1$ and $M = D + 1$.

$L_c = \lceil \frac{L_s-L_f}{D} \rceil + 1$, the model filters can actually be chosen to have more taps to improve the steady state performance ie the modelling capability of the delayless IFIR Structure. In case we add L_e taps more we can think of modelling a system $\mathbf{s}'$ of length $L_s + L_e$ whose first L_s elements are same as that of $\mathbf{s}$ and followed by DL_e zeroes.

Delayless IFIR Structure: Choice of interpolators

Paraunitary PRFB (M=D) synthesis filters are M in number and satisfy D shift orthogonality. In the case of delayless IFIR structure $M > D$ and hence paraunitary PRFB synthesis filters cannot be used. For any arbitrary interpolators chosen, the columns of $\mathbf{F}$ need not be orthogonal nor linearly independent. The choice of the interpolators determine the modelling capability of the IFIR structure and the convergence rate of the adaptive algorithm. The flexibility in choosing the interpolators is gained at the cost of losing the structure in $\mathbf{H}$ which allows it to be implemented as a filterbank.
In [7] (Scheme A) the interpolators were obtained by an offline optimization procedure minimising a cost function based on the condition number of an autocorrelation matrix of order ML_c to improve the convergence rate. A priori knowledge of input statistics is assumed for the optimization. In this paper we propose (Scheme B) to adapt the interpolators also along with the sparse filters using the same cost function J=$E[e^2(n)]$ as there are no conditions on them. Therefore no apriori input statistics is assumed. The performance of this scheme is verified using simulations in Section 6.

4. ADAPTATION ALGORITHM FOR THE IFIR STRUCTURE

We propose an online adaptation scheme for adapting model and interpolator filters in this section. Considering the usual mean squared error cost function, J=$E[e^2(n)]$ where $e(n) = d(n) - y(n)$

(A). SPARSE FILTERS

The derivative of J with respect to $\mathbf{c}_k$ is

$$\begin{aligned} \frac{\delta J}{\delta \mathbf{c}_k} &= 2E[e(n)\frac{\delta e(n)}{\delta \mathbf{c}_k}] \\ &= 2E[e(n)\mathbf{x}_k(n)]. \end{aligned} \quad (11)$$

where $\mathbf{x}_k(n)$=$[x_k(n), x_k(n-D), \dots, x_k(n-D_c)]^T$, with $D_c = (L_c - 1)D$

Using an NLMS type algorithm for adapting $\mathbf{c}_k$

$$\mathbf{c}_k(n+1) = \mathbf{c}_k(n) + \mu e(n)\frac{\mathbf{x}_k(n)}{\alpha + \mathbf{x}_k^T(n).\mathbf{x}_k(n)} \quad (12)$$

where α is a small positive constant.

The derivative of J with respect to $\mathbf{f}_k$ is

$$\begin{aligned}
\frac{\delta J}{\delta \mathbf{f}_k} &= 2E[e(n)\frac{\delta e(n)}{\delta \mathbf{f}_k}] \\
&= -2E[e(n)\frac{\delta y(n)}{\delta \mathbf{f}_k}] \\
&= -2E[e(n)\frac{\delta y_k(n)}{\delta \mathbf{f}_k}] \\
&= -2E[e(n)\frac{\delta \mathbf{c}_k^T \mathbf{U}(n)\mathbf{f}_k}{\delta \mathbf{f}_k}] \\
&= -2E[e(n)\mathbf{U}^T(n)\mathbf{c}_k] \qquad (13)
\end{aligned}$$

where $\mathbf{U}(n)$ is defined as follows.

Let $D_f = L_f - 1$.

Then $\mathbf{U}(n)$=

$$\begin{bmatrix} u(n) & u(n-1) & \dots & u(n-D_f) \\ u(n-D) & u(n-1-D) & \dots & u(n-D_f-D) \\ \vdots & \vdots & \vdots & \vdots \\ u(n-D_c) & u(n-1-D_c) & \dots & u(n-D_f-D_c) \end{bmatrix}$$

Let $\mathbf{gr}_k(n) = \mathbf{U}^T(n)\mathbf{c}_k$

Using an NLMS type algorithm for adapting $\mathbf{f}_k$

$$\mathbf{f}_k(n+1) = \mathbf{f}_k(n) + \beta e(n)\frac{\mathbf{gr}_k(n)}{\alpha + \mathbf{gr}_k^T(n).\mathbf{gr}_k(n)} \qquad (14)$$

5. SIMULATION

The identification of an FIR system of length $L_s = 128$ is considered. The input signal is a colored noise sequence generated by passing gaussian white noise by a first-order IIR filter with a pole located at $z = 0.9$. Simulations were carried out with the fullband NLMS, Scheme A and Scheme B. The step-sizes were chosen such that the best convergence rate were obtained in each case. The following are the design choices in Scheme B: $M = 3$, $D = 2$, $L_f = 8$ and $L_c = 66$. Scheme B employs an adaptation algorithm for both the sparse filter and the interpolator. The initial coefficients for the interpolators were chosen as cosine modulated versions of a prototype filter designed using the matlab routine fir1$(L_f, \frac{1}{M})$. μ was chosen to be 0.5 and β to be 0.005. After 100 modifications of the coefficients of the sparse filter $c_k(m)$ have been carried out, the interpolators are started to adapt. The adaptive filter $c_k(m)$ continues the adaptation algorithm. Fig.3 shows the learning curves. All the curves are an ensemble average of 25 independent runs. The learning curves clearly indicate the superior performance of our proposed structure.

6. CONCLUSIONS

We have proposed an online scheme for adapting the interpolators of a delayless IFIR structure. Therefore we do not require an offline optimization procedure to adapt the interpolators. The convergence rate is better than Scheme A due to this adaptation as shown by the simulation results.

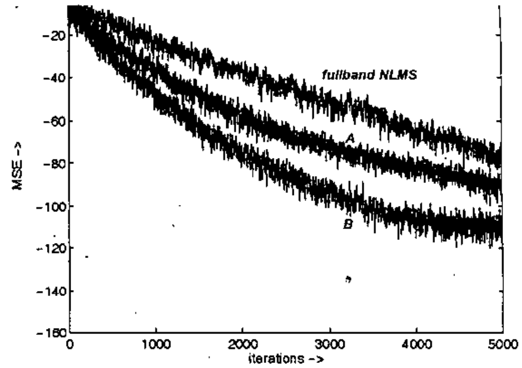

Figure 3. Learning Curve: Fullband NLMS versus the proposed delayless scheme

7. ACKNOWLEDGEMENTS

The authors thank Dr. Sridharan for useful discussions.

REFERENCES

[1] Widrow and Stearns, *Adaptive Signal Processing*, Prentice-Hall P T R, Englewood Cliffs, NJ, 1985.

[2] A.Gilloire and M.Vetterli, "Adaptive filtering in subbands," *IEEE Transactions on Signal Processing*, vol. 40, pp. 1862–1875, Aug. 1992.

[3] M.R.Petraglia, R.G.Alves and P.S.R.Diniz, "New structures for adaptive filtering in subbands with critical sampling," *IEEE Transactions on Signal Processing*, vol. 48, pp. 3316–3327, Dec. 2000.

[4] Yoon Gi Yang, Nam Ik Cho and Sang Uk Lee, "On the performance analysis and applications of the subband adaptive digital filter," *Elsevier Signal Processing*, vol. 41, pp. 295–307, Dec. 1995.

[5] D.R.Morgan and J.C.Thi, "A new delayless subband adaptive filtering architecture," *IEEE Transactions on Signal Processing*, pp. 1818–1830, Aug. 1995.

[6] M.R.Petraglia, R.G.Alves and M.N.S.Swamy, "A new open loop delayless subband adaptive filter structure," *Proc ICASSP*, vol. 2, pp. 1345–1348, 2002.

[7] M.K.Sridharan, "Subband Adaptive filtering algorithms and applications, PhD thesis, Indian Institute of Science," 2000.

[8] J.Dinesh Babu, "Delayless Subband Adaptive filtering structures and algorithms , PhD thesis, Indian Institute of Science," 2003.